Preface

The Government of Punjab has a strong desire to improve the quality of teaching and learning in the classroom. Various initiatives have been undertaken for provision of quality education to students in the Province. Provision of quality education at secondary level is an important step towards building an education system meant to contribute meaningfully towards development of our society. To achieve the desired goal, activity oriented training for secondary school teachers based on modern teaching methodologies has been considered imperative and crucial.

Directorate of Staff Development (DSD) has been training in-service and pre-service public school teachers and developing educational material since its inception. Considering the quality work produced over the years, the task of development of the Teachers' Guides for secondary school teachers in the subjects of English, Physics, Chemistry, Biology and Mathematics was assigned to the Directorate of Staff Development by the Provincial Government.

DSD worked in collaboration with over three hundred professionals i.e. Teachers, Book Writers and Teacher Trainers from both public and private educational institutions in the subject of English, Physics, Chemistry, Biology and Mathematics who worked in groups to develop these comprehensive Teachers' Guides. These Teachers' Guides with textbooks are aimed to achieve Students' Learning Outcomes (SLOs) through the teaching materials and methodologies which suit varying teaching and learning contexts of Punjab. These Teachers' Guides will help secondary school teachers to deliver and further plan their content lessons, seek basic information on given concepts and topics, and assess students' understanding of the taught concepts.

The DSD team acknowledges the cooperation extended by various public & private, national and international organizations in the preparation of Teachers' Guides. DSD is especially grateful to German International Cooperation Agency (GIZ) for extending its full cooperation and support in conduction of workshops, development of material, quality management, layout and designing of these Guides. DSD recognizes the contribution made by all developers and reviewers belonging to following organizations including Institute of Education and Research (IER) Punjab University, Government Science College, International School of Choueifat, Crescent Model Higher Secondary School, Punjab Textbook Board, Lahore Grammar School, Himayat-e-Islam Degree College, SAHE, PEAS, NEEC, HELP Foundation, Ali Institute of Education, Beaconhouse School System, ALBBS, The Educators, Divisional Public School, The City School, AFAQ, Portal, LACAS, Children's Library Complex (CLC) and GICW Lahore, Govt. Higher Secondary Schools and Govt. Colleges for Elementary Teachers in Punjab.

(Nadeem Irshad Kayani)
Programme Director
Directorate of Staff Development, Punjab

UNIT 1

Lesson Plan 1

TOPIC

Introduction to the Matrices

Matrices and Determinents

Grade IX

Students’ Learning Outcomes

Define

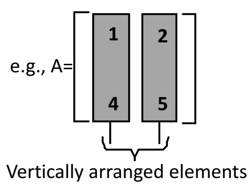
- A matrix with real entries and relate its rectangular layout (formation) with real life,
- Rows and columns of a matrix,
- The order of matrix,
- Equality of two matrices.

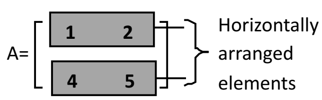
Information for Teachers

- A matrix is an ordered set of numbers listed in rectangular form.
- A general matrix can be written in a bracketed rectangular array of mxn elements, arranged in m rows and n columns as shown in fig.1

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ . & . & . & & . \\ . & . & . & & . \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$ Fig 1

- Usually capital letters such as A,B,C,X,Y, etc. are used to represent the matrices and small letters such as a,b,c,l,m,n, a_{12}, a_{13},........., to indicate the entries or elements of the matrices.
- Each matrix consists of horizontally and vertically arranged elements. Horizontally arranged elements are said to form **rows** whereas vertically arranged elements are said to form **column.**





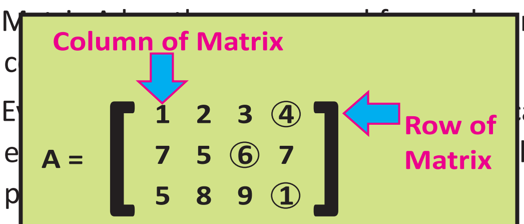
- **Order of a matrix:**

  If a matrix A has m number of rows and n number of columns, then the order of the matrix A is m x n. We read m x n as m cross n or m by n. It may be noted that m x n is not a product of m and n.

- **Equal matrices:**

  Two matrices A and B are said to be equal if and only if they are of the same order and their corresponding elements are equal. In this case we say that A=B.

Example. Let A denote the matrix.



- M ... ns. It c ...
- E ... alled e ... h the p ...

  Example $a_{2,3}$ is element of matrix in second row and third column which is 6. Similarly $a_{3,4}$= 1 and $a_{1,4}$=4.

- Matrices are used to organize information having two variables.

  Example: Marks of five groups of students in different subjects, Number of different grades books of different subjects in library etc.

Duration/Number of Period

80 mins/2period

Material/Resources required

Different grade level Books of different subject, worksheet, Calendar

Introduction

Activity

- Arrange students in groups of five students.
- Give worksheet (four by four tables) to students in groups (sample worksheet is provided below)
- Told them to ask and write time their group fellows spend for different daily activities
- Ask students to fill tables through the small scale survey.
- Use developed tables to paste on board.
- Discuss benefits of using table for information organization.

Sample Worksheet

	School	Games	Home work	Watch TV
Ali	6 hours	2 hours	3 hours	1 hours
Sana				
Zia				
Anum				

Development

Activity 1:

(Groups Activity)

- Arrange students in groups.
- Give few books of mathematics, science and English of grade 8^{th}, 9^{th} and 10^{th} to students in groups.
- Ask them to think different ways to record number of books using different ways.
- Encourage students to explore different data organization layouts.
- Ask groups to present their explored technique for data organization.
- If they used rectangular array then discuss, otherwise told them rectangular array to organize data.
- Told students that the number of categories horizontally and vertically are called rows and columns of the rectangular array and the rectangular array is called matrix.
- Discuss the size of matrix with the order which is 3x3.

Number of books in Library

	Maths	Science	English
8th	12	5	10
9th	11	5	9
10th	15	11	9

Activity 2:

(Individual Activity)

- Ask students to record marks of their last four weeks, test e.g, marks of mathematics, science, English and Urdu.
- Ask them to use rectangular array (matrix) to organize information.
- Ask them to write how many rows and column are required for the given situation.
- Also ask them how many entries will be required to fill information.
- Told them all entries are called elements of matrix.
- Ask students to determine order of the matrix used to organize information.

Activity 3:

(Group or individual activity)

- Give a calendar to students.
- Ask them to count and write how many Sunday, Saturday and Fridays in the month of June, November and March.
- Ask them to use rectangular array.
- Also ask them to count element, rows, column and order of matrix.
- Discuss the use of matrices to organize information in daily life context along with rows, column and order of matrices.

Activity 4:

(Group or individual activity)

- Discuss the equality of Matrices by taking different examples.

Conclusion/Sum up

- In the situations when information having two variables is involved we can use rectangular array called matrix.
- Each element of matrix associated with the categories in the row and column.
- Each element is also identified with row and column.
- Order of matrix is used to determine size of matrix which is number of rows by number of columns.

Assessment

Write more matrices and ask students to present matrices along with rows, columns, elements and order of given matrix.

- Write $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ on board and ask what could be the meaning of a matrix A. Ask about number of elements, rows and columns of A. Also ask about of order of A.
- Share population of Punjab, Sindh, Baluchistan in year 2000, 2005 and 2010 and ask students to organize information using matrix, ask them how many rows and columns will be used, also ask order of matrix.

Follow-up

- Ask students to select any situation from their surroundings where they could use rectangular array to organize information. Also ask them to determine rows, column, elements and order of the matrix used to organize information.
- Which of the following matrixes are equal and which of them are not?

$$A = \begin{bmatrix} 1 \\ 2 \end{bmatrix},\ B = \begin{bmatrix} 3 & 4 \end{bmatrix},\ C = \begin{bmatrix} 5 \\ 6 \end{bmatrix},\ D = \begin{bmatrix} 2+3 \end{bmatrix}$$

$$E = \begin{bmatrix} 5+1 \\ 2+2 \end{bmatrix},\ F = \begin{bmatrix} 2+1 & 3+2 \end{bmatrix},\ G = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix},$$

$$H = \begin{bmatrix} 1+2 & 2+1 \\ 2+1 & 2+3 \end{bmatrix},\ I = \begin{bmatrix} a & b \\ c & d \end{bmatrix},\ J = \begin{bmatrix} a+0 & b \\ c & d+0 \end{bmatrix}$$

- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

Lesson Plan 2

TOPIC

Addition and subtraction of Matrices

Grade IX

Students' Learning Outcomes

- Add and subtract matrices.

Information for Teachers

- Matrices are rectangular array used to organize information.
- Each element is associated with the two variables in accordance to horizontal and vertical categories.
- Two matrices of same order and same horizontal and vertical categories can be added (demonstrated below).

A =

	Maths	English	Science	Urdu
Group 1	21	31	11	20
Group 2	12	14	19	17
Group 3	24	20	14	30

B =

	Maths	English	Science	Urdu
Group 1	15	14	10	21
Group 2	24	14	11	14
Group 3	25	20	24	16

A+B=

	Maths	English	Science	Urdu
Group 1	21+15=36	45	21	41
Group 2	36	28	30	31
Group 3	49	40	38	46

- Similarly two matrices of same order and arrangement can be subtracted (demonstrated below).

A =

	Maths	English	Science	Urdu
Group 1	21	31	11	20
Group 2	12	14	19	17
Group 3	24	20	14	30

B =

	Maths	English	Science	Urdu
Group 1	15	14	10	21
Group 2	24	14	11	14
Group 3	25	20	24	16

A-B=

	Maths	English	Science	Urdu
Group 1	21-15=6	17	1	-1
Group 2	-12	0	8	3
Group 3	-1	0	-10	14

Duration/Number of Periods

80 mins / 2 period

Material/Resources required

Worksheets, three different grade books of different subjects

Introduction

Activity

- Arrange students into group of five students.
- Give worksheet (four by four tables) in groups of five students (sample worksheet is provided).
- Ask students to find time they spend for

different daily activities in two consecutive weeks from remaining four class fellows and fill tables through the small scale survey.

- Ask them to tell how much total time they spent for different daily activity in two weeks.
- Ask them to explain their working.
- Discuss how corresponding elements of two matrices added.
- Discuss addition of two matrices. Highlight order of the matrices should be same for addition.

Sample worksheet

Week 1=

Names	School	Games	Home work	Watch TV
Ali	6 hours			
Sana				
Sara				
Zia				

Week 2=

Names	School	Games	Home work	Watch TV
Ali	6 hours			
Sana				
Sara				
Zia				

Week 1 + Week 2 =

Names	School	Games	Home work	Watch TV
Ali	6+6=12 hours			
Sana				
Sara				
Zia				

Development

Activity 1

(Group Activity)

- Arrange students in groups.
- Give few books of maths, science and English of grade 8th, 9th and 10th to students in groups.
- Ask groups to organize number of books using 3x3 rectangular array.
- Arrange groups together and ask them to make rows and columns of their categories (sequence remain same).
- Add their matrices by adding corresponding elements of both matrices.
- After addition engage students in discussion about addition of matrices.
- Ask them that can we add two matrices of different order.

Activity 2

(Group or individual activity)

- Give following information to students and ask them to organize information with matrices.

Population in 2000			
	Kids	Women	Men
City A	2200	2152	2000
City B	1500	1780	1804
City C	980	789	753

Population in 2010			
	Kids	Women	Men
City A	4252	4752	4698
City B	3562	3625	3789
City C	2500	3687	3210

- Ask students to add both matrices after representation.
- Ask students to present addition of matrices in front of the class.
- Discuss what will happen if we interchange rows with columns or columns with rows in both matrices.
- Also ask students to describe increase of population in 2000 and 2010. For this ask students to subtract population of three city kids, women and men of 2010 from 2000 accordingly.
- Discuss addition and subtraction of matrices.

Activity 3

(Group activity)

- Arrange students into group of five.
- Ask them to prepare a drama in which they have to take a situation in which they have to raise value of matrices from daily life situation.
- Ask them to demonstrate addition and subtraction of matrices through drama.
- After the preparation they have to present drama in front of their fellows.
- Discuss application of addition and subtraction of matrices.

Conclusion/Sum up

- Two matrices can be added also subtracted.
- To add or subtract two matrices, order need to be same.
- To add or subtract corresponding variables should be in same sequence and will be operated accordingly.

Assessment

- Give two matrices of 2x2, 2x3, 3x2 and 3x3

  e.g $A=\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ and $B=\begin{bmatrix} 3 & 6 \\ 5 & 7 \end{bmatrix}$ or

  $C=\begin{bmatrix} 1 & 2 & 3 \\ 5 & 4 & 1 \\ 1 & 3 & 7 \end{bmatrix}$ and $D=\begin{bmatrix} 3 & 2 & 5 \\ 1 & 3 & 7 \\ 2 & 4 & 5 \end{bmatrix}$ and ask them to find sum and difference of two matrices.
- Provide information about population of male and female in year 2000, 2005 and 2010 of Punjab. Also provide same kind of information for any other province. Ask students to organize information using two matrices. Add and subtract both matrices.

Follow-up

1. Assign matrices of 2x2,3x3 and 4x4 order e.g

   $A=\begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix}$ and $B=\begin{bmatrix} 3 & 7 \\ 1 & 5 \end{bmatrix}$

   to students to find their sum and difference.
2. Ask students to search and note application of matrices addition and subtraction from daily life situation.

- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

Lesson Plan 3

TOPIC

Solution of Simultaneous Linear Equations

Grade IX

$3x + 2y = 7$ $9x + 4y = 9$

$-bx + 6y = 6$ $-bx + 5y = 8$

$$\underbrace{\begin{bmatrix}3 & 2\\ -b & 6\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}x\\ y\end{bmatrix}}_{x}=\underbrace{\begin{bmatrix}7\\ 6\end{bmatrix}}_{b} \qquad \underbrace{\begin{bmatrix}9 & 4\\ -b & 5\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}x\\ y\end{bmatrix}}_{x}=\underbrace{\begin{bmatrix}9\\ 8\end{bmatrix}}_{b}$$

Students' Learning Outcomes

- Solve a system of two linear equations and related real life problems in two unknowns using Cramer's rule.

Information for Teacher

- The theory of matrices and determinants is highly developed branch of Algebra.
- Simultaneous equation $C_1=a_1X+ b_1Y$ and $C_2=a_2X+ b_2Y$ can be represented and solved with the help of matrices and determinants.
- A determinant $\begin{vmatrix}2 & 3\\ 4 & 7\end{vmatrix}$ can be represented as (2x7 – 3x4) = (14-12)=2.
- Process of Cramer's Rule
  Given the system of linear equations
  $a_1x + b_1y = c_1$
  $a_2x + b_2y = c_2$

The above equation can be written in matrix form
as: Ax=B

where $A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$ $B = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \end{bmatrix}$

This system has the unique solution

$$x = \frac{D_x}{D} \quad \text{and} \quad y = \frac{D_y}{D}$$

where $|A|$ or $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ or D_2 $D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$ or D_2 $D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$ $D \neq 0$

When solving a system of equations using Cramer's Rule, remember the following:

1. Three different determinants are used to find x and y. The determinants in the denominators are identical.

2. The elements of D, the determinant in the denominator, are the coefficients of the variables in the system; coefficients of x in the first column and coefficients of y in the second column.

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$

3. D_x, the determinant in the numerator of x, is obtained by replacing the x-coefficients, a_1 and a_2, in D with the constants from the right sides of the equations, c_1 and c_2.

As $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ then $D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$

4. D_y, the determinant in the numerator for y, is obtained by replacing the y-coefficients, , b_1 and b_2, in D with the constants from the right side of the equation, c_1 and c_2.

As $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ then $D_y = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$

Duration/Number of Periods

40 mins / 1 Period

Material/Resources Required

Board, loose sheet, word problems sheet

Introduction

Activity

(Individual or group activity)

- Give simultaneous equations to students.
- Ask them to solve the equations with the help of different methods.

- Encourage them to represent simultaneous equation with the help of matrices.
- Ask them that they can also use concept of matrices and determinants to find the values of x and y.

Development

Activity 1

(Group Activity)

- Give simultaneous equations to students (y+2x=-3 and 3y-x=5).
- Ask students to write equations with the help of matrices.

$$\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} -3 \\ 5 \end{bmatrix} \text{i.e in } AX = .$$

or $X=A^{-1}B$

- Ask students to find determinant of A. *i.e* $|A|$ or 5
- Ask them to find D_1 and D_2
- Ask students to find $x = D_1/|A|$ and $y= D_2/|A|$.

Note:

The determinant D, is the determinant of matrix A so it can also be written as $|A|$

i.e $|A|$=D.

Similarly D_x and D_y can also be written as D_1 or D_2 i.e $D_x=D_1$,

Activity 2

- Take a box and put strips containing simultaneous equation on each of them.
- Ask students to draw two strips to get two equations.
- Ask them to use Cramer's rule to find value of two unknowns.
- After solving equations ask students to interchange their solved equations with fellow students.
- Involve students in peer checking and address mistakes if required.

Activity 3

- Arrange students in groups.
- Give some word problems to students to form two equations. E.g. 2 chair and 3 tables cost Rs. 2500 where as 4 chair and 1 table cost Rs. 3000. Find the cost of chair and table.
- Ask each group to find solution of problem using elimination, substitution methods and Cramer's Rule.
- Ask students to share their work with rest of the groups.

Conclusion/Sum up

- With the help of work produce by students in activities discuss with students stepwise process to apply Cramer's rule for solving simultaneous equations.
- Steps of Cramer's rule will be, conversion of equations into matrices, finding out determinant, D_1 and D_2, and value of x and y.
- Word problem related to simultaneous equation can be solved through Cramer's rule.

Assessment

- Give some simultaneous equations to students and ask them to use Cramer's rule to find out their solution e.g

  $y+2x=1$ and $2y-5x=7$

  or

  $45x-23y=21$ and $21x-24y=17$

- Give word problems and ask students to form simultaneous equation and with the help of Cramer's rule to find solution. e.g price of 15 notebooks and 12 books is Rs.1500/- and price of 18 notebooks and 15 books is Rs.2200. Find the price of a notebook and a book.

Follow-up

Ask students to develop some word problems which they can convert into matrices and apply Cramer's rule to find solution.

- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

UNIT 2

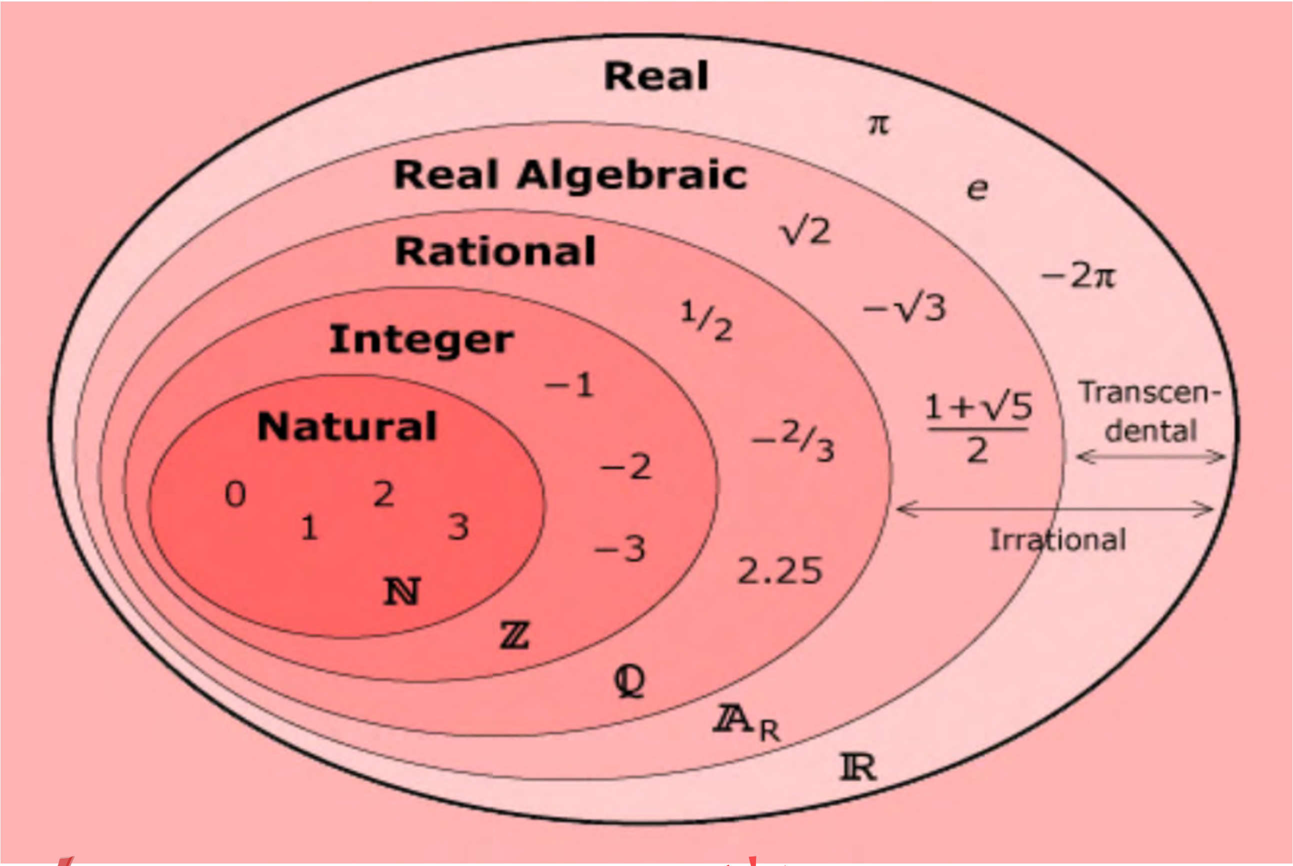
Lesson Plan 1

TOPIC

Real Numbers

Real and Complex Numbers

Grade IX



Students' Learning Outcomes

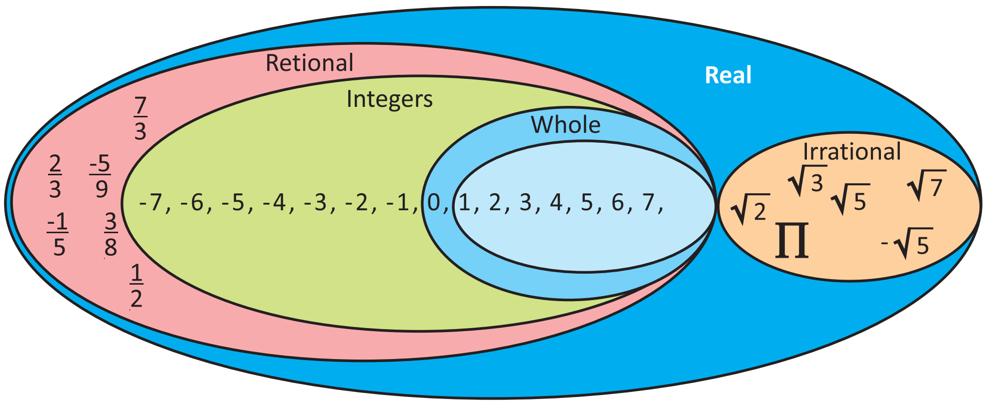
- Recall the set of real numbers as a union of sets of rational and irrational numbers.
- Depict real numbers on number line.
- Demonstrat a number with terminating and non-terminating recurring decimals on the number line.
- Give decimal representation of rational and irrational numbers.

Information for Teacher

- A real number is a value that represents a quantity, such as -5 (an integer), 4/3 (a rational number that is not an integer), 8.6 (a rational number given by a finite decimal representation), $\sqrt{2}$ (the square root of two, a number that is not rational) and π (3.1415926535...).
- Real numbers can be thought of as points on

an infinitely long line called the number line or real line, where the points corresponding to integers are equally spaced.

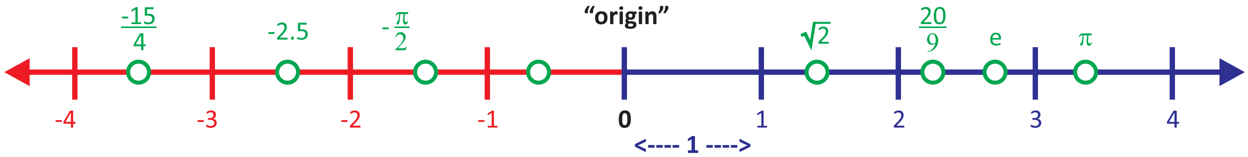
Real Number System



- Any real number can be determined by a possibly infinite decimal representation (such as that of π above).

The Real Number Line

A point is chosen on the line to be the **"origin"**, points to the right will be positive, and points to the left will be negative.



Duration/Number of Period

40 minutes / 1 Period

Material/Resources required

Diagram, chart paper, number flashcards.

Introduction

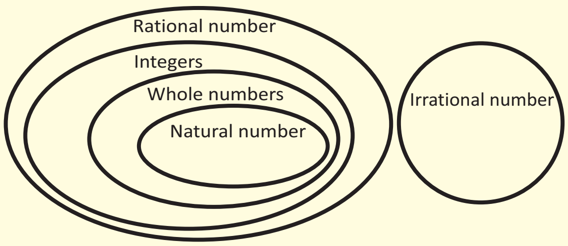
ACTIVITY

- Write different fractions or numbers on the board e.g. 3/7, 5/145, 4/5, $\sqrt{2}$, $\sqrt{6}$ etc.
- Ask students to use calculators to convert written numbers in decimal representations.
- Ask students to identify those values which are whole numbers, terminating decimal fractions, non-terminating with recurring, and non-terminating and non-repeating decimal fraction.

Development

Activity 1:

- Give following diagram to all students.
- Ask them to write numbers in appropriate place of diagram.
- Give a chart paper to students to draw diagram collectively.
- Hang chart paper in the class and discuss real numbers as union of different sets of numbers.



Activity 2:

- Take different number flashcards containing different kind of number.
- Put all number flashcards in a box
- Draw a number line on the board
- Ask different students to come in front of classroom to draw one card and paste on number line appropriately.
- After pasting all cards told students that they developed real number line.
- Depict and discuss real numbers on number line.

Conclusion/Sum up

- Numbers can be sorted into different sets, namely set of natural numbers, set of whole numbers, set of integers, set of rational and irrational numbers.
- Rational numbers are those which could be written as p/q form with q non-zero number and irrational numbers as never terminating, non-repeating decimal fraction.
- Real numbers are union of rational and irrational numbers.
- Different numbers can be depicted through real number line.

Assessment

- Ask the students to take any four natural numbers, whole numbers, integer, rational and irrational numbers. Ask them to develop real number line by drawing and rewriting their chosen numbers.
- Give following worksheet to students to fill with suitable titles.

Numbers	Title of number sets
0.45	Rational and real number
0	
$\sqrt{2}$	Whole, integer, rational and real number
-3/7	
1 ½	
-9/3	
3.1428571...	

Follow-up

- Ask students to search and write definition of natural, whole, rational numbers etc. along with examples.
- Give different numbers to students to identify name of sets they belong.
- Give different real numbers and ask them to write on real number line.
- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

UNIT 3

Logarithms

Lesson Plan 1

TOPIC

Logarithm

Grade IX

Students' Learning Outcomes

- Define common logarithm, characteristic and mantissa of log of a number.
- Differentiate between common and natural logarithm.

Information for Teacher

logarithms

- common (or Briggian) — The power to which a base of 10 must be raised
- natural (or Napierian) — The power to which the base ' e ' must be raised

- **Logarithms, Invention of** - The logarithm was invented by three people, independently of each other, between 1553 and 1614, German Augustine Monk Michael Stifel, Swiss astronomer and mathematician Joost Bürgi and Scottish Mathematician John Napier.
- **Logarithms (Concept)**-If you multiply a number raised to two different powers, the result is equal to the same number raised to the sum of the powers. This suggests a way in which multiplication, difficult with complicated numbers, can be replaced by addition, much easier even with complicated numbers.
- **Properties of Logarithm**

  Property 1: $\log_a 1 = 0$ because $a^0 = 1$

  Property 2: $\log_a 1 = 0$ because $a^1 = a$

  Property 3: $\log_a a^x = x$ because $a^x = a^x$
- **COMPONENTS OF LOGARITHMS-** The fractional part of a logarithm is usually written as a decimal. The whole number part of a logarithm is called the **CHARACTERISTIC.** This part of the logarithm shows the position of the decimal point in the associated number. The decimal part of a logarithm is called the **MANTISSA**.
- **Difference between Log and Natural log** - Log with base 10, or common log as it is popularly known is written as log x. There is another value of base that is very popular and is known as natural log.. Natural log of a number is the power to which e has to be raised to be equal to the number. We know that e X e = 7.389, hence In(7.389) = 2. On the other hand, 10 X 10 = 100 Hence, log 100 = 2

Duration/Number of Periods

80 mins/2period

Material/Resources required

Routine Classroom Resources, Log table, scientific calculator

Introduction

From History: (teacher's talk)

- **There was a boy named** Young Johnny Napier, he had to help his dad, who was a tax collector. Johnny got sick of multiplying and dividing large numbers all day and devised logarithms to make his life easier.
- They were a clever method of reducing long multiplications into much simpler additions (and reducing divisions into subtractions).
- The use of logarithms made trigonometry and many other fields of mathematics much simpler to calculate. When **calculus** was developed later in the century, logarithms became central to many solutions.
- The slide rule, once almost a cartoon trademark of a scientist, was nothing more than a device built for doing various computations quickly, using logarithms.

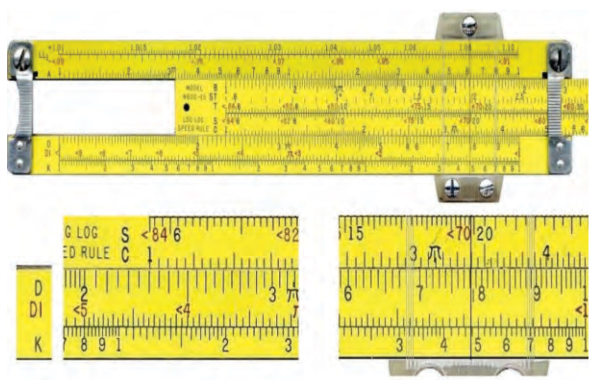


Figure 1 Slide Rule

- Logarithms are still important in many fields of science and engineering, even though we use calculators for most simple calculations.
- The modern calculators have borrowed the logic

from logarithms. In measuring

- 'Earthquake Intensity'
- PH value in Urine Test etc.
- Loudness of sound, brightness of light etc. logarithms is applied.

Development

Activity

- Write 1000 on board and ask what is it? May I write it as 10^3? Then tell its log is 3. Similarly complete the whole table with the class so they are easy with it.

Number	Exponential Expression	Logarithm
1000	10^3	3
100	10^2	2
10	10^1	1
1	10^0	0
1/10 = 0.1	10^{-1}	-1
1/100 = 0.01	10^{-2}	-2
1/1000 = 0.001	10^{-3}	-3

Write on the board. **LOGS ARE EXPONENTS** and ask students think in pairs and then share their understanding in open discussion.

Explain to them

1. Using $\log_{10}$ ("log to the base 10"):
   $\log_{10}100 = 2$ is equivalent to $10^2 = 100$
   where 10 is the base, 2 is the logarithm (i.e., the exponent or power) and 100 is the number.
2. Can you write the following in logarithmic form? (i) $4^3 = 1024$, (ii) $3^{-3} = 1/27$ (assign questions for independent work, the difficulty level should be raised gradually)
3. Questions are given in the book but at this point of time, do not ask them to open books rather keep them with you and give question on board. (similarly introduce exponential form)
4. Introduce Using natural logs ($\log_e$ or ln):
   Carrying all numbers to 5 significant figures,
   $\ln 30 = 3.4012$ is equivalent to $e^{3.4012} = 30$ or $2.7183^{3.4012} = 30$
5. Many equations used in chemistry were derived using calculus, and these often involved natural logarithms. The relationship between ln x and log x is: $\ln x = 2.303 \log x$

How to find ln by using calculator

If it's a scientific calculator with a button labeled "ln," you probably just need to type in the number you want to take the natural log of, and then hit the "ln" button.

If the calculator does not have a "ln" button, but it has a "log" button for log base 10, you can still calculate the natural log using the "change of base" formula. Let's write "log base a of b" as log_a(b). Then log_a(b) = log_c(b) / log_c(a). You want to take log_e(x), the natural log of a number x. That's equal to log_10(x) / log_10(e). With the calculator that has a log button but not a ln button, you can take the log base 10 of e and of the number you want to take the natural log of, and then use division to get the final answer. Just take the base ten log of the number, and divide it by the log base ten of e.

Why 2.303? Let's use x = 10 and find out for ourselves.

Rearranging, we have

(ln 10)/ (log 10) = number.

We can easily calculate that ln10=2.302585093... Or 2.303 and log 10 = 1.

So, the number has to be 2.303.

Using Calculators to find logs: (Hands on)

To find the logarithm of a number other than a power of 10, you need to use your scientific calculator or pull out a logarithm table. On most calculators, you obtain the log (or ln) of a number by

1. entering the number, then
2. Pressing the log (or ln) button.

- **Example 1:** log 5.43×10^{10} = 10.73479983...... (way too many significant figures)
- **Example 2:** log 2.7×10^{-8} = -7.568636236...... (Too many sig. figs.)
- Allow students to use calculator for the same.

How to consult the log table to find log

Give examples by considering the numbers 15, 150 and 1500

- 15 lies between 10 and 100

so its logarithm must be between 1 and 2, i.e, it is 1.*something*

- 150 lies between 100 and 1000

so its logarithm must be between 2 and 3, i.e, it is 2.*something*

- 1500 lies between 1000 and 10000

so its logarithm must be between 3 and 4, i.e, it is 3.*something*

In each case the ".*something*" has the same value and this is what is printed **in the table**

Example :To look up a logarithm of say, 15.27, you would

1. Run your index finger down the left-hand column until it reaches 15
2. Now move it right until it is on column 2 (it should be over 1818)
3. Using another finger, find the difference on column 7 of the differences (20)
4. Add the difference.

So the logarithm of 15.27 is

1.1818 + 0.0020 = 1.1838

So, let's look at the logarithm more closely and figure out how to determine the correct number of significant figures it should have.

For any log, the number to the left of the decimal point is called the **characteristic**, and the number to the right of the decimal point is called the **mantissa**.

- The characteristic only locates the decimal point of the number, so it is usually not included when determining the number of significant figures. The mantissa has as many significant figures as the number whose log was found. So in the above examples:
- **Example 1:** log 5.43×10^{10} = 10.735

  The number has 3 significant figures, but its log ends up with 5 significant figures, since the mantissa has 3 and the characteristic has 2.
- **Example 2:** log 2.7×10^{-8} = -7.57

  the number has 2 significant figures, but its log ends up with 3 significant figures.
- Assign some values to the students to work in pairs and find characteristic and mantissa.

Explain to the students that Natural logarithms work in the same way:

- **Example 3:**

  ln $3.95 \times 10^6 = 15.18922614... = 15.189$
- Assign question to identify characteristic and mantissa after finding the logarithm of numbers.

Conclusion/sum up

The general rule is that logs simply drop an operation down one level: exponents become multipliers; divisions become subtractions, and so on.

Assessment

Give following questions to complete

- Characteristic of 4567 = ______ ∴ $\log_{10} 45677$ ________ + .6597 = 3.6597
- Characteristic of 456.7 = ______ ∴ $\log_{10} 456.7 = 2 +$ __________ = 2.6597
- Characteristic of 45.67 = ______ ∴ $\log_{10} 45.67 =$ _________ + .6597 = 1.6597
- Characteristic of 4.567 = _______ ∴ $\log_{10} 4.567 = 0 + .6587 =$ ________________
- Characteristic of .4567 = _______ ∴ ___________ = -1 + .6597 = -0.3403
- Characteristic of .04567 = _______ ∴ ___________ = -2 + .6597 = -1.3403
- Characteristic of .0004567 = ______ ∴ $\log_{10}$ .0004567 = _________ + .6597 = -3.3403

Follow-up

Students will be given a project in a group of 4 or 5, to collect information, and present on a chart paper with proper pictorial representation (this projects will encourage the use of IT in their work), about following;

- slide rule,
- Function of slide rule
- Application of slide rule in logarithms
- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

UNIT 4

Lesson Plan 1

TOPIC

Algebraic Expressions

Algebraic Expression And Algebraic Formulas

Grade IX

Students' Learning Outcomes

- Know that a rational expression behaves like a rational number.
- Define a rational expression as the quotient $\frac{p(x)}{q(x)}$ of two polynomials p(x) and q(x) where q(x) is not the zero polynomial
- Examine whether a given algebraic expression is a
- o polynomial or not,
- o rational expression or not.

Information for Teacher

Rational number-Rational number is a number that can be in the form **p / q** where **p** and **q** are integers and **q** is not equal to zero

Rational Expression-The quotient of two polynomials is called a rational expression. A rational expression is the ratio

$$\frac{2x^2 + 3}{x^3 + 3x - 12}$$

of 2polynomials... We must be mindful of the final value of the denominator because the expression would be undefined whenever the denominator equals 0.

Polynomial-Polynomial is a mathematical expression involving a sum of powers in one or more variables multiplied by coefficients.

$$R(z) = \frac{P(z)}{P(z)}$$

A polynomial quotient of two polynomials *P (z)* and *Q (z)* is known as a rational function. The process of performing such a division is called long division, with synthetic division being a simplified method of recording the division.

Algebraic Expression- An **algebraic Expressions** is an expression formed from any combination of numbers and variables by using the operations of addition, subtraction, multiplication, division, exponentiation (raising to powers), or extraction of roots.

$7x, 2x - 3y + 1, \frac{5x^3 - 1}{4xy + □}$ $\pi r^□$, $\pi r \sqrt{r^2 + h^□}$ and are algebraic expressions. By an algebraic expression in certain variables, we mean an expression that contains only those variables, and by a **constant**, we mean an algebraic expression that contains no variables at all. If numbers are substituted for the variables in an algebraic expression, the resulting number is called the **value** of the expression for these values of the variables.

Algebraic expressions and polynomials- A **polynomial** is an algebraic sum, in which no variables appear in denominators or under radical signs, and all variables that do appear are raised only to

positive-integer powers. For instance, the trinomial $-2xy^{-1} + 3\sqrt{x} - □$ is not a polynomial; however, the trinomial $3x^2y^4 + \sqrt{2}xy - \frac{1}{□}$ is a polynomial in the variables x and y.

A term such as $-\frac{1}{□}$ which contains no variables, is called a **constant term** of the polynomial.The numerical coefficients of the terms in a polynomial are called the **coefficients** of the polynomial. The coefficients of the polynomial above are 3, $\sqrt{□}$ and $-\frac{1}{□}$

Algebraic expression and rational expression-Any algebraic expression, that is a quotient of two other algebraic expressions, is called a rational algebraic expression. Thus if P and Q are algebraic expressions then the expression $\frac{P}{1}$ is called a rational algebraic expression.

For example $\frac{g}{x}$ where $x \neq 0$, $\frac{-1}{x + □}$ where $x \neq -□$

and $\frac{x^2 + 5x}{x - □}$ where x □ $\frac{P}{1}$

We say that a rational algebraic expression is meaningless for those values of the variable for which the denominator Q is zero. To simplify rational algebraic expressions, we may be required to

Duration/Number of periods

80 min/ 2 periods

Material/Resources required

One set per group of algebra tiles (made by chart paper) which includes:

Three - "a" unit by "a" unit squares
Four - "a" unit by "b" unit rectangles
Three - "b" unit by "b" unit squares

"b" unit by "b" unit square

"b" unit by "a" unit rectangle

"a" unit by "a" unit square

Introduction

ACTIVITY:

Elicit previous knowledge about different sets of Numbers and note down on the board, with the help of that feed back.

