Course: Teaching of Mathematics (6409)
Semester: Autumn, 2021

ASSIGNMENT No. 1

Q. 1 Highlight the significant contributions of Islamic empire in the field of mathematics. How these contributions have impact on common person life?

Mathematics reveals hidden patterns that help us understand the world around us. Now much more than arithmetic and geometry, mathematics today is a diverse discipline that deals with data, measurements, and observations from science; with inference, deduction, and proof; and with mathematical models of natural phenomena, of human behavior, and of social systems.

As a practical matter, mathematics is a science of pattern and order. Its domain is not molecules or cells, but numbers, chance, form, algorithms, and change. As a science of abstract objects, mathematics relies on logic rather than on observation as its standard of truth, yet employs observation, simulation, and even experimentation as means of discovering truth.

The special role of mathematics in education is a consequence of its universal applicability. The results of mathematics--theorems and theories--are both significant and useful; the best results are also elegant and deep. Through its theorems, mathematics offers science both a foundation of truth and a standard of certainty.

In addition to theorems and theories, mathematics offers distinctive modes of thought which are both versatile and powerful, including modeling, abstraction, optimization, logical analysis, inference from data, and use of symbols. Experience with mathematical modes of thought builds mathematical power--a capacity of mind of increasing value in this technological age that enables one to read critically, to identify fallacies, to detect bias, to assess risk, and to suggest alternatives. Mathematics empowers us to understand better the information-laden world in which we live.

During the first half of the twentieth century, mathematical growth was stimulated primarily by the power of abstraction and deduction, climaxing more than two centuries of effort to extract full benefit from the mathematical principles of physical science formulated by Isaac Newton. Now, as the century closes, the historic alliances of mathematics with science are expanding rapidly; the highly developed legacy of classical mathematical theory is being put to broad and often stunning use in a vast mathematical landscape.

Several particular events triggered periods of explosive growth. The Second World War forced development of many new and powerful methods of applied mathematics. Postwar government investment in mathematics, fueled by Sputnik, accelerated growth in both education and research. Then the development of electronic computing moved mathematics toward an algorithmic perspective even as it provided mathematicians with a powerful tool for exploring patterns and testing conjectures.

At the end of the nineteenth century, the axiomatization of mathematics on a foundation of logic and sets made possible grand theories of algebra, analysis, and topology whose synthesis dominated mathematics research and teaching for the first two thirds of the twentieth century. These traditional areas have now been supplemented by major developments in other mathematical sciences--in number theory, logic, statistics, operations research, probability, computation, geometry, and combinatorics.

In each of these subdisciplines, applications parallel theory. Even the most esoteric and abstract parts of mathematics--number theory and logic, for example--are now used routinely in applications (for example, in computer science and cryptography). Fifty years ago, the leading British mathematician G.H. Hardy could boast that number theory was the most pure and least useful part of mathematics. Today, Hardy's mathematics is studied as an essential prerequisite to many applications, including control of automated systems, data transmission from remote satellites, protection of financial records, and efficient algorithms for computation.

Islamic mathematicians gathered, organised and clarified the mathematics they inherited from ancient Egypt, Greece, India, Mesopotamia and Persia, and went on to make innovations of their own. Islamic mathematics covered algebra, geometry and arithmetic. Algebra was mainly used for recreation: it had few practical applications at that time. Geometry was studied at different levels. Some texts contain practical geometrical rules for surveying and for measuring figures. Theoretical geometry was a necessary prerequisite for understanding astronomy and optics, and it required years of concentrated work. Early in the Abbasid caliphate (founded 750), soon after the foundation of Baghdad in 762, some mathematical knowledge was assimilated by al-Mansur's group of scientists from the pre-Islamic Persian tradition in astronomy. Astronomers from India were invited to the court of the caliph in the late eighth century; they explained the rudimentary trigonometrical techniques used in Indian astronomy. Ancient Greek works such as Ptolemy's Almagest and Euclid's Elements were translated into Arabic. By the second half of the ninth century, Islamic mathematicians were already making contributions to the most sophisticated parts of Greek geometry. Islamic mathematics reached its apogee in the Eastern part of the Islamic world between the tenth and twelfth centuries. Most medieval Islamic mathematicians wrote in Arabic, others in Persian.

