Course: General Mathematics and Statistics (6401)

Semester: Autumn, 2021

ASSIGNMENT No. 1

Q.1 Discuss different type of banking systems used in Pakistan. Also, what are the types of cards used by banks? What is finance?

There are five common types of bank accounts offered by banks in Pakistan.

- Basic Banking Account
- Current Account
- Savings Account
- Foreign Currency Account
- Fixed Deposit Account

BASIC BANKING ACCOUNT

Almost all major commercial banks in Pakistan offer the Basic Banking Account (BA) to their clients. As the name suggests, it is a simple bank account that provides basic banking facilities.

BA is ideal for account holders who don't need to make frequent transactions.

Salient Features of Basic Banking Account

These are some of the main features of a Basic Banking Account:

- No account maintenance fee
- No interest rate, which means it's a non-profit bearing account
- Exempt from Zakat deduction
- The account may be closed if the balance remains 'nil' for a consecutive period of six months
- Maximum four transactions, i.e. two deposits and two withdrawals via check, are allowed free of charge on a monthly basis. Account-holders will have to pay a small fee for additional transactions
- Unlimited ATM service. While withdrawals from the bank's ATM are free of charge, ATMs of other banks may charge a transaction fee
- Customers can switch to BA from any other type of account

Minimum Account Opening Amount: PKR 1000 (may vary with each bank)

Minimum Balance Amount: None

CURRENT ACCOUNT

The current account is easily one of the most popular types of bank accounts in Pakistan, widely used by working individuals, businessmen and commercial entities. This account is ideal for making business transactions on a day-to-day basis, which means you can deposit and withdraw your money at any time.

You can open a current account in any private or public sector bank, most of which offer a number of different current accounts to their customers.

Salient Features of a Current Account

These are some of the key features of current account:

- No-interest bearing account
- Exempt from annual Zakat deductions
- Debit card service with unlimited transactions. However, the amount you can withdraw via ATM per day may vary with each bank.
- Free of charge phone banking service
- No restriction on the number of deposits and withdrawals via checks
- Free online and internet banking
- SMS and email alerts for every transaction

Minimum Account Opening Amount: PKR 1000 (may vary with each bank)

Minimum Balance Amount: None (may vary with each bank)

SAVINGS ACCOUNT

As the name suggests, savings accounts are meant for securing your savings. In addition to that, unlike the current account, they also allow you to earn a certain percentage of interest over time. It means the amount you deposit in your savings account will accumulate a modest profit.

Moreover, banks in Pakistan offer a number of different savings accounts for individuals who want to earn income through interest.

Salient Features of a Savings Account

Here are some of the main features of saving accounts:

- Profit-bearing account
- Most banks calculate profit on a monthly average basis.
- Zakat and withholding tax are deducted
- Funds can be accessed at any given time
- Banks periodically credit the profit to the client's account
- Debit cards can be used to make unlimited transactions
- Free online banking services
- Free of charge phone banking services

Minimum Account Opening Amount: PKR 100 (varies with banks)

Minimum Balance Amount: None (varies with banks)

FOREIGN CURRENCY ACCOUNT

This type of bank account is usually maintained by overseas Pakistanis, dual national citizens, charitable institutions and commercial entities. It only allows customers to deposit money in foreign currency. Most banks in Pakistan offer both savings and current foreign currency accounts where customers can deposit money in US Dollar, Euro, Japanese Yen and British Pound.

Salient Features of a Foreign Currency Account

Here are some of the most important features of a foreign currency account.

- Only deposits amount in a foreign currency
- Can transfer funds abroad
- Depositors can earn interest if they choose a savings account
- Zakat and other taxes to be deducted from foreign currency savings accounts.
- Availability of Traveler's checks and other remittance services
- Its credit card can be utilized in and outside Pakistan.
- Transfer amount from one account to another
- Non-residents don't have to pay withholding tax and Zakat, depending on the bank.