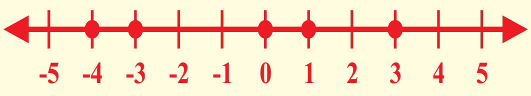
Now Explain Rational Numbers on the Number Line:

A **number line** is a visual representation of the numbers from **negative infinity** to **positive infinity,** which means it extends indefinitely in two directions.

The numbers on the number line can be grouped into different categories. The **natural numbers** are the numbers in the set {1, 2, 3, 4, 5, ...}. The **whole numbers** are the numbers {0, 1, 2, 3, 4, ...}. The set of integers is {...,- 2,- 1, 0,+ 1,+ 2, ...}.A **rational number** is any number that can be expressed as a fraction whose denominator is not equal to zero. For example, $-\frac{2}{3}, \frac{4}{5}, \frac{30}{10}$, and $\frac{9}{2}$ are all rational numbers. The rational numbers can also be expressed in decimal form. More specifically, the decimal equivalent of any rational number will terminate or will repeat. If the decimal repeats it should be written with bar notation. Notice that $-\frac{2}{3}=0.\overline{6}$, $\frac{4}{5}=0.8$, $\frac{30}{10}$'$=3$, and $\frac{9}{2}=4.5$.

Example

a. Name the set of numbers graphed.



The graph shows the set: {- 4, - 3, 0, 1, 3}.

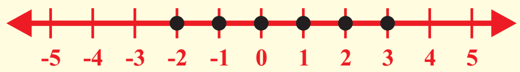
b. Find the absolute value.

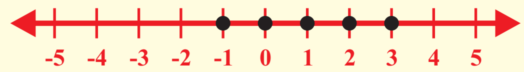
|10|

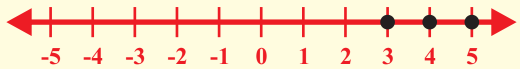
10 is ten units from zero in the positive direction.

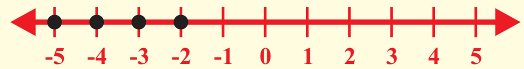
Practice

Name the set of numbers graphed.

1. 

2. 

3. 

4. 

Development

ACTIVITY 1

Do Brain storming activity about algebra, note down all responses on the board and then explain algebraic expression and polynomial

- Do you know what algebra is?
- *'Ilm algebra wa'lmugabalah* (Arabic)
- The science of reintegration and equation = The reunion of broken parts.
- We have been studying algebra from grade six, but today we are going to see its geometric representations. We are going to see algebra as broken parts.

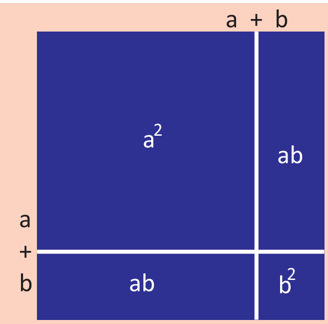
ACTIVITY 2

Stage 1: Polynomials

- Before distributing the algebra tiles talk to the whole class. Hold the b^2 tile and ask "what is this, they will answer square." Now ask if one side of this square length 'b' what is its area? (b^2)
- Take the next rectangle followed by larger square and do the same so that they know the areas are (b^2, ab, a^2 respectively)
- Now divide the class into groups and distribute each set of tiles and follow the procedures.

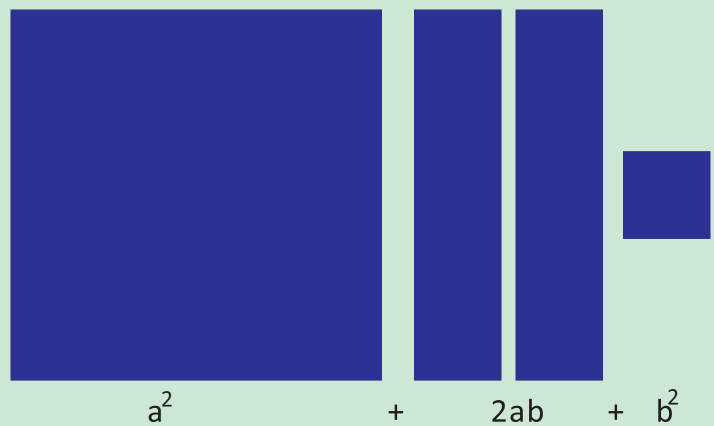
Procedure 1: Ask the students to form a square or rectangle using the algebra tiles that shows

$(a + b)(a + b)$ or $(a + b)^2$ $= a^2 + 2ab + b^2$



- **One possibility** would look like this:
- **Questions to Ask:**
  - What figure does this make? A square or a rectangle? Why?
  - What is the length of each side?
  - If you rearrange the squares and rectangles making up the larger square, what do you have?

Answer:

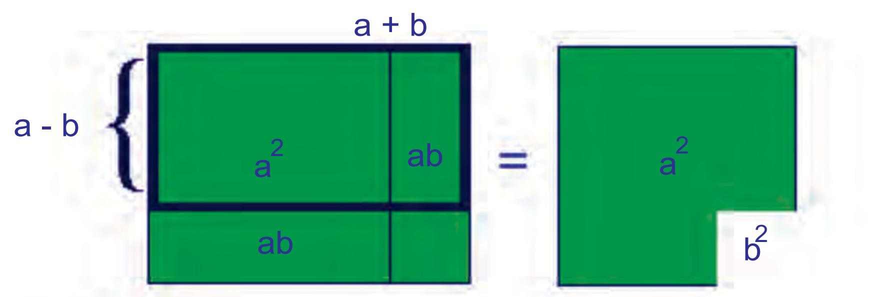


Procedure 2:

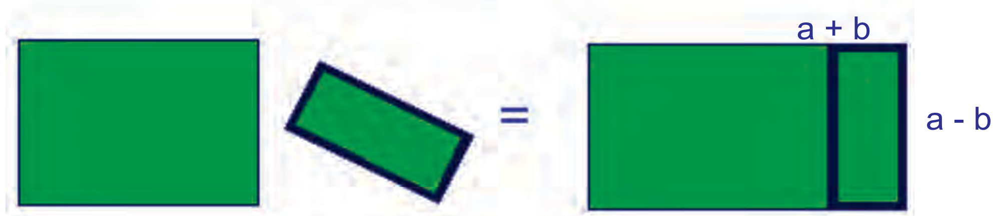
Ask the students to form a square or rectangle using the algebra tiles that shows

$$(a - b)(a + b) = a^2 + ab - ab - b^2 = a^2 - b^2$$

One possibility would look like this:



"a" squared minus "b" squared



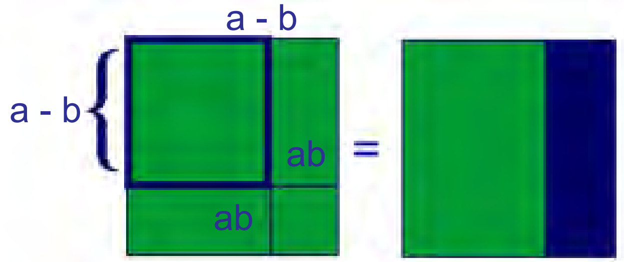
Now rotate the "ab" rectangle 90 degrees and place on the other side.

Procedure 3:

Ask the students to form a square or rectangle using the algebra tiles that shows

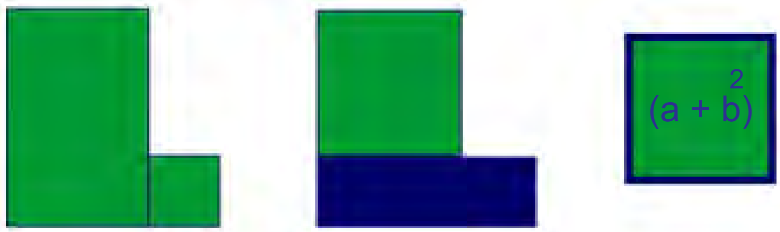
$$(a - b)(a - b) \text{ or } (a - b)^2 = a^2 - 2ab + b^2$$

One possibility would look like this:



The two rectangles "ab" overlap

Start with "a" squared and subtract one of the "ab" rectangles



In order to take another "ab" rectangle away. a "b" squared must be added first.

When the second "ab" rectangle is taken away. only the (a-b) square remains.

- **Teacher's talk**: With this exercise we have seen algebraic expressions put in equations with equal signs, like we used in grade 7 and 8 as well.
- **WHAT these expressions actually are**? We can define that an algebraic expression that has only one term is known as monomial. For example, $2x$, $3y^2$, $-4a$, x^3y^2z.
  - A ***polynomial*** is an algebraic expression that has more than one term. **Examples**

Ask the following questions before explaining the rational expressions

- **What is a rational number?**
- **What is an expression?**
- **What is an algebraic expression?**
- **What is a polynomial?**

we can say that **Rational number** is a number that can be written as a fraction with a numerator that is an integer and a denominator that is an integer other than 0."

- We now need to look at rational expressions. A **rational expression** is nothing more than a fraction in which the numerator and/or the denominator are polynomials. Here are some examples of rational expressions. [write on the board]

$$\frac{6}{x-1} \qquad \frac{z^2-1}{z^2+5} \qquad \frac{m^4+18m+1}{m^2-m-6} \qquad \frac{4x^2+6x-10}{1}$$

- The last one may look a little strange since it is more commonly written $4x^2+6x-10$
  However, it's important to note that polynomials can be thought of as rational expressions if we need to, although they rarely are.

Some example questions are given below, give more examples depending on the availability of the time.

Identify the P and Q in the following rational expressions

(a) $\frac{1}{x-1}$ $P =$, $Q =$

(b) $\frac{2xy-x^2}{2x^2-\text{و}}$ $P =$, $Q =$

(c) $\frac{2wy-7x}{3yz+\text{ی}}$ $P =$, $Q =$

(d) $\frac{m^4+18m+5}{mn+8}$ $P =$, $Q =$

ACTIVITY 3

- Ask the students to look at the rational expressions written on the board and identify the polynomials in them. Give time to think and then mark the answers on board with whole class discussion.
- Invite a student on board and ask to reduce it to lowest term.

  not reduced to lowest term $\longrightarrow \frac{12}{8} = \frac{\cancel{(4)}(3)}{\cancel{(4)}(2)} = \frac{3}{2} \longleftarrow$ reduced lowest term

- Now tell that ,with rational expression it works exactly the same way.

  not reduced to lowest term $\longrightarrow \frac{\cancel{(x+3)}(x-1)}{x\cancel{(x+3)}} = \frac{x-1}{x} \longleftarrow$ reduced lowest term

- As we know that $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$ similarly can somebody write -(x+3)/ (x+1)

Conclusion / Sum up

Today we have seen the geometrical representation of algebraic expressions. We have also seen rational expressions. Towards the end we have seen that simplifying rational expressions is just as simplifying rational numbers.

Remember: rational expression is undefined for any value of the variable that makes the denominator equal to 0. So we say that the **domain** for a rational expression is all real numbers except those that make the denominator equal to 0.

Assessment

Graph each set of numbers on a number line.

1. {integers from -2 to 6, inclusive}

2. {-4, -3, -2, -1}

3. {integers less than 1 but greater than -4}

4. {integers greater than 2}

5. {integers less than or equal to 3}

6. {integers less than -4 +(-1)}

7. Standardized Test Practice Which number shows the absolute value of -30?

A. $|-30| = -30$ **B.** $|-30| = 30$ **C.** $|-30| = \frac{1}{30}$ **D.** $|-30| = -\frac{1}{30}$

Give a concept example and then give a question to solve. For example:

<u>**Concept Example:**</u> $2/3 \bullet 3/8 = ¼$ can u solve $7x^2/15y^3 \bullet -3y/14x^3$

<u>**Concept Example:**</u> $\frac{15}{35} = \frac{3}{7} \cdot \frac{5}{5} = \frac{3}{7}$ can you simplify $\frac{15x^2y}{35xy^2}$

Problem: After the first two terms in the following sequence. each number is the sum of the preceding two terms. Find the missing numbers.

<u>4</u> ___ ___ ___ ___ <u>67</u>

Solution: let x represent the second term of the sequence. Then missing terms will be

<u>4</u> <u>x</u> <u>$x+4$</u> <u>$2x+4$</u> <u>$3x+$</u> <u>$5x+12$ or 67</u>

i.e, $5x + 12 = 67 \implies x = 11$

Check: <u>4</u> <u>11</u> <u>4+11</u> <u>11+15</u> <u>26+41</u>

<u>4</u> <u>11</u> <u>15</u> <u>26</u> <u>41</u> <u>67</u>

This will also help the student in understanding next concepts.

Follow-up

Give following table as the follow up activity and then assign questions according to these rules.

- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

Algebra of Rational Expressions

Rule	Example
Multiplication: $\frac{P}{Q} \times \frac{R}{S} = \frac{PR}{QS}$	$\frac{x+1}{x} \times \frac{(x-1)}{2x+1} = \frac{(x-1)(x+1)}{x(2x+1)} = \frac{x^2-1}{x(2x+1)}$
Addition with Common Denominator: $\frac{P}{Q} + \frac{R}{Q} = \frac{P+R}{Q}$	$\frac{y}{xy+1} + \frac{x-1}{xy+1} = \frac{x+y-1}{xy+1}$
General Addition Rule: (works with or without common denominator) $\frac{P}{Q} + \frac{R}{S} = \frac{PS+RQ}{QS}$	$\frac{y}{x} + \frac{x-1}{y} = \frac{y^2 + x(x-1)}{xy}$
Subtraction with Common Denominator: $\frac{P}{Q} - \frac{R}{Q} = \frac{P-R}{Q}$	$\frac{y}{x^2-1} - \frac{x-1}{x^2-1} = \frac{-x+y+1}{x^2-1}$
General Subtraction Rule: (works with or without common $\frac{P}{Q} - \frac{R}{S} = \frac{PS-RQ}{QS}$	$\frac{y^2}{x} - \frac{xy}{y+1} = \frac{y^2(y+1) - x^2y}{x(y+1)}$
Reciprocals: $\frac{1}{\left[\frac{P}{Q}\right]} = \frac{Q}{P}$	$\frac{1}{\frac{x+1}{y-1}} = \frac{y-1}{x+1}$
Cancellation: $\frac{PR}{QR} = \frac{P}{Q}$	$\frac{y^2(xy-1)}{x\,(xy-1)} = \frac{y^2}{x}$

- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

UNIT 7

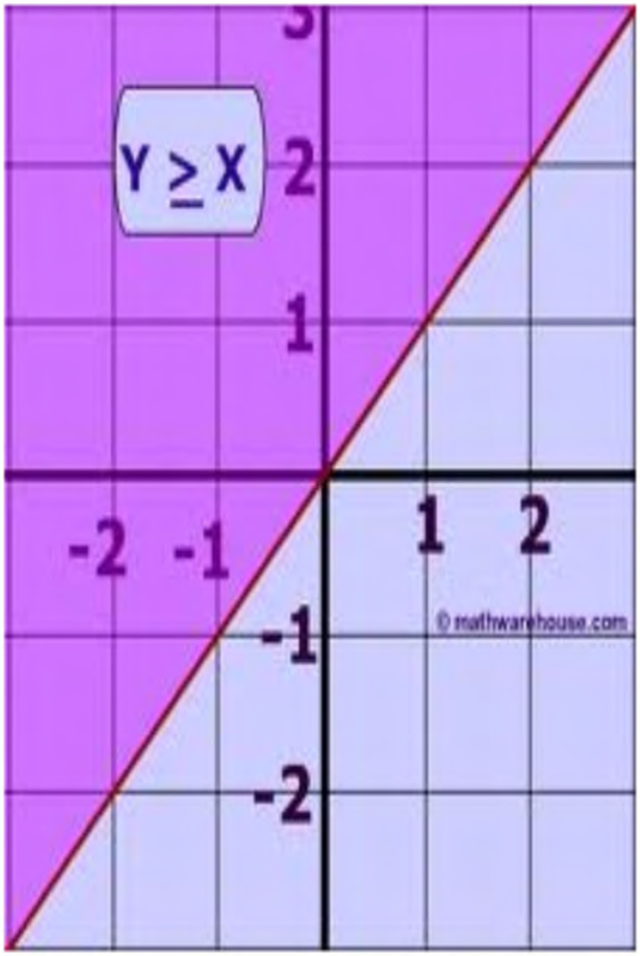
Lesson Plan 1

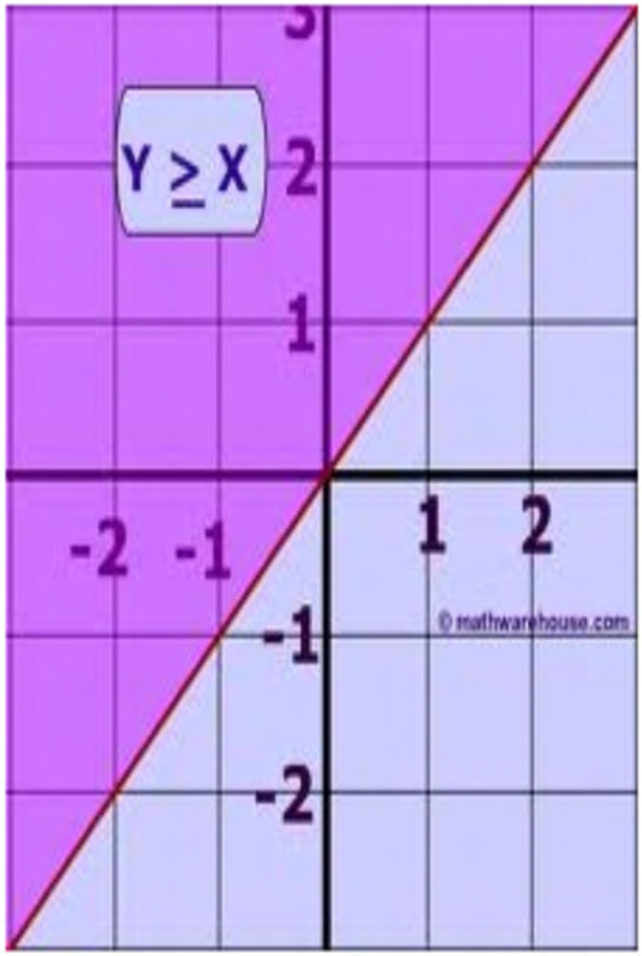
TOPIC

Linear Equations

Linear Equations and Inequalities

Grade X





Students' Learning Outcomes

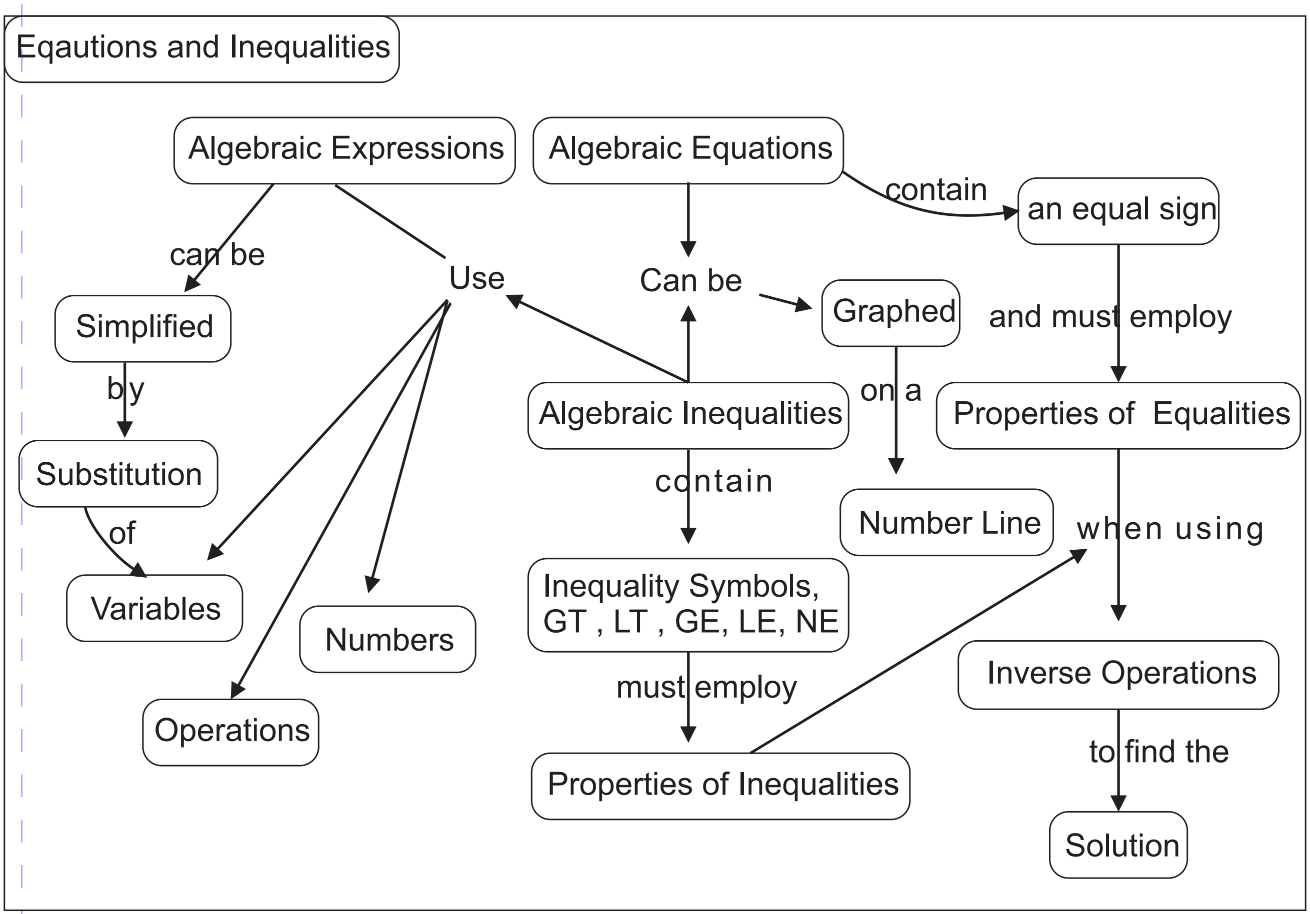
- Recall linear equation in one variable
- Solve linear equation with rational coefficients.
- Define inequalities (> , <) and (≥, ⊗).