Al-Khwarizmi (8th–9th centuries) was instrumental in the adoption of the Hindu-Arabic numeral system and the development of algebra, introduced methods of simplifying equations, and used Euclidean geometry in his proofs.He was the first to treat algebra as an independent discipline in its own right, and presented the first systematic solution of linear and quadratic equations. Ibn Ishaq al-Kindi (801–873) worked on cryptography for the Abbasid Caliphate, and gave the first known recorded explanation of cryptanalysis and the first description of the method of frequency analysis. Avicenna (c. 980–1037) contributed to mathematical techniques such as casting out nines. Thābit ibn Qurra (835–901) calculated the solution to a chessboard problem involving an exponential series. Al-Farabi (c. 870–950) attempted to describe, geometrically, the repeating patterns popular in Islamic decorative motifs in his book Spiritual Crafts and Natural Secrets in the Details of Geometrical Figures.[36] Omar Khayyam (1048–1131), known in the West as a poet, calculated the length of the year to within 5 decimal places, and found geometric solutions to all 13 forms of cubic equations, developing some quadratic equations still in use. Jamshīd al-Kāshī (c. 1380–1429) is credited with several theorems of trigonometry, including the law of cosines, also known as Al-Kashi's Theorem. He has been credited with the invention of decimal fractions, and with a method like Horner's to calculate roots. He calculated π correctly to 17 significant figures.

Sometime around the seventh century, Islamic scholars adopted the Hindu-Arabic numeral system, describing their use in a standard type of text fī l-ḥisāb al hindī, (On the numbers of the Indians). A distinctive Western Arabic variant of the Eastern Arabic numerals began to emerge around the 10th century in the Maghreb and Al-Andalus (sometimes called ghubar numerals, though the term is not always accepted), which are the direct ancestor of the modern Arabic numerals used throughout the world.

Q. 2 Creativity is the outcome of mathematics teaching, justify this statement by providing examples.

Math is very useful in everyday life. Math can help us do many things that are important in our everyday lives. Here are some daily tasks for which math is important:

- Managing money
- Balancing the checkbook
- Shopping for the best price
- Preparing food
- Figuring out distance, time and cost for travel
- Understanding loans for cars, trucks, homes, schooling or other purposes
- Understanding sports (being a player and team statistics)
- Playing music
- Baking
- Home decorating
- Sewing
- Gardening and landscaping

Parents can help teens connect math they learn in school and their everyday lives. As a parent, you could talk to your teen about how you use math in your daily life. You could also ask family members and friends how they use math in their daily lives. Please talk to your teens about these math connections to real world. Share with your child the examples of everyday math applications, which are listed below. When your teens hear how math can be used every day, they will be more likely to view math as important and valuable. They may also become more interested in mathematics. Remember that you as a parent can greatly influence how your child thinks about mathematics.

The testimonials included on this website give brief examples of how people use math in their daily lives. Please watch these. You can share information from these videos with your teen.

Examples of Math Connections to Daily life

Managing Money

Your teen will learn skills in algebra class that will help them with money. One important skill they will learn is how to calculate interest and compound interest. Your teen can use this skill to manage their money now and when they grow up. This skill also will help them pick the best bank account. It will also help them decide which credit card is best to have. People who take out loans need to understand interest. It will also help them figure out the best ways to save and invest money.

Recreational Sports

Geometry and trigonometry can help your teens who want to improve their skill in sports. It can help them find the best way to hit a ball, make a basket or run around the track. Basic knowledge of math also helps keep track of sports scores.