Minimum Account Opening Amount: USD 250 or equivalent (may vary with banks)

Minimum Balance Amount: USD 1000 or equivalent (may vary with banks)

Q.2 Write down different types of sets each with example. Also discuss operation on sets.

A set is well defined as the collection of data that does not carry from person to person.

1. Empty Sets -

The set, which has no elements, is also called a Null set or Void set. It is denoted by {}.

Below are the two example of empty set.

Example of empty set: Let, Set A = {a : a is the number of students studying in Class 6th and Class 7th}. Since we all know, a student cannot learn in two classes, therefore set A is an empty set.

Another example of empty set is, set B = {a: 1 < a < 2, a is a natural number}, we know natural cannot be a decimal, therefore set B is a null set or empty set.

2. Singleton Sets-

The set which has just one element is named a singleton set.

For Example: Set A = { 8 } is a singleton set.

3. Finite and Infinite Sets-

A set which has a finite number of elements is known as finite sets, whereas the set whose elements can't be estimated, but it has some figure or number, which is large to precise in a set which is known as Infinite Set.

For Example: Set A = {3,4,5,6,7} is a finite set, as it has a finite number of elements.

Set C = {Number of Cows in India} is an infinite set, there is an approximate number of Cows in India, but the actual number of cows cannot be expressed, as the numbers could be very large and counting all cows is not possible.

4. Equal Sets-

If every element of set A is also the elements of set B and if every element of set A is also the elements of set A are called equal sets. It means set A and set B have equivalent elements and that we can denote it as:

A = B

For Example: Let A = {3,4,5,6} and B = {6,5,4,3}, then A = B

And if A = {set of even numbers} and B = { set of natural numbers} the A ≠ B, because natural numbers consist of all the positive integers starting from 1,2,3,4,5 to infinity, but even numbers start with 2,4,6,8, and so on.

5. Subsets-

A set S is said to be a subset of set T if the elements of set S belong to set T, or you can say each element of set S is present in set T. Subset of a set is denoted by the symbol (⊂) and written as S ⊂ T.

We can also write the subset notation as;

$S \subset T$ if $p \in S \Rightarrow p \in T$

According to the equation given above, "S is a subset of T only if "p" is an element of S as well as an element of T."Each set is a subset of its own set, and a void set or empty set is a subset of all sets.

6. Power Sets-

The set of all subsets is known as power sets.We know the empty set is a subset of all sets, and each set is a subset of itself. Taking an example of set X = {2,3}. From the above-given statements, we can write,

{} is a subset of {2,3}

{2} is a subset of {2,3}

{3} is a subset of {2,3}

{2,3} is also a subset of {2,3}

Therefore, power set of X = {2,3},

P(X) = {{},{2},{3},{2,3}}

7. Universal Sets-

A set that contains all the elements of other sets is called universal sets. Generally, it is represented as 'U.'

For Example: set A = {1,2,3}, set B = {3,4,5,6} and C = {5,6,7,8,9}

Then, we will write universal set as, U = {1,2,3,4,5,6,7,8,9,}

Note: According to the definition of the universal set, we can say that all the sets are subsets of the universal set.

Therefore,

$A \subset U$

$B \subset U$

And $C \subset U$

8. Disjoint Sets

If two sets X and Y do not have any common elements, and their intersection results in zero(0), then set X and Y are called disjoint sets.It can be represented as; $X \cap Y = 0$.

All of the operations on sets are ways for us to take two or more sets and make a new set using the elements of the ones we have.

The most basic of these operations are union, intersection, absolute complement, and relative complement (also called set difference). Then there are a little more complicated operations like the symmetric difference and Cartesian product.

To start, suppose A, B, and CC are sets.

Union:

Technically we need the axiom of union and axiom of pairing to define this. What I have below is the general idea.

The union of A and B is the set

A∪B={x:x∈A∪B={x:x∈A or x∈Bx∈B}}.