Information for Teachers

Use the key words. (For concept map)

GT	greater than	>
LT	less tan	<
GE	greater and equal	≥
LE	less and equal	≤
NE	not equal	≠

Concept Map



Duration/Number of Periods

80 mins/2period

Material/Resources required

A bag of 40 chips (20 red, 20 yellow cut outs of chart paper)

10 paper cups (or any other identity to represent variables)

Dry-erase boards or large sheets of paper As in the picture ,

red chips represent negative the yellow positive.

Introduction

Activity

 OR  compare 

Write words, equations and inequalities on the board and ask students to share their understanding of these terms in their daily life. Note: Write down main points of discussion on board. Conclude discussion with the help of the following talk.

- Many of us use inequalities every day without even knowing we are. When we go into the store and we are looking at earrings, shoes, food, or any item for that matter we think about the amount of money we have (we usually estimate our total) and then we calculate in our heads the largest amount of the items that we can buy with the money we have. We also use the logic of inequalities when trying to meet certain criterion either in competitions or when applying for the scholarship you are hoping to receive. We calculate in our heads what obstacles we must overcome and then come up with the solution that solves the question of whether we need to be above or below the guidelines required (greater than or less than the guidelines).
- Let us think of x students attending a class. If there are 80 students we will say **$x=80$**. **This is an equation** and there is only one answer, i.e., there are 80 students. Now if I say that there are less than 80 (**$x< 80$**) students that mean it can be any numerical value ranging from 0 to 79 inclusive. [In this example negative value of x has no meaning]. This is an **inequality. It has many answers.**

Development

Activity 1

Stage 1:

Present the following equations for students to solve: $x + 10 = 15$, $y - 3 = -1$, $5 - m = -2$, $w + 4 = -5$. (Students know how to solve linear equation)

Have students compare and discuss their solutions with a partner. For any problem with which students had difficulty, ask several students with different answers to present their solutions on the board and help them clarify their understanding.

Stage 2:

1. Distribute a bag of chips, a set of cups, and a large sheet of paper or chart paper piece to each group of students.
2. Explain that students will be using cups and chips activity to solve the equation $2X + 6 = 12$.
3. Present the following directions to students:

- If the variable is positive, place the cup(s) facing up.
- If the variable is negative, place the cup(s) facing down.
- The coefficient of the variable indicates the number of cups to use.

Then, ask students to show you the representation of 2x using the cups. They should all place two cups facing up on top of their paper or chart paper sheet. Explain the following:

- The chips represent the numbers.
- If a number is positive, the chip should be yellow side up.
- If a number is negative, the chip should be red side up.

Have students use six yellow chips to represent +6. They should place these chips next to their two cups. Then, have they drawn an equal sign (=) to the right of the two cups and six yellow chips. Explain that they can represent +12 by placing 12 yellow chips on the other side of the equal sign.

4. Ask students what can be done to both sides of the equation to get rid of the six yellow chips (+6) on one side of the equation. Elicit from students that -6 should be added to each side (i.e., add six red chips to both sides); alternatively, +6 could be subtracted from each side (i.e., take away six yellow chips from each side).
5. On the overhead, add six red chips to the side with six yellow chips. Also add six red chips to the side with 12 yellow chips, and have students repeat these actions in their groups. Ask, "When you pair each red chip with a yellow chip, what happens?" Call on a student to explain that each pair is equal to 0.
6. Have students remove the pairs of red and yellow chips, leaving just two cups facing up and six yellow chips. Ask, "What equation do we have now?" Elicit from students that the cups represent 2x, the remaining yellow chips represent +6, and the equation now left is 2x = 6. Write this new equation on the board below the original equation.
7. Ask, "If two cups equal six chips, what does that tell us about one cup?" They should notice that there are three chips for each cup.
8. Demonstrate that the final equation is now x = 3, and write this equation on the board below the equation 2x = 6.
9. Give students the following problems to solve in their groups using cups and chips:

   5m + 1 = -9, 2x + 3 = 4 (allocate time)
10. Review the solutions to the problems with the class. For the second problem, be sure to discuss the final step, when students arrive at the equation 2x = 1. Ask, "Were you actually able to use the cups and chips to solve the problem? When you had 2x = 1, what operation did we have to do?" Elicit from students that both sides had to be divided by 2 (or that the chip needed to be split in half), to yield the answer x = ½.

12. Explain to students that you want them to try a problem with a negative coefficient. Give students the problem -2x + 3 = -5 to solve.
13. Ask, "What was the first step in solving this problem?" The students should notice that the first step is to subtract 3 from (or add -3 to) both sides of the

equation, yielding -2x = -8.

14. Ask, "What is the next step to balance the equation and get x by itself?" Students may note that both sides need to be divided by -2, yielding x = 4. They may also state or demonstrate that they can turn over both the cups and the chips on both sides of the equation, which would represent multiplication by -1.

15. Ask, "How can we check this to make sure it is the correct answer?" Obtain from students that the value x = 4 can be substituted into the original equation to show that it works: -2(4) + 3 = -5.

Conclude the activity and tell that the cup was the variable.

Stage 3: [practice the skill]

Below are some equations you might use (make sure some of the variables have negative and fractional coefficients):

- 3x + 2 = 14
- -3m - 1 = -10
- -7x + 5 = 12
- -w + 13 = 9
- ½d + 7 = 10

here you may assign more questions to ensure that they are skilled in solving equations.

Ask what is this? (Write x+3 = 7) they will say it's an equation. Then write

x+3 < 7 on board and ask the same question. Let them speak up! Then tell it's an inequality.

Explain to students that now that they have solved some equations using cups and chips in algebra; it's time to try solving similar equations rather **inequalities.** [refer to the discussion done in the beginning x<80]

When solving linear inequalities, we use a lot of the same concepts that we use when solving linear equations. Basically, we still want to get the variable on one side and everything else on the other side by using inverse operations. The difference is, when a variable is set equal to one number, that number is the only solution. But, when a variable is less than or greater than a number, there are an infinite number of values that would be a part of the answer.

Tell them the inequalities are solved in similar way with some rules.

- We can add and subtract on both sides.
- We can multiply or divide both sides with positive numbers
- We will have to **flip** the inequality sign when multiply or divide with a negative number.

Activity 2

Draw the following picture on a chart paper and show to students and take their feed back in terms of inequation or COMPARISON

Ask as many questions as time allow like following:

Who is taller than 60"?

Is it one person or more?

How many are less than 60"?

Solve x+3 < 7 on board and tell that since we get X<4 its means any number less than 4. It has unlimited answers.

Assign them some inequalities to solve in pairs. (X+3 <7 , 2X-1 > 5 , 6-X >4) (allocate time)

Invite different pairs on board to discuss the answers.

Conclusion/Sum up

Once students have answered all questions, ask them to summarize the process of solving inequalities. Solicit input from several students, and relate their descriptions to the cups and chips activity. Emphasize the need to add or subtract and multiply or divide, and be sure to stress that the final step should always be to check the answer in the original equation and inequalities.

Assessment

- Write linear equations and in-equalities on board to be solved independently.
- Call weak students on the board to show their worked solutions.
- Following type of question must be involved in assessment.

1. If a ي then what are the possible values of a.
2. If $n \otimes$ ي what are the possible values of n, where $n \in \beta$.
3. If $7 - x$ وو what are the values of x where x O integers.

Follow-up

- Find the solution sets of the following

  a. $-5 \le a + 1$ لآو

  b. $\frac{1}{2} \le x < -\frac{1}{\text{و}}$

Senior class is planning a festival in school. The price of a festival ticket is Rs.23 per person. The cost of renting the building is Rs1375, Rs50 per security, Rs575 for the DJ and Rs225 for the light show. How many students need to purchase tickets so that the juniors make a profit of at least Rs 2750?

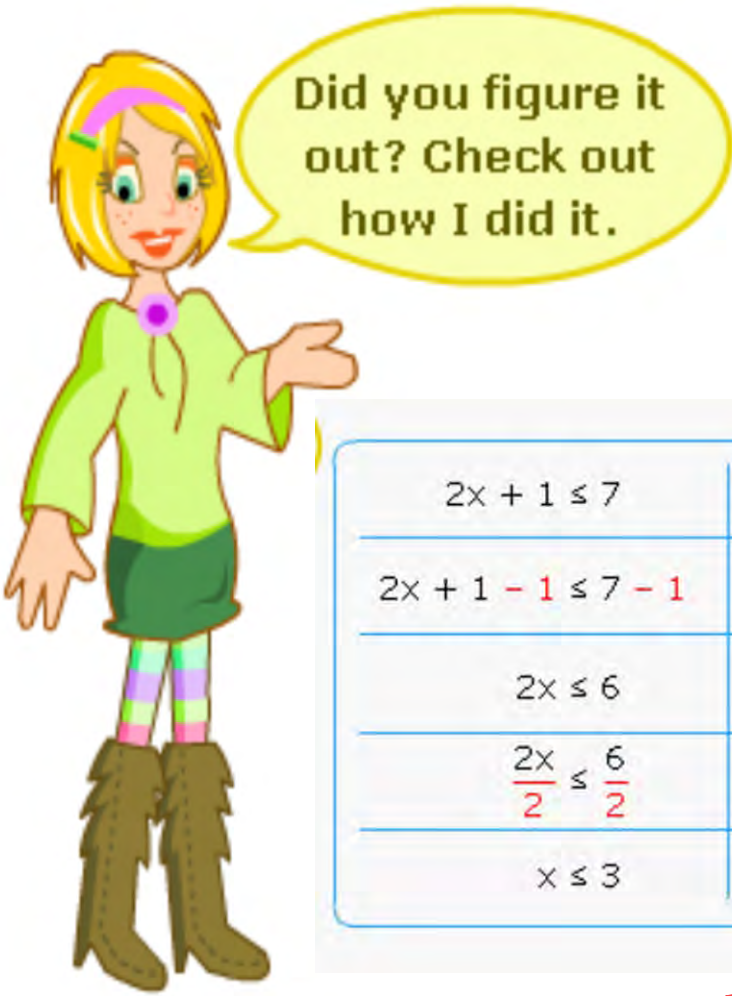
- What is the cost of the festival regardless of how many students come?
- In order to make a profit of at least Rs2750, the senior class needs to collect Rs2750 plus enough to cover their costs. How much do they have to take in?
- How much will the senior class get each ticket sold?
- What is the inequality that shows the senior class making at least or more than Rs2750 as a profit?

Lesson Plan 2

TOPIC

Linear Inequalities

Grade IX



$2x + 1 \le 7$	Original Inequality.
$2x + 1 - 1 \le 7 - 1$	Subtract 1 from each side to undo the addition.
$2x \le 6$	Simplify.
$\frac{2x}{2} \le \frac{6}{2}$	Divide each side by 2 to undo the multiplication.
$x \le 3$	Simplify.

Students' Learning Outcomes

- Recognize properties of inequalities (i.e, trichotomy, transitive, additive and multiplicative).
- Solve linear inequalities with rational coefficients.

Information for Teacher

- **a < b** a is less than b
  a ≤ b a is less than or equal to b
- **a > b** a is greater than b
  a ≥ b a is greater than or equal to b

- Trichotomy Property: For any two real numbers a and b ,exactly one of the following is true: a< b , a = b , a > b .
- Transitive Properties of Inequality:
  If a < b and b < c , then a< c .   If a > b and b > c , then a > c .
- Addition Properties of Inequality:
  If a < b , then a + c < b + c
  If a > b , then a + c > b + c
- Subtraction Properties of Inequality:
  If a < b , then a - c < b - c
  If a > b , then a - c > b – c
- Inequality Properties of Opposites
  If a > 0 , then - a < 0
  If a < 0 , then - a > 0

- Multiplication and Division Properties of Inequalities for positive numbers:

- If a < b and c > 0 , then ac < bc and $\frac{a}{c} < \frac{b}{c}$
- If a > b and c > 0 , then ac > bc and $\frac{a}{c} > \frac{b}{c}$

- Multiplication and Division Properties of Inequalities for negative numbers:

- If a < b and c < 0 , then ac > bc and $\frac{a}{c} > \frac{b}{c}$
- If a > b and c < 0 , then ac < bc and $\frac{a}{c} < \frac{b}{c}$

Duration/Number of Periods

80 mins / 2 period

Material/Resources required

3 X 5 cards, enough for the class, Activity Time: Half to one period. Before class, teacher makes up the 3 X 5 cards. On the back of two cards, write the letter "A."Do the same with "B" and "C" and so on until you have enough for each student to get one card. The purpose of this is so that each student will be paired up with another student that has that same letter. scissors, hard sheet of any material, two paper clips, two pencils, coin, timer

Introduction

Activity 1

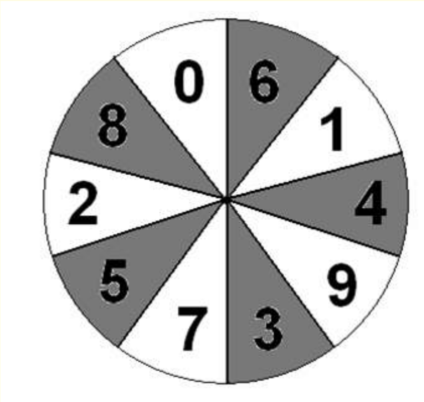
Write the following on board and ask their meaning to the whole class. (≤, ≥, =, <, >)

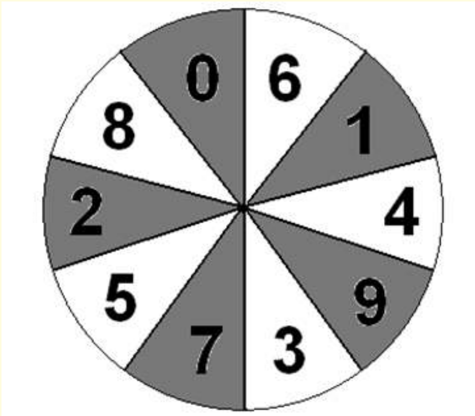
Write some of the statements on board and ask the quiet members of the class so that you ensure that they know the meanings.

5----8 , 3---- -3, 0.5 ---- 0.4 , -5/6 --- -6/5 , 5+x ---7+x

[give them time to apply all the symbols]

Activity 2





1. Draw the spinners on the board and ask students to make these spinners of some suitable size (by working in pairs) and cut out the spinners. Use a pencil and paper clip to complete each spinner.

   Numbers on un shaded sectors are positive.

   Numbers on shaded sectors are negative.

2. Player 2 spins both spinners, flips the coin, and then starts the timer.
3. Player 1 develops an inequality using the two numbers spin and the coin flip as follows:

   Heads: x ± Spinner 1 > Spinner 2

   Tails: x ± Spinner 1 < Spinner 2

   Example: Mehmood spins a 6 on Spinner 1 and ali spins –4 on Spinner 2, and flips tails. The inequality is x + 6 < – 4.
4. Player 1 has 40 second to develop and solve the inequality. If the solution is correct, the player is awarded 2 points.
5. Play moves to Player 2.
6. The winner is the first player to score 20 points by the end of a round

Take feedback from the students that how it helped them to learn more about solving inequalities.

Development

Activity 1

- Recall the inequalities did in the previous session.
- Write the RULES discusses on side of the board.
- **Distribute the cards prepared.**
- Students are then to pair up with the person that has the same letter on the back of their card. They put them together and solve the inequality. For example, one "A" had **5x + 3 <**, and the other "A" had **35**, their equation would be 5x + 3 < 35. They then solve for x.

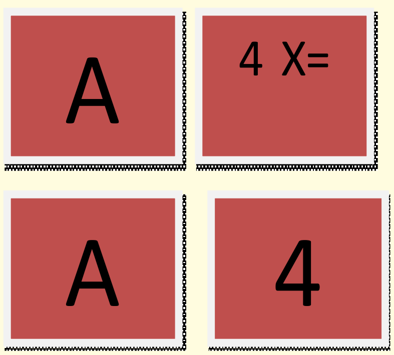
- Teacher then picks random pairs to present their inequality on the board.
- Extensive discussion will help them understand clearly.
- ASK if I have some money, Ali has 10 rupees more than me and Khurram has 40 rupees more than Ali then can I say that Khurram is richer than me? [repeat if needed]

On one the cards, write an inequality [LEFT SIDE along with sign] that your level can solve such as:

$4-x \quad 4,$

$4-4x \quad -5\,,$

$\frac{2}{3}(x-2) \quad 1$



EACH SIDE of TWO CARDS

On the corresponding card for that letter, write a number or expression. Mix up the cards at random, making sure that half the kids will get an inequality, and half the kids will get a number. [raise the difficulty level as per your students level]

- If there is an odd number of students, just add another"A" card and put a number on the back of it. Three people would then be grouped as "A's". They would then combine their numbers together. In other words, using the above example, let's say you add another"A" card and write -24 on it. Then the three "A" cards would make: 5x+3 > 35-24.
- Then introduce the TRANSITIVE property that is if X>Y and Y>Z then X>Z too.
- In the same pairs ask the class to 'Find examples to show the same property from real life examples'. For example, **IF POTATO IS EXPENSIVE THAN ONION AND EGGS ARE EXPENSIVE THAN POTATOS THEN "EGGS ARE EXPENSIVE THAN ONION TOO"**

Let them share as many as time allows.