Home Decorating and Remodeling

Calculating areas is an important skill. It will be useful for your teen in remodeling future homes and apartments. It will help your teen find how much paint they need to buy when repainting a room. It is also an important skill for anyone who wants to install new tiles in a bathroom or a kitchen. Knowing how to calculate perimeters can help your child when deciding how much lumber to buy for floor or ceiling trim.

Cooking

People use math knowledge when cooking. For example, it is very common to use a half or double of a recipe. In this case, people use proportions and ratios to make correct calculations for each ingredient. If a recipe calls for 2/3 of a cup of flour, the cook has to calculate how much is half or double of 2/3 of a cup. Then the cook has to represent the amount using standard measures used in baking, such as ¼ cup, 1/3 cup, ½ cup or 1 cup.

Shopping

Your teen will use math when buying different items. When buying a new computer, your child will need to figure out which store offers the best price or best financing. Math is useful in finding the best deal for food items. For example, your teen will need to decide which pack of soda to buy when given a choice of 20 oz., 2-liter, 12 pack, or 24-pack. Stores often have sales that give a percentage off an original price. It is helpful for people to know how to figure out the savings. This math skill is very useful because it helps us calculate discounts so we can buy an item for the best price offered.

Q. 3 discuss the role of misconceptions and errors by students in teaching of mathematics.

Many children find fractions particularly hard. Though frustrating, the reason for this becomes obvious once you consider just how different fractions are from the kinds of numbers children have hitherto worked with.

As "experts" who "get" fractions, we forget quite how different fractions actually are.

Most misconceptions in fractions arise from the fact that fractions are not natural numbers.

Natural numbers are the positive whole numbers that we count with, e.g. 1, 2, 3, 97, 345, 234,561 etc. These are the kinds of numbers children spend most of their time learning.

Crucially, this means that by the time we start to teach them fractions, they'll have a highly developed schema about how natural numbers work.

Then we teach them fractions, which are rational numbers (ratio-nal numbers). Fractions work differently to natural numbers, but children will try to shoe-horn fractions into their pre-existing understanding. As they assume numbers all work the same way. But this is simply not the case.

To get to the root of how to fix this, we have to understand how natural and rational numbers differ.

First of all they natural and rational numbers differ with regards to value. With natural numbers one number is always linked to one value. For example, 3 is always 3, 26 is always 26 and so on. In fractions, value is a lot more complicated.

In a fraction, value is governed by relationship between the two numbers in the fraction and the 'whole'. The value of 3 in a fraction is not simply linked to the natural number 3.

Its value not only depends upon whether it is the numerator, the denominator or the whole, but also upon what the other numbers in the fraction are. It is the relationship between these three numbers that determines the value.

To make things even more complicated, it is not always obvious what the "whole" is. When we start teaching fractions, quite often we start off with examples where the "whole" is one.

For example, dividing shapes into halves and quarters assumes that the whole is "one shape". Strictly speaking, we should say that this is half of one circle, not "half a circle" or just "half".

After all, things are always "half" of something (even 0.5 is half of one) but we often miss that bit out, then wonder why children are confused. When we shift to examples where the example is not one, if we omit to flag that up and make a big deal of it, then children will get confused.

But, alas, it is even more complicated than that. For though the 'whole' is not always one, the final answer is almost always expressed with reference to one.

A friend was doing some training about fractions and asked the teachers what 2 pizzas divided between 3 people would be.

There was a big argument. Some people said as you are dividing by 3 then everybody would get a third of the pizza whereas others said no, each person would get two thirds of the pizza.

They had to resort to cutting up paper plates before they were satisfied they understood who was right. Because, of course, both answers are correct.

Each person would get a third of the pizza (all the pizza, both of them) or, alternatively expressed, each person would get 2/3 of one pizza – which is how we usually express fractions.

In this case the "whole" is 2 as there are 2 pizzas, but we express the answer with reference to 1 pizza (each person gets 2/3 of a pizza). So you can see how a child's pre-existing schema about number value struggles when presented with something quite so different.