What this is saying is that we take all of the elements that are in A and all of the elements that are in B, then we put them into a new set that we denote A∪B. And if there's any repeated elements that happen to be in both A and B we just have one of them in their union, because we view something like {1,2,2} as the same as {1,2}.

The union operation is both associative and commutative.

Associative means, for any three or more sets that you union, you can group them however you want and get the same thing. In symbols that is:

$(A\cup B)\cup C=A\cup(B\cup C)$

Commutative means, it doesn't matter which set comes first, because it comes out to the same thing in any order you want to do it. That looks like this:

A∪B=B∪A∪B=B∪A

Intersection:

The intersection of A and B is the set

$A\cap B=\{x\in A\cup B:x\in A \text{ and } x\in B\}$

This means the new set we make has only the elements that A and B have in common. Like the union, this is also both associative and commutative. Those words have the same meaning as before. Here is what they look like in practice:

$(A\cap B)\cap C=A\cap(B\cap C)$

$A\cap B=B\cap A$

Absolute complement:

This can only make sense if all of the sets you are working with are known to be a subset of an ambient set. Suppose our sets A,B, and C are all subsets of a different set E. Then we denote each of their complements as Ac, Bc, and Cc. That is defined to be every element of E that is not in the subset and it looks like this:

$Ac=\{x\in E:x\notin A\}$

This is different from the relative complement, because it is assumed that EE contains all of the elements we're working with. One can't stress this point enough, since there is no universal set that contains every element and every set, this notation only makes sense if we state first that the sets we're working with are all subsets of a common ambient set. If this is not the case, then the operation to use is called the relative complement or sometimes set difference.

Relative complement:

The relative complement is defined to be all of the elements of the first set that are not elements of the second and it looks like this:

$A\setminus B=\{x\in A:x\notin B\}$

The absolute and relative complements are the same if B is a subset of A, but that doesn't have to be the case here.

This operation is neither associative nor commutative. It is pretty common for people to refer to it as set difference.

Symmetric difference:

There's two ways that you can get the symmetric difference. One way is to take the union of the two sets and then do the set difference with their intersection. That looks like this:

$A\oplus B=(A\cup B)\setminus(A\cap B)$

The other way, which gives you the same result, is to union the relative complement when A is the first set with the relative complement when B is the first set.

$A\oplus B=(A\setminus B)\cup(B\setminus A)$

This operation is both associative and commutative.

$(A\oplus B)\oplus C=A\oplus(B\oplus C)$

$A\oplus B=B\oplus A$

Cartesian product:

For this operation, it's easiest to just talk about the case with two sets. The Cartesian product between two sets is the set of all ordered pairs where the first element of the pair is from the first set and the second element is from the second set.

$A\times B=\{(a,b):a\in A\ b\in B\}$

This operation is not commutative because $(a,b)\neq(b,a)$ if $a\neq b$

Q.3 Use Cremer's rule to solve the following system of equation. Give the reason where solution is not possible;

$$2x = 13 - 5y$$
$$2y = 17 - 5x$$

The equations can be expressed as
$2x + 5y - 13 = 0$

$5x + 2y - 17 = 0$

Use Cramer's Rule to find the values of x, y, z.

$$\frac{x}{D_x} = \frac{-y}{D_y} = \frac{1}{D}$$

$$D_x = \begin{vmatrix} 5 & -13 \\ 2 & -17 \end{vmatrix}$$

$= 5 \times (-17) - (-13) \times 2$

$= -85 + 26$

$= -59$

$$D_y = \begin{vmatrix} 2 & -13 \\ 5 & -17 \end{vmatrix}$$

$= 2 \times (-17) - (-13) \times 5$

$= -34 + 65$

$= 31$

$$D = \begin{vmatrix} 2 & 5 \\ 5 & 2 \end{vmatrix}$$

$= 2 \times 2 - 5 \times 5$

$= 4 - 25$

$= -21$

$$\frac{x}{D_x} = \frac{-y}{D_y} = \frac{1}{D}$$

$$\therefore \frac{x}{-59} = \frac{-y}{31} = \frac{1}{-21}$$

$$\therefore \frac{x}{-59} = \frac{1}{-21}, \frac{-y}{31} = \frac{1}{-21}$$

$$\therefore x = \frac{-59}{-21}, y = \frac{-31}{-21}$$

$$\therefore x = \frac{59}{21}, y = \frac{31}{21}$$

Q.4 (a) Simplify $(a^2 + 3a^2b + 3ab^2 + b^3) \div (a - b)$.