- Now ask if there are two numbers X and Y. I say X=Y and X=9 what can be Y? They will answer 9.
- Then ask if there are two numbers X and Y. I say X<Y and X=9 what can be Y? they will answer any number greater than 9.[infinity]
- Then ask if there are two numbers X and Y. I say X>Y and X=9 what can be Y? they will answer any number LESS than 9.[infinity, including negative numbers]
- NOW introduce the **Law of Trichotomy. It says that if we consider two numbers there can be ONE AND ONLY ONE possibility of being true.** (X=Y OR X<Y OR X>Y)
- Ask students to give some examples from daily life
- Supplement their understanding by giving some mathematical sentences
- Introduce open and closed intervals

with the help of the examples

- Ask them students to give examples where the inequality goes to infinity
- Make sure of the maximum involvement of the students.

Conclusion/Sum up

Summarize the lesson with the quick review of the properties of the inequalities. Involve students to give examples

1. If a b and c is any real number, then
   a + c b + c
   For example, -3 -1 implies -3+4 -1 + 4
2. If a b and c is positive, then ac bc.
   For example, 2 3 implies 2(4) 3(4)
3. If a b and c is negative, then ac bc.
   For example , -1/2 2 and 2 8/3 imply -1/2 8/3

Assessment

Assign questions for independent practice. As per difficulty level.

Q1:
- If x>y then –x/2 ---- -y/2
- If p<q and q<0 then p---- 0
- If m-2 = n then m-----n

Q2: Give some inequalities to solve, e.g,
- 3X – 2 > 18- 2X
- x/2+ x/3 = 10
- (x + 3)/4 – (4x - 5)/5 = – 1

Follow-up

For each question solve the pair of inequalities using the guess and check method. State whether the answers for (a) are the same as (b).

If yes, describe how (b) is derived from (a).

1. a) $4x < 24$ where x is positive integer
   b) $x < 6$ where x is a positive integer
2. a) $6x$ -18 where x is negative integer
   b) x -3 where x is negative integer
3. a) $\frac{x}{2}$ 4 where x is positive integer
   b) x 8 where x is positive integer
4. a) $\frac{x}{3} > -2$ where x is negative integer
   b) $x > -6$ where x is negative integer

- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

UNIT 10

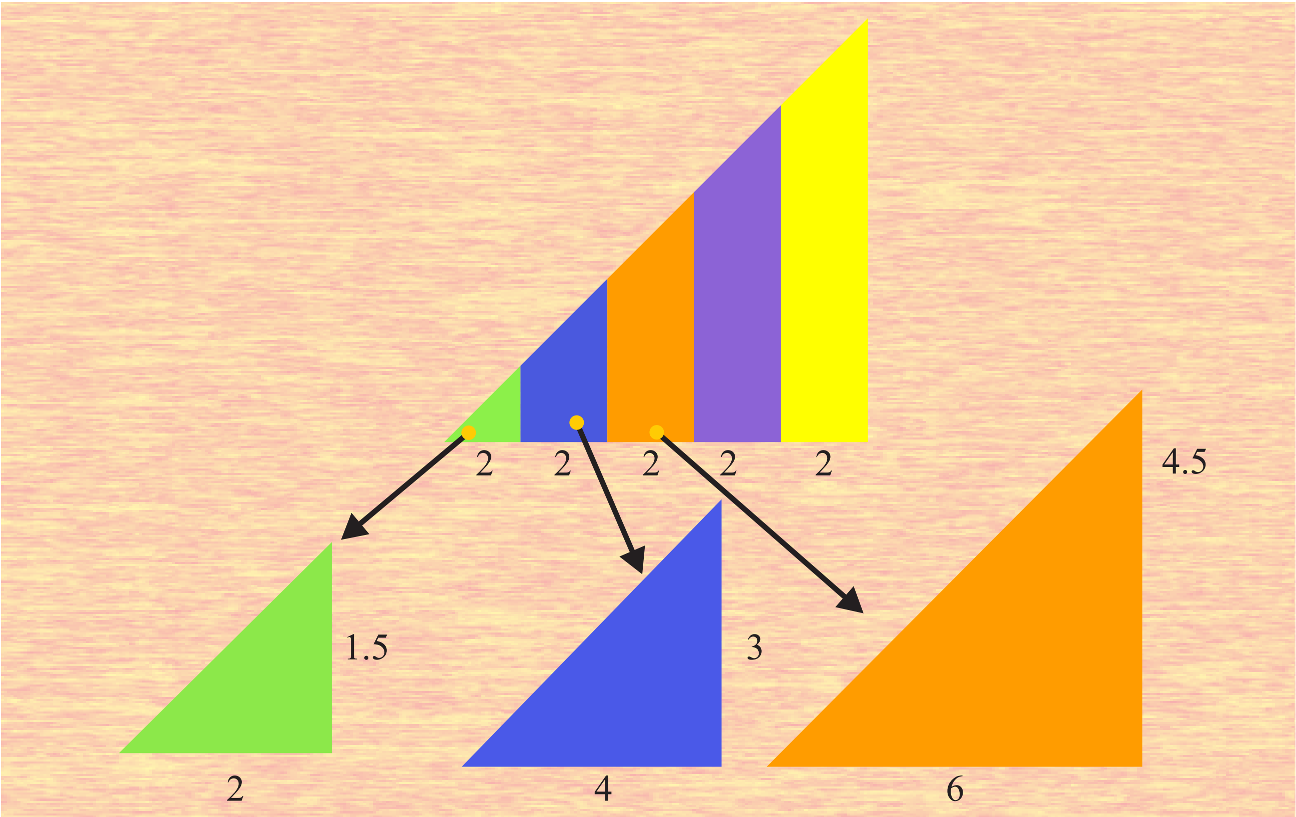
Variations

TOPIC

Joint Variation

Lesson Plan 1

Grade X



Students' Learning Outcomes

- Define joint variation.
- Solve problems related to joint variation.

Information for Teachers

- Joint variation is the same as direct variation with two or more quantities. That is: Joint variation is a variation where a quantity varies directly as the product of two or more other quantities.

or

- It is a relation between two quantities such that an increase in any quantity causes an increase in the other quantity or decrease in one quantity causes a decrease in the other quantity.
- Direct variation between variables ***x*** and ***y*** can be expressed as:

  $y = kx$, where '**k**' is the constant of variation and $k \neq 0$.

y = kxz represents joint variation. Here, y varies jointly as x and z.

More Examples on Joint Variation

- $y = 7xz$, here y varies jointly as x and z.
- $y = 7x^2z^3$, here y varies jointly as x^2 and z^3.
- Joint Proportional – occurs when you have 2 or more variables **directly** proportional to another variable. y = kzx (k is a constant)
- **Combine Proportional** – occurs when variables are related **directly** and **inversely** to each other.

(k is a constant) $y = \frac{kz}{\hat{s}}$

Duration/Number of Periods

80 mins/2period

Material/Resources Required

Different coloured chalks / marker, coloured chart paper stripes, coloured pencils, scissors – UHU

Introduction

Discuss with the students about the following:

a. We all know the formula for area of triangle and area of rectangle. They both are the examples of

joint variations. Area of a triangle = $\frac{1}{و}$ *bh* an example of joint variation. Here the constant is 1/2. Area of a triangle varies jointly with base 'b' and height 'h'.

b. Area of a rectangle = L x w represents joint variation. Here the constant is 1. Area of a rectangle varies jointly with length 'L' and width 'w'.

c. Do you know the Newtons's law of motion?

Force = mass x acceleration. The force exerted on an object varies jointly as the mass of the object and the acceleration produced. It is an example of joint variation. Here 'm' mass is constant.

Activity

- Ask students to give few examples from daily life of joint variation / direct variation.
- Divide them in groups / pairs and ask them to write down examples of joint variation on flip chart / chart paper.
- Ask few groups / pair to present their work.

Development

Activity 1

- Demonstrate to the students the Five step method to solve the joint variation. This will make it easy for them to understand and apply. (use colors to make them understand)

Assume ***a*** varies jointly with ***b*** and ***c***. If

b = 2 and c = 3, find the value of ***a***. Given that a = 12 when b = 1 and c = 6.

Solution:

Step 1: First set up the equation. ***a*** varies joint with ***b*** and ***c*** ***a*** = kbc

Step 2: find the value of the constant, k.

Given that a=12 when b=1 and c=6

a = kbc

12 = k x 1 x 6

K = 2

Step 3: Rewrite the equation using the value of the constant 'k' a = 2bc

Step 4: Using the new equation, find the mission value.

If b = 2 and c = 3, then a= 2 x 2 x 3 = 12.

Step 5: So, when ***a*** varies jointly with ***b*** and ***c*** and

If b = 2 and c= 3, then the value of a is 12.

Activity 2

- Groups will solve the problems / questions by using all five steps.
- Groups will explain their findings and steps involved in solution of each problem

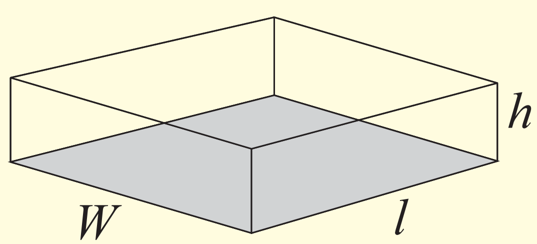
Activity 3

Guided practice:

- Assign questions from the textbook to be done in pairs
- Pairs will discuss their work with each other to clarify their concept

Activity 4

- Example to explain: Draw the figure given and tell that it shows a rectangular solid with a fixed volume. Express its width, 'w' as a joint variation in terms of its length, 'l' and height, h.



Solution: In other words, the longer the length / or the height h, the narrower is the width ***w***.

- Now give values for the variables in above figure to solves as a whole class.
- (independent practice) Assignment # _____

Homework: page no. from book

- (Guided practice) Review homework.

Assessment questions

Conclusion/Sum up

Conclude the lesson by recapping.

- The definition of joint variation
- Basic steps involved in solving problem related to joint variation.

Assessment

Use following sums to assess students

- The area of a rectangular plot is 1200 m^2. Into how many sub-plots can it be divided such that each sub-plot is a square, of 400 m^2 (area)? (answer is 3)
- A sprinter completes a 200 m race in 5.5 seconds and wins a silver medal. If the winner of the gold medal completes the race in 0.3 seconds earlier than silver medalist, then find the average speed of gold medalist. (Answer is 38.46 m/s)

Follow-up

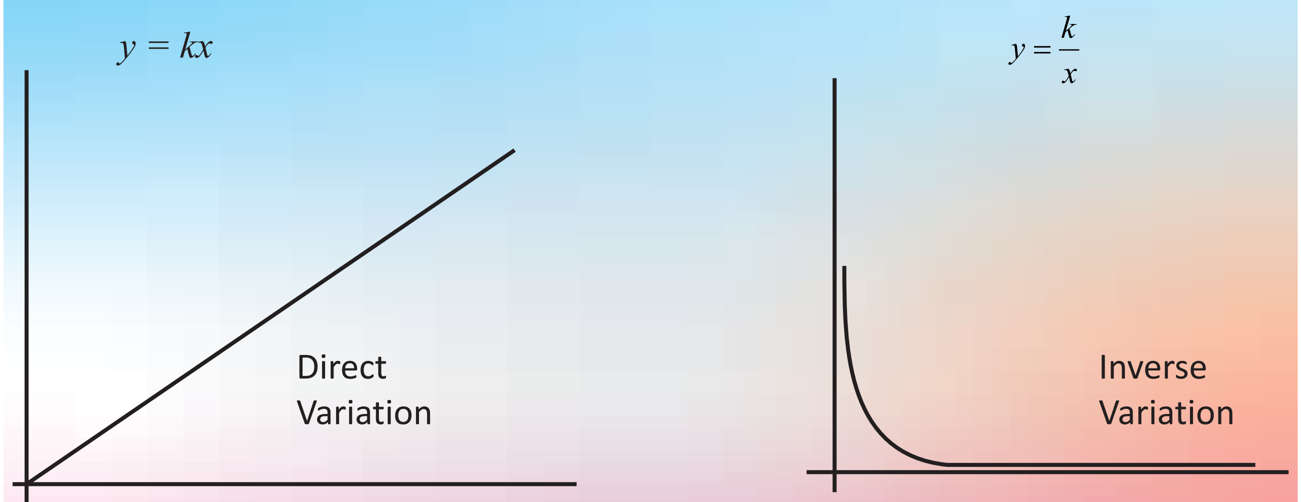
- The force with which the earth attracts an object above the surface of the earth varies inversely with the square of the distance of the object from the center of the earth. If an object 4000 miles from the center of the earth is attracted with a force of 160 pounds, find the force of attraction if the object were 6000 miles from the center of the earth.
- You are given a string of 12cm. you are to make quadrilaterals whose sides are integers and whose areas are 9cm^2, 5cm^2, 4cm^2, 3cm^2 (square, rectangle etc).

Lesson Plan 2

TOPIC

K-Method

Grade X



number	x	.25	.5	1	2	3	4
direct	$y = \frac{2}{\hat{s}}$	.5	1	2	4	6	8
inverse	$y = \frac{2}{\hat{s}}$	8	4	2	1	.67	.5

Students' Learning Outcomes

- Use K-method to prove conditional equalities involving propotions
- Solve real life problems based on variations

Information for Teachers

- A direct variation is an equation where both variables either increase at the same time or decrease at the same time.
- "K" method is used to solve the problems having these variations
- It is a constant of proportionality or the shortcut method which involves writing a proportion and solving it.

Duration/Number of Periods

80 mins/2period

Material/Resources Required

Flash card stated with problems, coloured charts, chalk / board

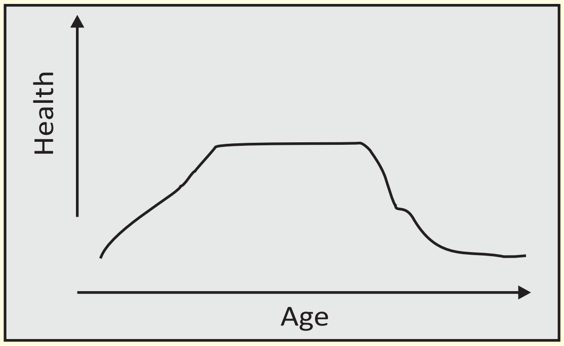
Introduction

Activity 1

- We have our parents at home, we have our siblings too. Most of us would have grand father, mother (dada, dadi, nana, nani) as well.
- Have you ever noticed that ever since a child is born s/he gets taller, stronger and smarter as every year pass by? After 14 or 15 years the child becomes a young individual. And then have you noticed that year by year we find our grandparents becoming weaker, catch cold quickly. Have you noticed your parents who are almost the same for past seven to eight years?

Now explain it on the board with the help of students

- Childhood to Youth --------------------- health improves with age
- In youth to middle age ----------------- fitness stabilizes
- From middle to old age ---------------------- - health decreases with age



- Well that was just a general observation.
- Of course, one can keep his youth as longer as one exercise and have healthy stuff.

Activity 2

- Involve the students in a real life situation.
- Ask them that an object falling from rest in a vacuum is directly proportional to the time it has fallen. After an object has falling for 1.5 sec, its speed is 14.7m/s. what is its speed after it has fallen 5 sec?
- First we will write an equation for this situation.
- Let us take 's' stand for speed and 't' for time. So we get the following equation

  S=Kt
- First we substitute the value of 's' and 't' and solve for the value of 'k'
- So we get

  14.7 = k (1.5)

  49/5 = k
- Now substitute the value of 'k' back in the original equation along with the second value of time (t) and then solve for the speed 's' we get

$$S = \frac{49}{5}(5$$

- S = 49 m/s
- Ask the students that by this way k-method is used to solve variation problems.

Activity 3

Draw a cup of tea on board. Suppose that for one cup of tea you need ½ spoon tea-leaves and 1 spoon sugar.

For two cups of tea you would need -------- spoons of sugar and --------- spoon of tea-leaves.

(ask students-2 and 1)

For three cups of tea you would need ---------- (3) spoons of sugar and ----------- (1.5) spoons of tea-leaves.

For Four cups of tea you would need ---------- - (4) ------------ spoons of sugar and ----------- (2) spoon of tea-leaves.

Ask following questions

- "Will the taste be same if we make four cups or only one cup with the same method?
- Can we say that 0.5:1 = 1:2 = 1.5:3 = 2:4? (why do we say so?)

Answer by students: ½ and 1.5/3 and 2/4 gives the same result as 0.5 or (1/2)

- Do you remember equivalent fractions done in class 4, 5 and 6?
- Can we say that these are equivalent fractions and can we also call them equivalent ratios?

Note: Conclude the discussion on the definition of ratios.

'Relation of correspondence between two values'

Development

Activity 1

Demonstrate the application of K-method by solving following on the board.

Students will be involved at different steps.

$\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ than prove that $\frac{x^3+y^3+z^3}{a^3+b^3+c^3} \quad \frac{xyz}{abc}$

Solution: $\frac{x}{a} = \frac{y}{b} = \frac{z}{c} \quad k$

Step 1: Taking K as constant

$$\frac{x}{a} = k \Rightarrow x \quad ak$$

Similarly, $y = bk$ and $z = ck$

Step 2: putting the values of x, y and z in one side of the equation

$$\frac{x^3+y^3+z^3}{a^3+b^3+c^3} = \frac{(ak)^3+(bk)^3+(ck)^3}{a^3+b^3 \quad c^3}$$

$$= \frac{k^3(a^3+b^3+c^3)}{a^3+b^3 \quad c^3}$$

$$\therefore \frac{x^3+y^3+z^3}{a^3+b^3+c^3} \quad k^3 \longrightarrow \text{eq (i)}$$

Now consider

Step 3: putting the values of x, y and z in the other side of the equation

$$\frac{xyz}{abc} = \frac{ak \times bk \times ck}{abc}$$

$$= \frac{abck^3}{abc}$$

$$\therefore \frac{xyz}{abc} \quad k^3 \longrightarrow \text{eq (ii)}$$

Step 4: From equations (i) and (ii), we have or comparing results of both sides.

$$\frac{x^3+y^3+z^3}{a^3+b^3+c^3} = \frac{xyz}{abc}, \text{if } \frac{x}{a} = \frac{y}{b} \quad \frac{z}{c}$$

Activity 2

- Involve the students in discussion

  There were some guests at home and you got to buy some Pepsi-bottles. How much you have to pay for three if each bottle cost Rs. 15. How much you have to pay for 5 of them. (let them answer)

  Can we say that 'amount you have to pay for Pepsi bottles is "PROPORTIONAL" to Number of bottles you buy? **Mathematically** we write it as:

- Total amount ∞ number of bottles (keeping the cost per bottle constant)
- Total amount = cost per bottle (constant) number of bottles x

- Now explain that in this example the cost per bottle is not changing or is CONSTANT.
- As we have concluded that total amount VARIES as the number of bottle VARIES the concept is also termed as 'VARIATION' so we have DIRECT variations and in this situation.
- Ask the students to share an example of inverse variation.

Activity 3

- Divide the class in groups
- Assign cards/paper strips with two questions on each
- Ask students to solve it in groups by using K-method
- Have few examples for reference

Example 1. If $\frac{a}{b} = \frac{c}{d}$ than prove that

$$\frac{3ac + 4bd}{3ac - 4bd} = \frac{3a^3 + 4b^2}{3a^2 - 4b^2}$$

Example 2. If $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ than prove that

$$\frac{x - y}{a - b} = \frac{y - z}{b - c} \quad \frac{z - x}{c - a}$$

Activity 4

- Ask the students to solve the problems given in their textbook by applying K-method.
- Observe their response and guide them if they have any difficulty.

Conclusion/Sum up

Recap the definition of k- method and steps involved in solving problems using k-method.