Sadly, that's not the end of it, at all. Natural numbers also differ from rational numbers in other ways.

Overcoming fractions misconceptions in maths: Understand number order

The second way natural and rational numbers differ is in terms of order. With natural numbers, order is simply linked to magnitude; each number in the counting sequence is one more than its predecessor. Whereas ordering fractions does not work like that at all.

For a start, bigger denominators actually mean numbers are getting smaller (schema alert!), and that's without even considering the added complications of considering the numerator as well.

Really, it's no wonder so many children struggle to understand that bigger denominators do not mean bigger values. Even when we show them practically that 1/10 of a cake is a lot smaller than 1/3. Their pre-existing schema can't cope. Plus, as soon as the cake is no longer physically visible, the old schema reasserts itself and imposes its familiar way of understanding ordering onto fractions.

Of course, that's before we consider equivalence (who said there wasn't a reason teaching Maths can be so frustrating at times!) With natural numbers, the number label indicates a fixed quantity. 2 is always 2. But with rational numbers, the same number label can represent different quantities.

For example, if I have half of £10 and you have half of £100, we do not have the same amount. Yet we still talk about a half. For some children they assume that a half is a fixed quantity, because this is what their schema tells them.

Conversely, with fractions different number labels can equal the same quantity (2/4 being the same as ½). This never happens with natural numbers where different number labels never equal the same quantity, so children struggle with it.

But there is still more! With fractions, in English at any rate, some of the number names are not obviously linked to their natural number counterparts. Half, third and a quarter do not call to mind 2, 3 and 4. Then we have number like 'fifth' which might mean the ordinal position fifth or might mean the fraction 1/5.

The best ways to avoid common fractions misconceptions

Because of all these substantial differences between natural and rational numbers, to truly overcome fractions misconceptions, teachers need to:

- Resist the temptation to rush through material and just leave it on a knowledge organiser for pupils to look over.
- Bear in mind the cognitive load inherent in what we are teaching (only try and overcome one potential misconception at a time)
- Take small steps – even when it seems easy
- Make sure all children really master these simple steps before moving on
- Use a model you can build on later

To see what this kind of teaching really looks like, watch this clip of a video I produced as part of Third Space Learning's Maths Masterclass series. In it, I break the initial teaching of fractions down into very small steps, to avoid engendering the kind of misconceptions we are all familiar with.

Fore more free CPD for teachers head over to the Third Space maths hub

The principles of how to overcome these misconceptions in fractions can then be applied to most other misconceptions in mathematics.

Common maths misconceptions in place value

Naturally, once we appreciate the important role our prior learning plays in understanding new learning, we can identify pitfalls in other areas of Maths.

Then we can take more care to directly teach children why their previous learning needs adapting in any particular case.

For instance, some children struggle with place value because it contradicts their previously built schema about how the number system works. They see each number as a number in its own right and not as a partition-able quantity, where the value of each digit is determined by its place.

So the number 10 might be seen not really as a 1 and a 0, but as a distinct sign – as if the two digits were joined up together.

Or maybe their schema is fixed on 1 meaning 1, so the idea that 1 can refer to 1 group of something doesn't really take root. They might just cope with the idea of 1 group of apples, 2 groups of apples but baulk when the groups are groups of a given number. So the concept of 1 group of 10 can't quite get traction. How can you have a group of a number? What does that even mean?

We all too readily ask children to understand the idea of groups, and particular groups of a number. If you think about it, the word order doesn't help. It would be more straightforward if we thought first about what's in our group – oh yes, tens – and then thought about how many of these groups we had.

Even when children have incorporated an understanding of countable groups into their schema, there are more icebergs ahead. Since place isn't really important when we first teach children numbers, their schemas are resistant to the idea that, suddenly, place has a crucial meaning that alters the value of previously familiar digits.

Overcoming place value misconceptions:

Understand 'lots' and 'groups'

Rushing through early place value work without really spending quality time on the vital concept of 1 lot of 10 is a false economy.