$$\frac{3a^2b + a^2 + 3ab^2 + b^3}{a - b} = 3ab + a + 6b^2 + b + \frac{7b^3 + b^2}{a - b}$$

Answer: $\dfrac{3a^2b + a^2 + 3ab^2 + b^3}{a - b} = 3ab + a + 6b^2 + b + \dfrac{7b^3 + b^2}{a - b}$

(b) The product of two polynomials is $9x^4 - 4x^2 + 15x + 10$. If one polynomial is $3x + 2$, then find the other polynomial.

$$
\begin{array}{r|llllll}
 & 3x^3 & -2x^2 & +5 & & \\
\hline
3x+2 & 9x^4 & +0x^3 & -4x^2 & +15x & +10 \\
- & 9x^4 & +6x^3 & & & \\
\hline
 & & -6x^3 & -4x^2 & +15x & +10 \\
- & & -6x^3 & -4x^2 & & \\
\hline
 & & & 0 & 15x & +10 \\
- & & & 15x^2 & +10x & \\
\hline
 & & & -15x^2 & +5x & +10
\end{array}
$$

Hints

$\frac{9x^4}{3x} = 3x^3$

$3x^3(3x+2) = 9x^4 + 6x^3$

$\frac{-6x^3}{3x} = -2x^2$

$-2x^2(3x+2) = -6x^3 - 4x^2$

$\frac{0}{3x} = 5$

$5(3x+2) = 15x^2 + 10x$

Therefore, $\frac{9x^4 - 4x^2 + 15x + 10}{3x+2} = 3x^3 - 2x^2 + 5 + \frac{0}{3x+2} = 3x^3 - 2x^2 + 5$

Answer: $\frac{9x^4 - 4x^2 + 15x + 10}{3x+2} = 3x^3 - 2x^2 + 5 + \frac{0}{3x+2} = 3x^3 - 2x^2 + 5$

Other Polynomial is =

$3x^3 - 2x^2 + 5$

Q.5 State the proof of Quadratic formula i.e,

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$ax^2 + bx + c = 0$

Divide both sides of the equation by a so that you can complete the square.

$$x^2 + \frac{b}{a}x + \frac{c}{a} = \frac{0}{a}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Subtract c/a from both sides

$$x^2 + \frac{b}{a}x + \frac{c}{a} - \frac{c}{a} = 0 - \frac{c}{a}$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Complete the square: The coefficient of the second term is b/a. Divide this coefficient by 2 and square the result to get $(b/2a)^2$ Add $(b/2a)^2$ to both sides:

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

Since the left side of the equation right above is a perfect square, you can factor the left side by using the coefficient of the first term (x) and the base of the last term(b/2a) Add these two and raise everything to the second.

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

Get the same denominator on the right side:

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Now, take the square root of each side:

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

Simplify the left side:

$$x + \frac{b}{2a} = +- \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = +- \frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

Rewrite the right side:

$$x + \frac{b}{2a} = +- \frac{\sqrt{b^2 - 4ac}}{2a}$$

Subtract b/2a from both sides:

$$x + \frac{b}{2a} - \frac{b}{2a} = -\frac{b}{2a} +- \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} +- \frac{\sqrt{b^2 - 4ac}}{2a}$$

Adding the numerator and keeping the same denominator, we get the quadratic formula:

$$x = \frac{-b +- \sqrt{b^2 - 4ac}}{2a}$$