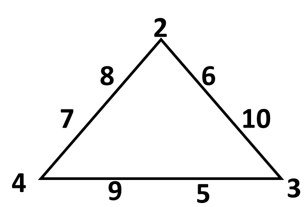
Assessment

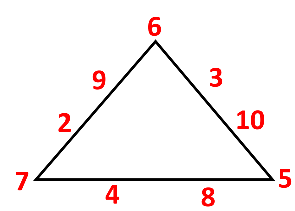
- The ratio among the sides of a triangle are 3,4 and 5. Its perimeter is 156m. Find the length of each side.
- The ratio between two numbers is 5:6. If 4 is added to each of them. Then new ratio becomes 7:8. Find the numbers.

Follow-up

- Assign some questions as follow as homework

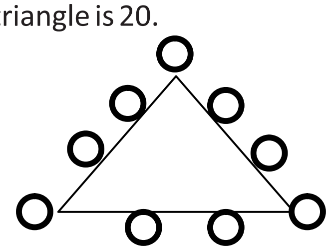
1. Find the least number of boys needed to form the following arrangements: "two boys in front of a boy, two boys behind a body and a boy between two boys."
2. The sum of the numbers on each side of the triangle given below is 21. Rearrange the numbers such that the sum on each side is now 24





Answer

3. Write each of the digits 1 to 9 in each circle below so that the sum of the numbers on each side of triangle is 20.



UNIT

12

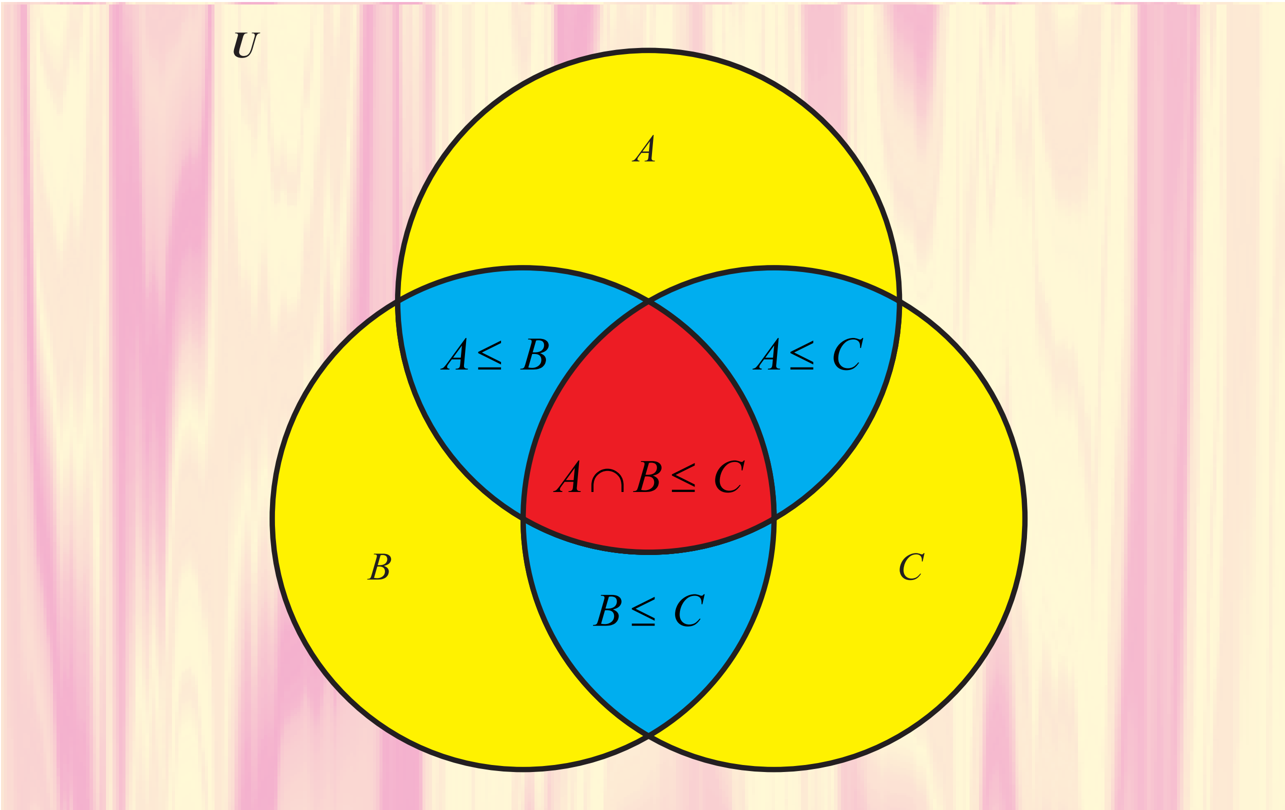
TOPIC

Lesson Plan 1

Venn Diagram

Set and Functions

Grade IX



Students' Learning Outcomes

- Use Venn diagram to verify
  - Commutative law for union and intersection of sets.
  - De Morgan's laws.
  - Associative laws.
  - Distributive laws.

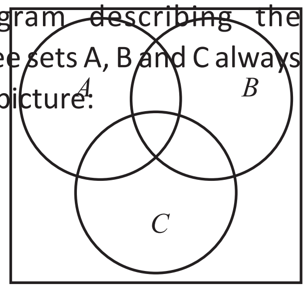
Duration/Number of Periods

80 mins/2period

Information for Teachers

- **Venn diagram**

Relationships between multiple sets are sometimes graphically described using **Venn diagram**. A Venn diagram describing the relationship between three sets A, B and C always begins with the following picture:



The rectangle "framing" the picture denotes the universal set; all things not in A, B or C are in the area surrounding them inside the frame.

- **Commutative property**

  (a) $A \cup B = B \cup A$

  (b) $A \cap B = B \leq$!

- **Associative property**

  (a) $(A \cup B) \cup C = A \cup (B \cup C)$

  (b) $(A \cap B) \cap C = A \cap (B \leq A$ﷲ

- **Distributive property**

  (a) $A \cup (B \cap C) = (A \cup B) \cup (A \cup C)$

  (b) $A \cap (B \cup C) = (A \cap B) \cup (A \leq C$ﷲ

Material/Resources Required

Chart having Venn diagram, Coloured Chart Papers,

Introduction

Activity 1

Display the chart having venn diagram in the class then discuss the following terms to the students.

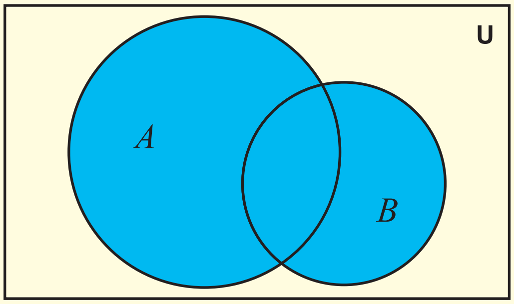
Venn Diagram:

The union, intersection, difference and complement of sets can be depicted graphically by means of Venn diagram. In a Venn diagram the universe U is represented by points within a rectangle and sets A, B, C, etc. are represented by points inside circles within the rectangle. Below, Figure 1 graphically depicts the union A U B of two sets A and B, Figure 2 depicts the intersection A4 B of two sets A and B, Fig. 3 depicts the difference A - B of two sets A and B and Fig. 4 depicts the complement of a set A.

Union of sets

The union of two sets A and B is the set consisting of all elements in A plus all elements in B and is denoted by AUB

Fig. 1



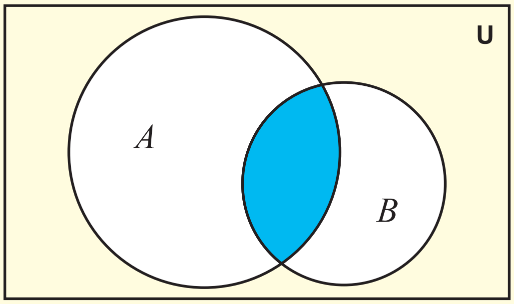
AUB

Example If A = {a, b, c, d} and B = {b, c, e, f, g} then A U B = {a, b, c, d, e, f, g}.

Intersection of sets

The intersection of two sets A and B is the set consisting of all elements that occur in both A and B (i.e. all elements common to both) and is denoted by A4B

Fig. 2



$A \leq B$

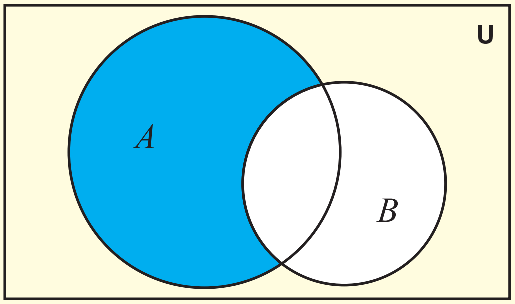
Example

If A = {a, b, c, d} and B = {b, c, e, f, g} then

$A \leq .$ ={b, c}.

Difference of two sets

The set consisting of all elements of a set A that do not belong to a set B is called the difference of A and B and denoted by A - B

Fig. 3


A - B

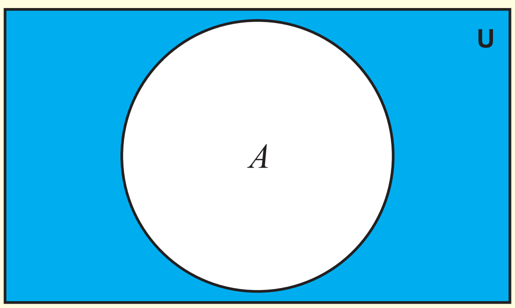
Example

If A = {a, b, c, d} and B = {b, c, e, f, g} then

A - B = {a, b}.

Complement of set

The complement of a set A is all of the elements in the universal set except those in A, and is denoted by A'

Fig. 4


Complement of A

Activity 2

Discuss with students commutative, associative, distributive properties and De morgan's law of union and intersection, and assign them in groups to prove these properties by taking different sets.

Then discuss the results with the whole class.

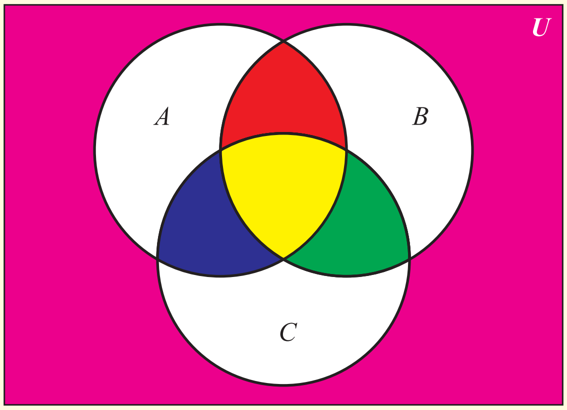
Development

Activity 1

Display the following chart of Venin diagram in the class or draw the same on the board. Explain this diagram to the students.

Ask the students to draw the same diagram on their notebooks. Observe their response and ask them to share with each other.

Ask the students that with the help of this diagram deduce the commutative and associative properties. First demonstrate the idea to the students and let them to reach the maximum solutions as:



- A U B is the whole of the white areas represented by letters A, B and C together with the red, yellow, green and blue areas
- $A \leq .$ is the red area and the yellow area
- B U C is the whole of the white areas represented by the letters B and C, together with the red, yellow, green and blue areas
- $B \leq /$ is the yellow area and the green area
- A U C is the whole of the white areas represented by the letters A and C, together with the red, yellow, green and blue areas
- $A \leq /$ is the yellow area and the blue area
- A U B U C is everything except the magenta area
- $A \cap B \leq /$ is the yellow area

- A^c or A(complement) is the total of the white areas containing the letters B and C, together with the green area and the magenta area
- B^c or B(complement) is the total of the white areas containing the letters A and C, together with the blue area and the magenta area
- C^c or C (complement) is the total of the white areas containing the letters A and B, together with the red area and the magenta area
- $(A \cup B)^c$ is the total of the white area containing the letter C and the magenta area
- $(B \cup C)^c$ is the total of the white area containing the letter A and the magenta area
- $(A \cup C)^c$ is the total of the white area containing the letter B and the magenta area
- $(A \cup B \cup C)^c$ is the magenta area
- $(A \leq B)'$ is everything except the red and yellow areas
- $(B \leq C)'$ is everything except the yellow and green areas
- $(A \leq C)'$ is everything except the yellow and blue areas
- $(A \cap B \leq C)'$ is everything except the yellow area
- $B \cap C \leq A'$ is the yellow area
- $A \cap B \leq C'$ is the yellow area
- $A \cap C \leq B'$ is the yellow area

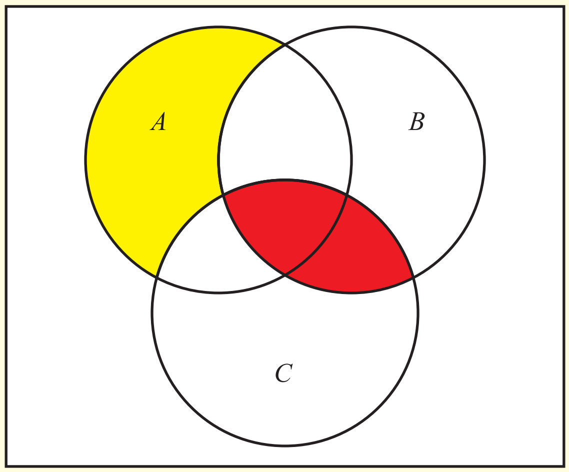
Activity 2

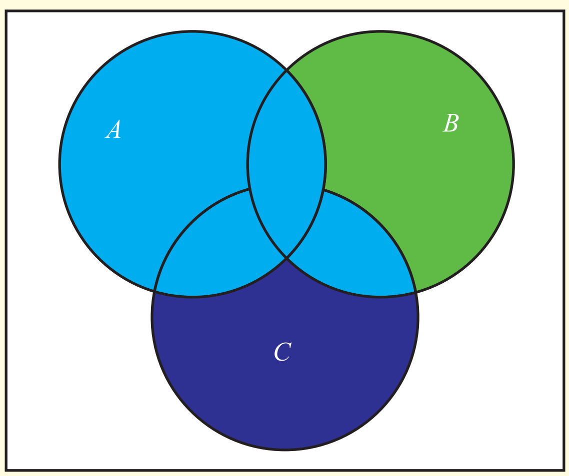
Explain distributive property with the help of following diagram.

Suppose that certain types of people in a given population are assigned to the sets A, B and C. For instance, set A could be the set of males in the population, set B could be the set of people under the age of 21 and set C could be the set of people who drink milk:

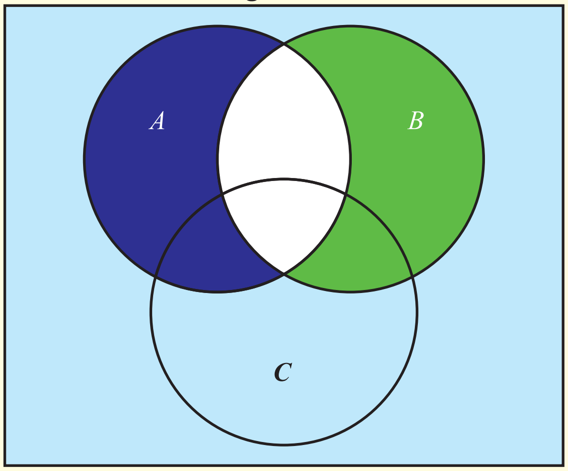
1. What does each of the eight regions in the above Venn diagram represent?
2. Why were none of the sets used for females, people of age 21 or older, or for those who do not drink milk?

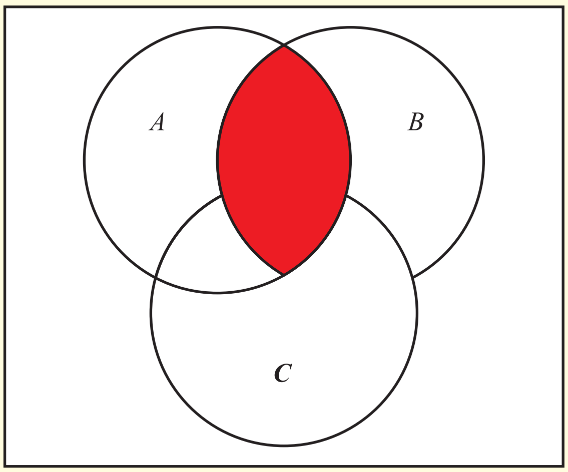
3. We can represent these using Venn diagrams, as shown by our depiction of the first distributive property:



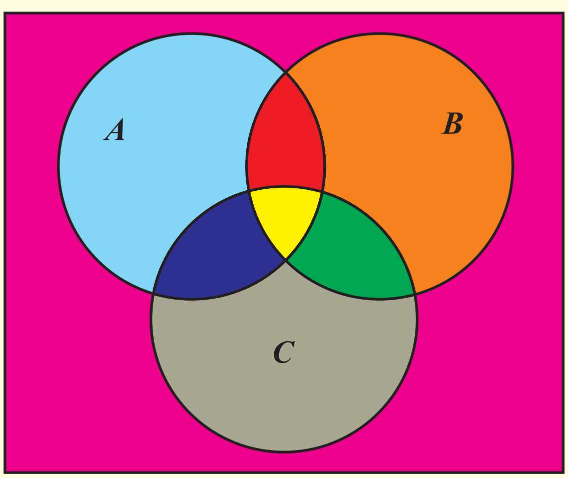


$A \cup (B \leq C$ ﷲs the sum of the yellow and red areas, while $(A \cup B) \cap (A$ C ﷲs the cyan (light blue) area (cyan is the sum of green and blue in the RGB, or Red-Green-Blue, color model) and by our depiction of the second De Morgan's Law:





$(A \leq B)^{/}$ is the white area on the left, while A^{c} U B^{c} is everything except the white area on the right.



yellow	abc
red	$abc^{/}$
blue	$ab^{/}c$
cyan	$ab^{/}c^{/}$
green	$a^{/}bc$
orange	$a^{/}bc^{/}$
grey	$a^{/}b^{/}c$
magenta	$a^{/}b^{/}c^{/}$

Activity 3

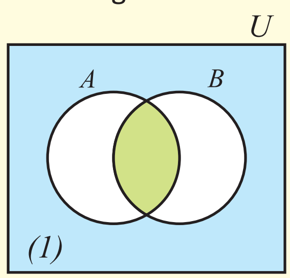
Ask the students to verify the de Morgan's laws by using Venn diagram. After taking their response demonstrate the following on the board:

De Morgan's laws for complementation

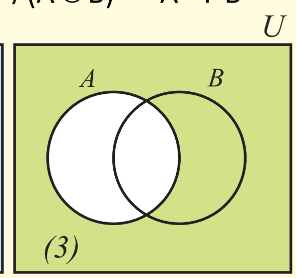
Let U be the universal set containing sets A and B. Then

i) $(A \cup B)^{/} = (A^{/} \cap B^{/})$ ii) $(A \cap B)^{/} = (A^{/}$: $B^{/}$Å

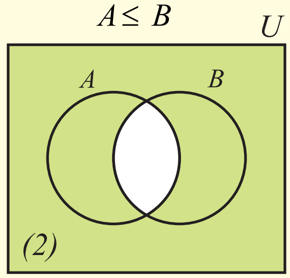
Venn diagrams to verify $(A \cup B)^{/} = A^{/}$ 4 B^{C}



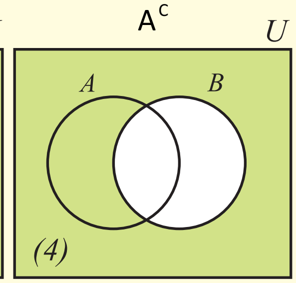
$A \leq B$



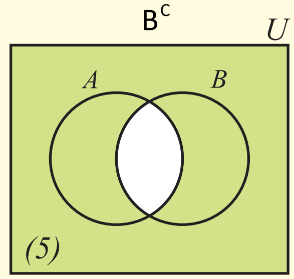
A^{C}



(A 4 B)^{C}



B^{C}



$A^{/}$ 4 B^{C}

From (2) and (5) it follows that $(A \cap B)^{/} = A^{/}$ 4 $B^{/}$

Conclusion/Sum up

Summarize the formation of Venn diagram and step by step procedure of use of Venn diagrams to prove different set operations.