Instead, spend time getting children to bundle straws into groups of 10 and draw circles around groups of 10 objects before using more abstract apparatus such as dienes or numicon – where the individual ones comprising the tens are less obvious.

As teachers, it can be hard to remember this because lots of children grasp this easily. Meaning the important groundwork that many other children really need is not laid.

This is counterproductive and is likely to lead to misconceptions that will be resistant to correction later on. The trick is to get everybody getting it the first time!

Other common maths misconceptions in KS1 and KS2

Decimals misconceptions in KS2 Maths

Children think that 0.10 is more than 0.2 because their schema tells them that 10 is more than 2. The fact they we rarely tell them that 0.2 is also the same as 0.20 doesn't help.

In fact, our whole teaching as zero as a place holder would be better if we explained that place value columns continue into infinity so in theory, we could write 20 as 020 or 000020 or 20.0000 or even 000000020.0000000. If children were introduced to the concept of redundant zeros (alongside lots more work on place value) that would help.

Subtraction misconceptions in KS1 math's

We spend a lot of time teaching adding then after that, teach subtraction, usually spending a lot less time. We are then surprised when children revert to adding when faced with a subtraction.

Overcome this common misconception by teaching the part, part, whole model; where the underlying model of both addition and subtraction are clear.

This is much better than rushing through, doing examples with bigger and bigger numbers. As it means children really understand how subtraction and addition fit together.

It is much better if a child really understands that 5-2=3 and how this relates to 3+2=5 and 5-3=2 and 2+3=5 than if a child can do equations with numbers to 20.

Money misconceptions in KS1 Maths

We spent so much time in the early years building one to one correspondence. Then we introduce money, and this concept is comprehensively undermined. Personally I think our curriculum does this far too early and it would be better taught much later.

However, until that happens, we should really explicitly teach why the old rule doesn't apply. Getting children to see how heavy, awkward and fiddly carrying lots of 1p coins around and showing them how coins are a helpful solution to this problem will probably help. So don't start with 2p, start with 10p.

Q. 4 elaborate the advantages and limitations of problems solving method of teaching mathematics.

Higher-order thinking takes thinking to a whole new level. Students using it are understanding higher levels rather than just memorizing facts. They would have to understand the facts, infer them, and connect them to other concepts.

Here are 10 teaching strategies to enhance higher-order thinking skills in your students.

1. Help Determine What Higher-Order Thinking Is

Help students understand what [higher-order thinking] is. Explain to them what it is and why they need it. Help them understand their own strengths and challenges. You can do this by showing them how they can ask themselves good questions. That leads us to the next strategy.

2. Connect Concepts

Lead students through the process of how to connect one concept to another. By doing this you are teaching them to connect what they already know with what they are learning. This level of thinking will help students learn to make connections whenever it is possible, which will help them gain even more understanding. For example, let's say that the concept they are learning is "Chinese New Year." An even broader concept would be "Holidays."

3. Teach Students to Infer

Teach students to make **inferences** by giving them "real-world" examples. You can start by giving students a picture of a people standing in line at a soup kitchen. Ask them to look at the picture and focus on the details. Then, ask them to make inferences based on what they see in the picture. Another way to teach young students about how to infer is to teach an easy concept like weather. Ask students to put on their raincoat and boots, then ask them to infer what they think the weather looks like outside.

4. Encourage Questioning

A classroom where students feel free to ask questions without any negative reactions from their peers or their teachers is a classroom where students feel free to be creative. Encourage students to ask questions, and if for some reason you can't get to their question during class time, show them how they can answer it themselves or have them save the question until the following day.

5. Use Graphic Organizers

Graphic organizers provide students with a nice way to frame their thoughts in an organized manner. By drawing diagrams or mind maps, students are able to better connect concepts and see their relationships. This will help students develop a habit of connecting concepts.

6. Teach Problem-Solving Strategies

Teach students to use a step-by-step method for solving problems. This way of higher-order thinking will help them solve problems faster and more easily. Encourage students to use alternative methods to solve problems as well as offer them different problem-solving methods.