Assessment

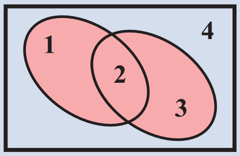
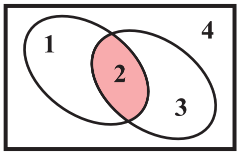
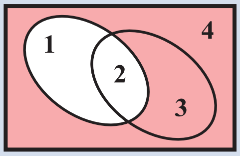
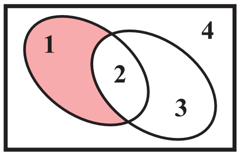
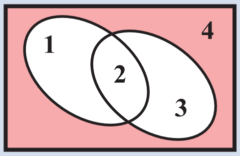
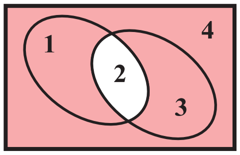
Ask the students to locate all this information appropriately in a Venn diagram.

A = {1,2,3,4}, B = {1,3,5,7}, and C = {7,9,3}, and the universal set U = {1,2,3,4,5,6,7,8,9}.

Follow-up

In group assign class poster to draw. Best presented poster will be displayed in the class.

- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

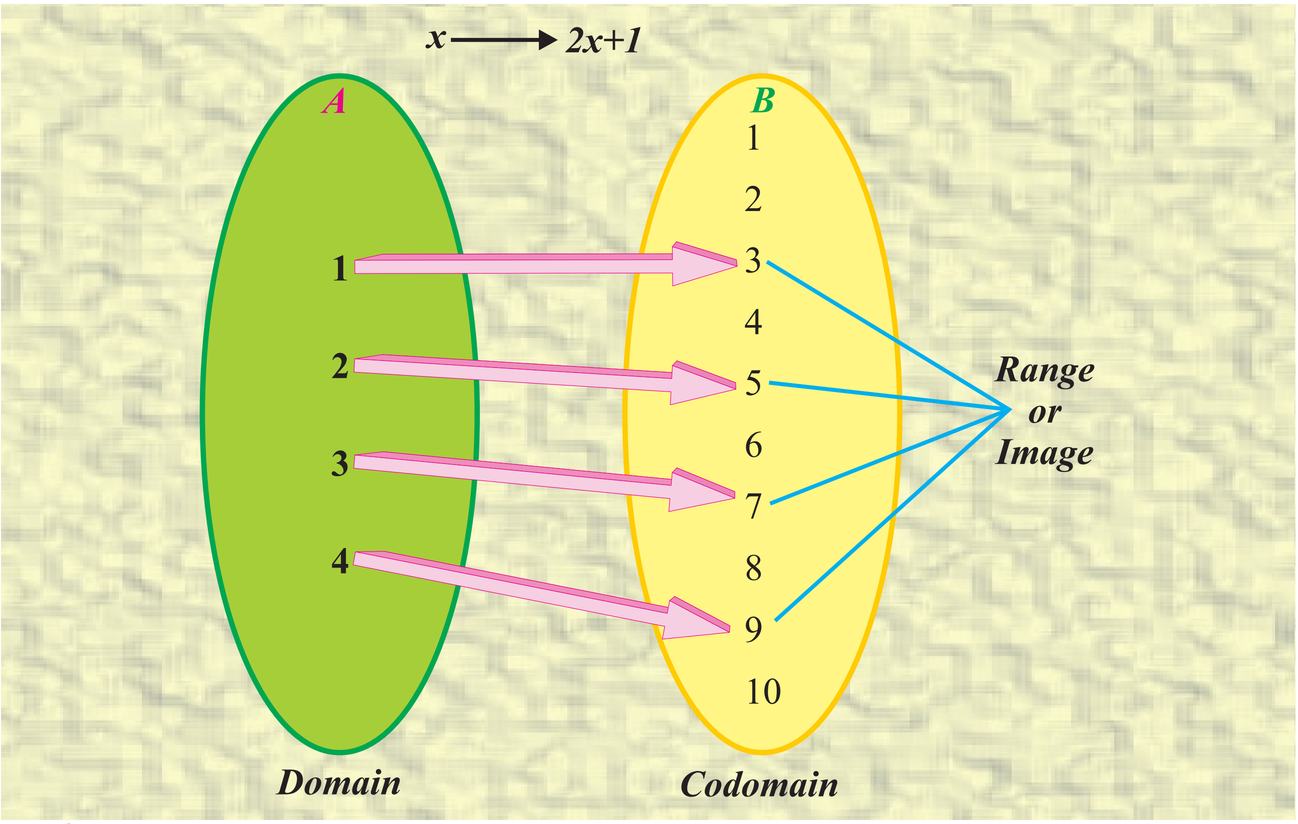
set notation	pronunciation	meaning	Venn diagram	answer
A* U *B	"*A* union *B*"	everything that is in either of the sets		{1, 2, 3}
A* ^ *B **or** A ∩ B	"*A* intersect *B*"	only the things that are in both of the sets		{2}
A^c **or** ~***A***	"*A* complement", or "not *A*"	everything in the universe outside of *A*		{3, 4}
A* – *B	"*A* minus *B*", or "*A* complement *B*"	everything in *A* except for anything in its overlap with *B*		{1}
~(*A* U *B*)	"not (*A* union *B*)"	everything outside *A* and *B*		{4}
~(*A* ^ *B*) **or** **~(** A ∩ B **)**	"not (*A* intersect *B*)"	everything outside of the overlap of *A* and *B*		{1, 3, 4}

Lesson Plan 2

TOPIC

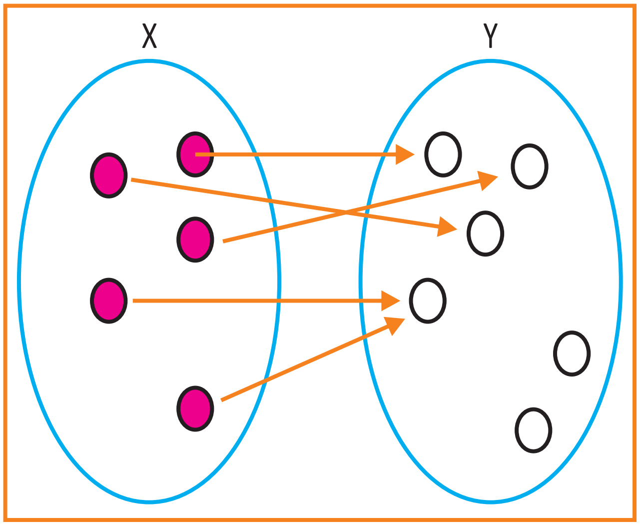
Function

Grade X



Students' Learning Outcomes

- Define function and identify its domain, co-domain and range.



Information for Teachers

- A function relates an input to an output.
- A function relates each element of a set with exactly one element of an other set.
- What can go into a function is called domain

- What may possibly come out of a function is called the co-domain?
- What actually comes out of the function is called the range.

Duration/Number of Periods

80 mins/2period

Material/Resources Required

coloured chart papers, picture of a tree, class posters, number cards, chalk, board

Introduction

Activity 1

- Brainstorm about the term "function"
- Ask following (explanation) example from students after showing picture.
- This tree grows 20 cm every year what will be its height after

a. Five years

b. Ten years

c. Twenty years

- Explain one example by following procedure on the board

***h*(age) = age x 20cm**

So, if the age is 10 years, the height is

$h(10) = 10 \times 20 = 200$cm

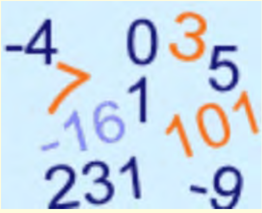
Saying **"*h*(10) = 200"** is like saying 10 is somehow related to 200. Or 10 ---> 200

- The function may not work if you give it the wrong values (such as a negative age), and knowing the values that can come out (such as always positive) can also help.

So we should really say all the values that ***can go into*** and ***come out of a*** function.

Activity 2

- Divide the class in groups and give them the following number Grades:

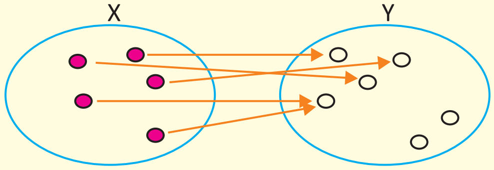


- Ask the students to make different sets
- Groups will present their sets e.g. set of even numbers, set of odd numbers, set of prime numbers and set of positive multiples of 3 or so on.
- Wrap up by explaining 'function is defined in term of sets".

Development

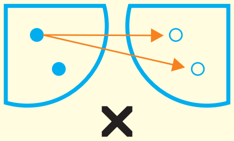
Activity 1

Draw the following figure on the board:

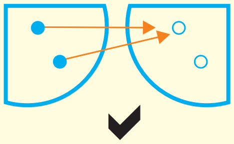


Explain to the students that:

- ○ A function relates each element of a set with exactly one element of another set.
- Ask them to tell about "each element" and "exactly one" in the above definition. Let them to brainstorm and come to answer.
- After taking their response, explain to them that
- ○ "Each element" means that every element in 'x' is related to some element in 'y'. We say that function covers 'x'.
- ○ "exactly one" means that a function is single valued. It will not give back 2 or more results for the same input.
- Enhance their understanding by showing



one-to-many
(This is not function)



may-to-one
(This is function)

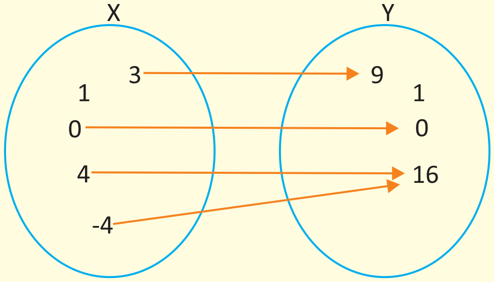
- Ask the students what they can conclude from this result?

Tell them that if a relationship does not follow these two rules, then it is not a function.

Activity 2

Write the following relationship on the board:

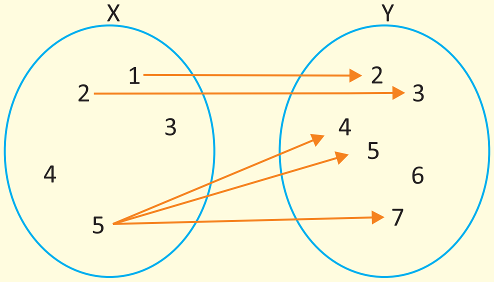
$f(x) \dashrightarrow x^2$



- Can also be written in table

$X = X$	$Y = X^2$
3	9
1	1
0	0
4	16
-4	16

- Ask the students to guess that the relation is a function or not.
- After taking their response, explain them that it is function because:
  - ○ Every element in **x** is related to **y**
  - ○ No element in **x** has two or more relationships.
- Now ask the students to guess for another relation.



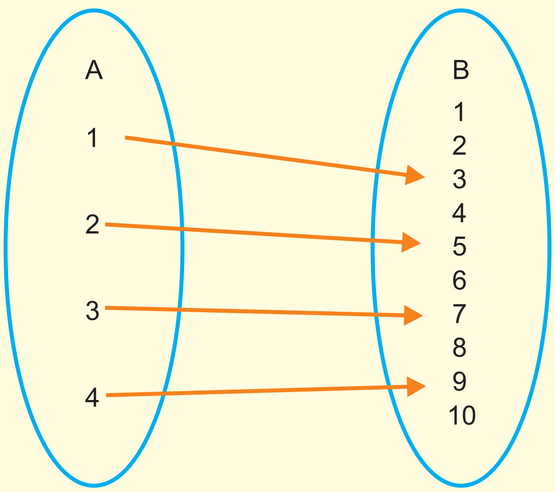
- Let them to reach the answer.
- After taking their response, tell them that
  - ○ 3 in **x** has no relationship in **y**
  - ○ 4 in **x** has no relationship in **y**
  - ○ 5 is related to more than one value in **y.**
- So it is not a function
- Ask the students to solve the problems about function given in their textbooks.

Activity 3

- Identifying domain, codomain and range.

- Write the following on the board:
  - What can go into a function is called **domain.**
  - What may possibly come out of a function is called **codomain.**
  - What actually comes out of a function is called the **range.**
- Draw the following illustration on the board

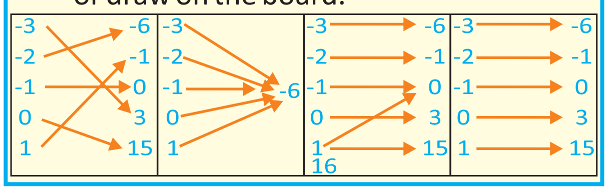
  X ----> 2x + 1



- Ask the students to identify "domain", "codomain" and "range".
- After taking their response, tell them that set 'A' is the domain, set B is the codomain and set of elements that get pointed to in B are range.
  - Domain : {1, 2, 3, 4}
  - Codomain: {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
  - Range: {3, 5, 7, 9}

Activity 4

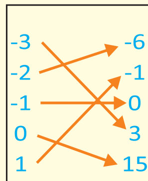
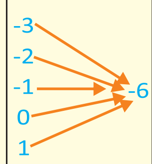
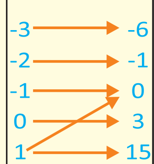
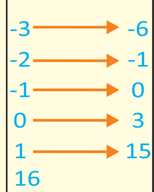
- Paste the chart having following figures or draw on the board.



- Ask the students to observe the figures which are showing **domain** and **range.**
- Ask students "do you think these are function or not.
- Ask them to explain

  "why ?"

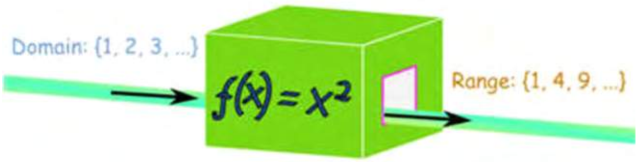
  "why not ?"
- Let the students come up with every possible answer so that confusion can be addressed.
- At the end explain the figures to the students.

<table>
<tr><td>
</td><td>This is a function. You can tell by tracing from each x to each y. there is only one y for each x; there is only one arrow coming from each x</td></tr>
<tr><td>
</td><td>This is a function! There is only one arrow coming from each x; there is only one y for each x. it just so happens that it's always the same y for each x, but it is only that one y. so this is a function; it's just an extremely boring function!</td></tr>
<tr><td>
</td><td>This one is not a function: there are two arrows coming from the number 1; the number 1 is associated with two different range elements. So this is a relation, but it is not a function.</td></tr>
<tr><td>
</td><td>Okay, this one's a trick question. Each element of the domain that has a pair in the range is nicely well-behaved. But what about that 16? It is in the domain, but it has no range element that corresponds to it! This won't work! So then this is not a function. check, it isn't even a relation!</td></tr>
</table>

Conclusion/Sum up

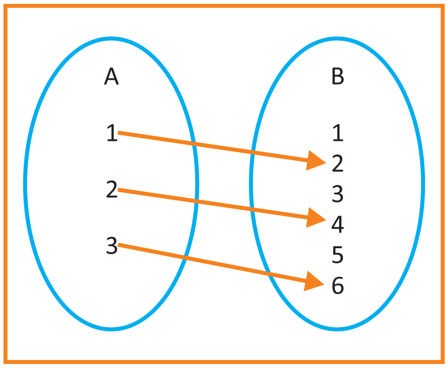
Summarize the following:

- A function must be ***single valued.*** It cannot give back 2 or more results for the same input.
- The domain is an essential part of the function.
- Different domain will produce different function
  - Domain (what goes in)
  - Range (what goes out)
- Share example: a simple function like $f(x) = x^2$ can have the ***domain*** (what goes in) of just the counting numbers {1, 2, 3,}, and the **range** will therefore be the set {1, 4, 9,...}



Assessment

Ask the students to state the domain and range of the following relation. Is the relation a function?



Follow-up

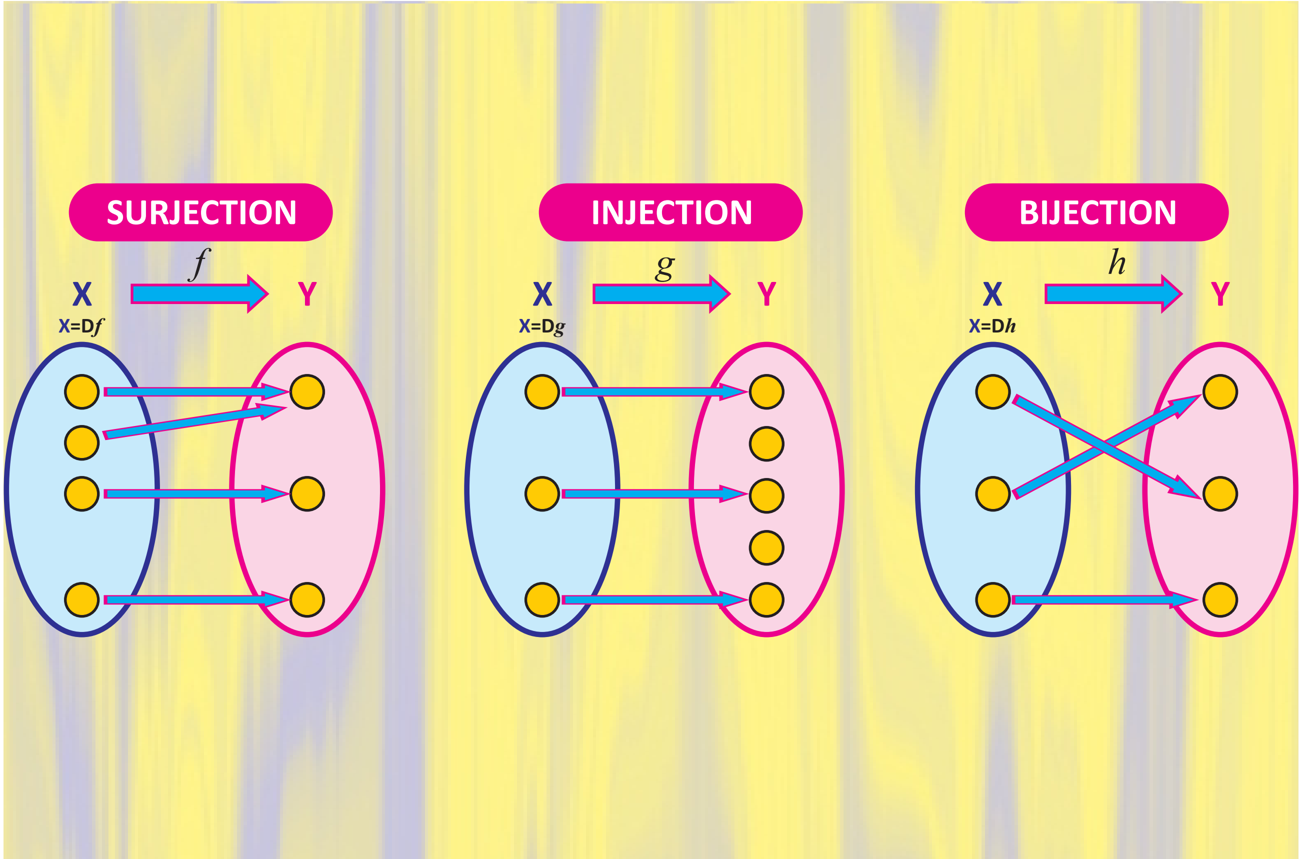
- Ask students to show following with examples:
  a. Function
  b. Domain
  c. Codomain
  d. Range
- Ask students to make their own questions for the above and solve them.
- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

Lesson Plan 3

TOPIC

Function

Grade IX



Students' Learning Outcomes

Demonstrate the following:

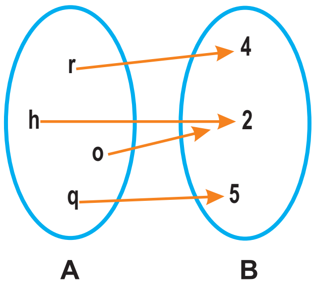
- Into function,
- One - to – one function,
- Into and one –to-one function (Injective function),
- Onto function (surjective function),
- One-to-one and onto function (bijective function).