7. Encourage Creative Thinking

Creative thinking is when students invent, imagine, and design what they are thinking. Using creative senses helps students process and understand information better. Research shows that when students utilize **creative higher-order thinking skills**, it indeed increases their understanding. Encourage students to think "outside of the box."

8. Use Mind Movies

When concepts that are being learned are difficult, encourage students to create a movie in their mind. Teach them to close their eyes and picture it like a movie playing. This way of higher-order thinking will truly help them understand in a powerful, unique way.

9. Teach Students to Elaborate Their Answers

Higher-order thinking requires students to really understand a concept, not repeat it or memorize it. Encourage students to elaborate their answers by asking the right questions that make students explain their thoughts in more detail.

10. Teach QARs

Question-Answer-Relationships, or QARs, teach students to label the type of question that is being asked and then use that information to help them formulate an answer. Students must decipher if the answer can be found in a text or online or if they must rely on their own prior knowledge to answer it. This strategy has been found to be effective for higher-order thinking because students become more aware of the relationship between the information in a text and their prior knowledge, which helps them decipher which strategy to use when they need to seek an answer.

Q. 5 explain the role of learning cantered activities in teaching of mathematics.

The mathematics learning aids is a place where anybody can experiment and explore patterns and ideas. It is a place where one can find a collection of games, puzzles, and other teaching and learning material. The materials are meant to be used both by the students on their own and with their teacher to explore the world of mathematics, to discover, to learn and to develop an interest in mathematics. The activities create interest among students or in anybody who wants to explore, and test some of their ideas, beliefs about mathematics.

The activities in the math aids should be appealing to a wide range of people, of different ages and varying mathematical proficiency. While the initial appeal is broad-based, the level of engagement of different individuals may vary. The math aids activities listed here have been done with students and teachers of different grade levels. The activities are intended to give children an experience of doing mathematics and not merely for the purpose of demonstration. The math aids provides an opportunity for the students to discover mathematics through doing. Many of the activities present a problem or a challenge, with the possibility of generating further challenges and problems. The activities help students to visualize, manipulate and reason. They provide opportunity to make conjectures and test them, and to generalize observed patterns. They create a context for students to attempt to prove their conjectures. It is important to note that while in science experiments provide evidence for hypotheses or theories, this is not so in mathematics. Observed patterns can only suggest mathematical hypotheses and conjectures, not provide evidence to support them. (Sometimes, they may help to disprove a conjecture through a counter-example.) Mathematical truths are accepted only on the basis of proofs, and not through experiment. Mathematics learning aids is a place to enjoy mathematics through informal exploration. It is a place where anyone can generate problems and struggle to get a answer. It is a space to explore and design new mathematical activities. So, the math aids should not be used to assess students' knowledge of mathematics. Often mathematics lab takes students knowledge beyond the curriculum. Mathematics learning aids is a self-explanatory lab with activities, in which students could come anytime (free to them) and engage in the work, continue working on the problems/tasks, and use teachers as and when they are stuck. In this way, the role of the teacher is not to teach how to progress in the activity but to facilitate inquiry with the mathematics in it. The facilitation could be done either by probing questions, giving an extra resource or asking to follow or discuss with peers. Mathematics Learning aids Manual In this report we have described some activities which could typically be included in a school mathematics learning aids. The activities are suitable for students of class 6 to class 10. We have also included a couple of activities suitable for a lower level — the place-value snake and the fraction chart. The items have been grouped under two broad headings: (i) activities and (ii) games and puzzles. The activities could be done individually by students, with guidance from a teacher, or could be used for demonstration with a small group of students. Some of the activities could also be used as teaching aids in a classroom. The games and puzzles are fun to do individually and all of them contain some element of mathematics which can be explored while doing them or as a sequel. Some of these items have been developed by the authors at HBCSE. Others have been taken from articles and books and have been modified or developed further.