Information for Teachers

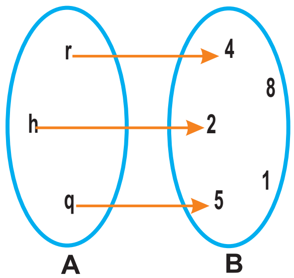
A function f: A⟶B is said to be into function.

Onto Function/Surjective

A function *f* from *A* to *B* is called onto if for all *b* in *B* there is an *'a'* in *A* such that *f (a)=b*. All elements in B are used. Such functions are referred to as surjective.



"Onto"
(all elements in B are used)



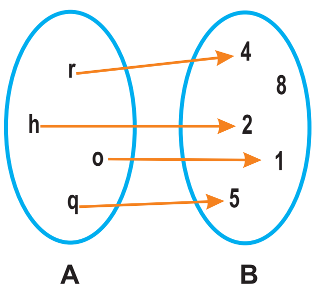
NOT "Onto"
(the 8 and 1 in Set B are not used)

When working in the coordinate plane, the sets A and B become the Real numbers, stated as

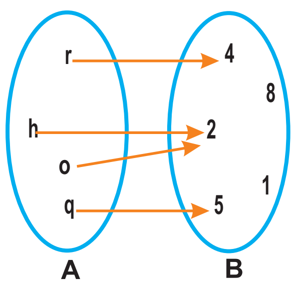
f: R→R

One-to-One Function/Injective

A function *f* from *A* to *B* is called one-to-one (or 1-1) if whenever *f* (a)= *f* (b) then a=b. No element of B is the image of more than one element in A. In a one-to-one function, given any *y* there is only one *x* that can be paired with the given *y*. Such functions are referred to as injective.



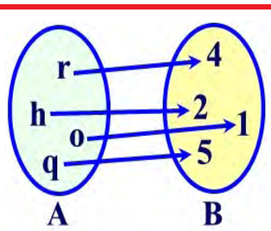
One - to - One



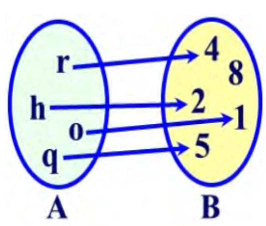
NOT "One - to - One"

Both/Bijective/One-to-one and onto

Functions can be both one-to-one and onto. Such functions are called bijective. Bijections are functions that are both injective and surjective.



"Both"



NOT "Both" - not Onto

Duration/Number of Periods

80 mins/2period

Material/Resources Required

Power point presentation, Posters charts (class posters are attached) Caloured carts / papers / pencils

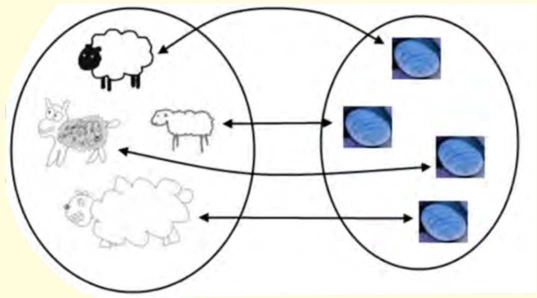
Introduction

Activity 1

- Use Power Point presentation to start.
- Narrate different stories to explain injective, surjective and bijective function.
- Follwoing is one example for reference:

Talking about bijection:

Imagine a shepherd tending a flock of a few dozen sheep. In the morning, the sheep get out of the farm to do sheepish things. In the evening, the shepherd wants to make sure he got all his sheep back. One problem: he can't count. What's a shepherd to do? One solution would be to keep track of the sheep as they go out.



For example, he could throw a pebble into a bucket for each sheep leaving the farm. In the evening, a pebble goes out for each returned sheep. At any given time, the number of pebbles in the bucket is exactly equal to that of outstanding sheep. You can picture the sets of sheep and pebbles like so: Each sheep is paired with a distinct pebble and there are no left overs on either side. In set theory this is a bijection.

Activity 2

- First of all only draw the following (Fig.1, Fig.2, Fig.3) on the board.
- Ask the students to observe the figures and let them come up with possible solutions.
- At the end explain the figures to the students as follows.

i. **Onto (Surjective) function:** if a function f:**A**$\to$ **B** is such that Ran ***f*** **= B** i.e., every element of **B** is the image of some elements of **A**, then f is called an **onto** function or a surjective function.

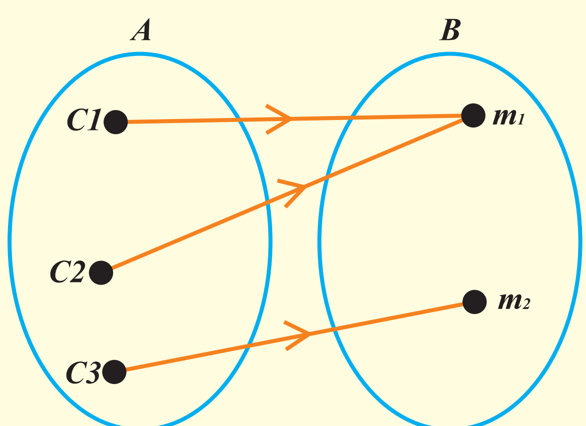


Fig (1)

$f = \{(c_1,m_1),(c_2,m_1),(c_3,m_2)\}$

ii. **(1-1) and into (Injective) function:** If a function ***f*** from **A** into **B** is such that second elements of no two of its ordered pairs are equal, then it is called an injective (1-1, and into) function. The function shown in fig (2) is such a function.

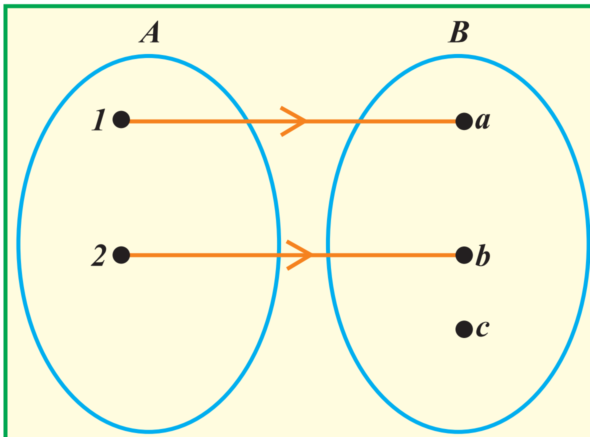


Fig (2)

$f = \{(1,a),(2,b)\}$

iii. **(1-1) and Onto function (bijective function):** If ***f*** is a function from **A** onto **B** such that second elements of no two of its ordered pairs are the same, then ***f*** is said to be (1-1) function from **A** onto **B** such a function is also called a (1-1) correspondence between A and B. It is also called a bijective function. Fig(3) shows a (1-1) correspondence between the sets **A** and **B**.

(a, z), (b, x) and (c, y) are the pairs of corresponding elements i.e, in this case f={(*a, z*), (*b, x*), (*c, y*)} which is a bijective function or (1-1) correspondence between the sets **A** and **B**.

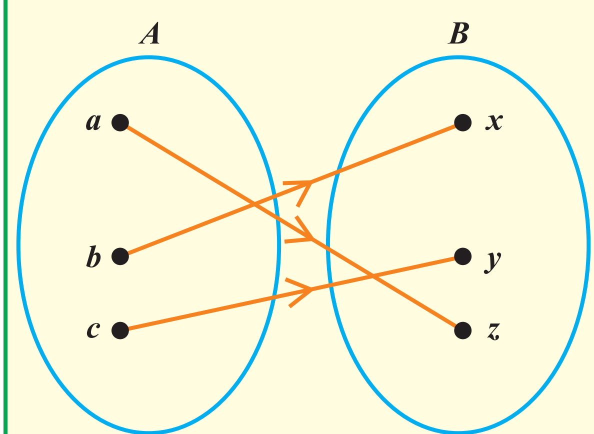


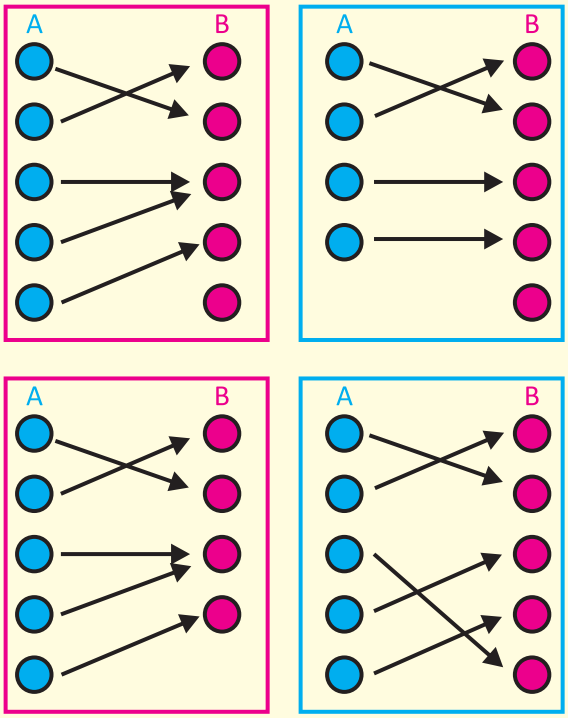
Fig (3)

$f = \{(a,z),(b,x),(c,y)\}$

Development

Activity 1

Draw following on the board:



- Ask group to
  - a. Figure out what are these?
  - b. What does each relation shows
  - c. Explain general function
  - d. Reason out to prove them injective, surjective and bijective
- Groups to present their work for whole class followed by questions/answers session to clarify concepts.

Activity 2

- The teacher poses the following problem:

  My uncle, who is a rancher, needs me to feed one carrot to each of his llamas. Now the thing is, I don't know how to count, and I'm not sure if I have enough carrots to feed each of the llamas. How can I know if I have more carrots or llamas without counting?
- Ask groups to work for the given:

Tasks

Group 1:

Aim: To motivate the definition of a function

Goal: To feed (associate) one carrot to one llama

Question: Could we have fed the same carrot to more than one llama?

Group 2:

Aim: To motivate the definition of injectivity (one-to-oneness)

Question: Could we have fed one llama more than one carrot? Would we be able to tell of which we had more that way?

Group 3:

Aim: To establish a concept of size of a set

Goal: To look for left over after an injective pairing.

Question: After I am done feeding each carrot to a distinct llama, how will I know if I had more carrots or llamas? What if there are exactly the same number of carrots and llamas?

Group 4:

Aim: To motivate the definition of surjectivity.

Question: Can someone draw me a picture for the situation when there are more carrots than there are llamas? What do we notice? Can the rule be one-to one? Could we have a situation where every llama is feed exactly one carrot? What would that mean about the number of carrots and the number of llamas?

Group: 5

Aim: To motivate the definition of bijectivity

Goal: There is a function that is one-to-one and onto if and only if two sets is of the same size.

Question: So, how can we prove that two sets are the same size? Think about the carrots and llamas. What will happen if we have exactly the same number of carrots as llamas?

Groups 6

Aim: To establish the relationship greater (less) than or equal to

Question: How can we show that one collection of things is greater than or equal to another group of things?

Examples: How can we prove that any natural number is greater than or equal to zero? How can we show that any number is equal to itself? Is there more than one way to do it?

Group 7:

Aim: To count the number of function from a set into itself.

Question: How many functions are there from one set to itself? How many bijection are there?

Conclusion/Sum up

Sum up the lesson by recaping the following:

- Bijections are the function that are both surjective and injective
- Defining injective and sujective function

Assessment

- Draw the following on the board and ask the students to represent functions and of which type?

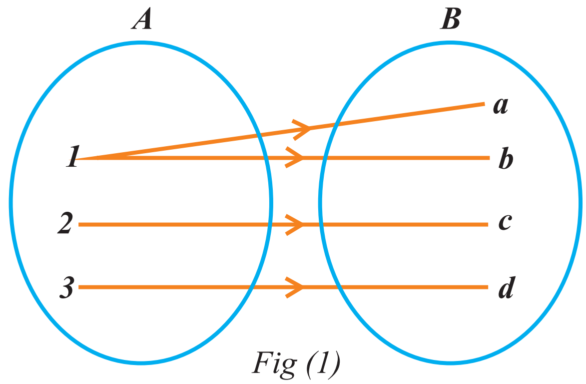


Fig (1)

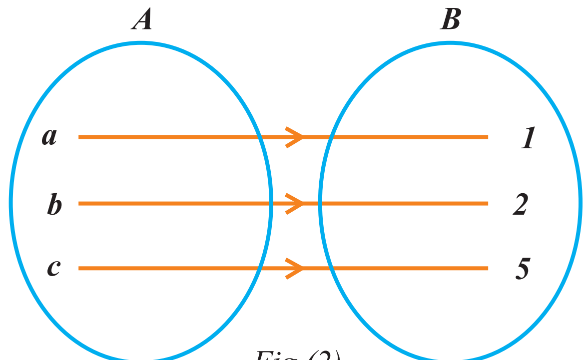


Fig (2)

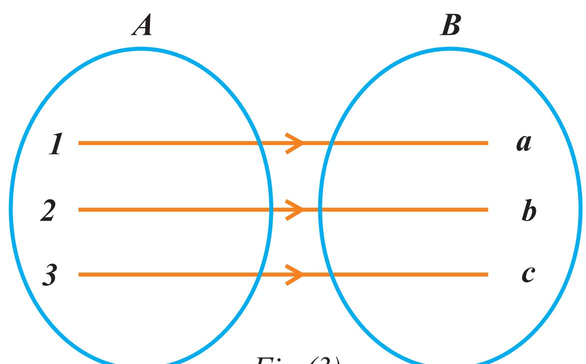


Fig (3)

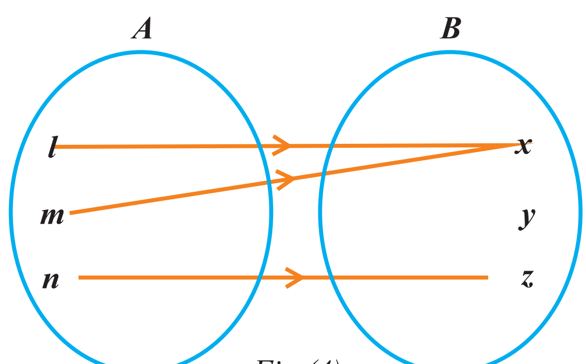


Fig (4)

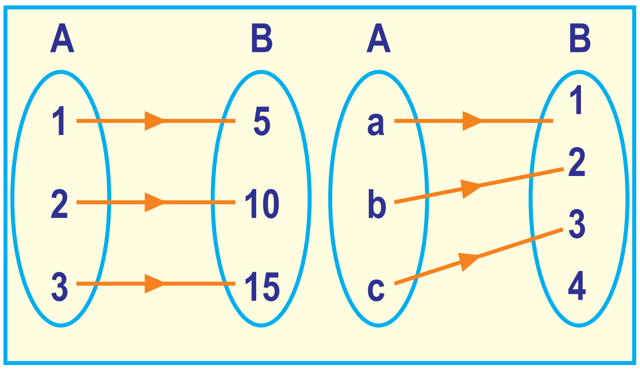
Follow-up

- Find he inverse of each of the following relations. Tell whether each relation and its inverse is a function or not:
  - i. $\{(2,1),(3,2),(4,3),(5,4),(6,5)\}$
  - ii. $\{(1,3),(2,5),(3,7),(4,9),(5,11)\}$
  - iii. $\{(x,y)y = 2x + 3, x \in \exists\ \}$
  - iv. $\{(x,y)y^2 = 4ax, x\ \ 0\}$
  - v. $\{(x,y)x^2 + y^2 = 9x \le 3, y\ \ \}$

Making posters of type of functions.

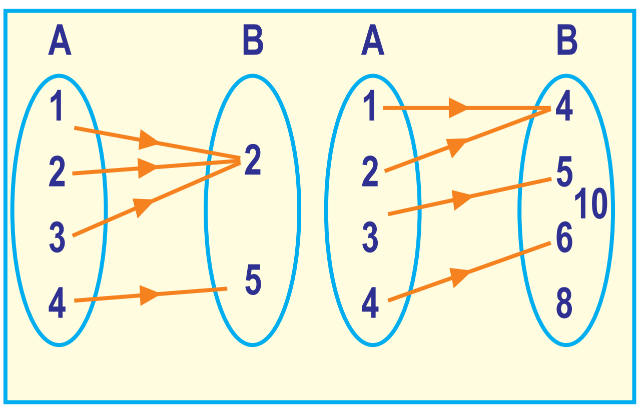
One-one function

There is one-one correspondence between the elements of the set A and the set B



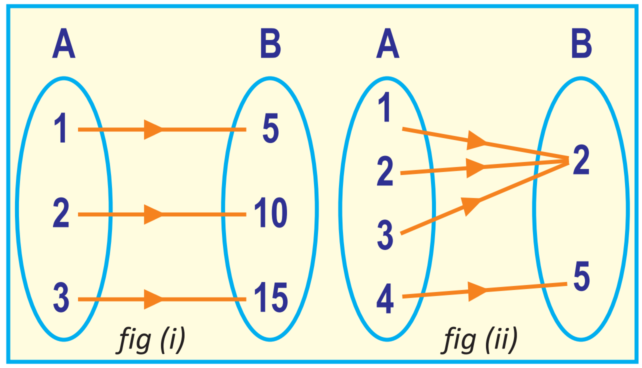
Many-one function

There is many-one correspondence between the elements of the set A and the set B.



Onto function

Every element of the set B has at least one pre-image.

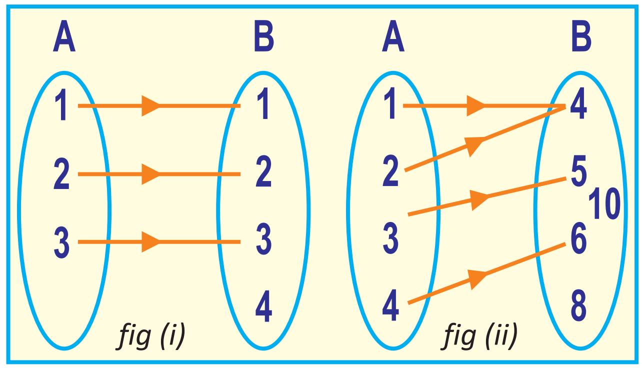


Note:

In above fig. (i), the function is one-one and onto, while in fig. (ii) the function is many-one and onto

Into function

There is at least one element of B which has no pre-image.



Note:

In above fig. (i) the function is one-one and into, while in fig. (ii) The functionis many-one and into.

Note:

For types of functions, the four arrow diagrams given for one-one and many-one are repeated for ONTO and INTO functions because each function is always one-one onto or one-one into; many-one onto or many-one into.

- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

UNIT 13

Lesson Plan 1

TOPIC

Frequency Distribution - I

Basic Statistics

Grade X

2	4	1	9	5	2	1	3	6	2	2	10	6	4	11	12	9
9	12	7	2	8	12	1	3	5	1	6	12	8	7	12	12	1
11	10	8	5	7	3	5	8	9	9	8	10	9	6	5	5	7
10	3	1	3	10	1	11	5	4	9	5	5	11	10	2	8	1
6	9	1	3	12	3	3	2	4	6	1	4	10	12	1	11	2
10	4	5	6	5	1	2	7	12	11	5	11	12	9	6	11	7
12	5	9	9	12	1	5	11	12	11	11	5	2	4	1	8	8
5	12	12	10	7	8	5	1	3	8	1	8	12	11	1	9	8
11	1	9	5	6	6	10	6	12	9	7	6	9	10	3	1	12
11	6	11	9	2	7	5	5	4	11	12	12	3	8	12	8	10

0/97

Students' Learning Outcomes

- Construct grouped frequency table.

Information for Teacher

- In statistics sometime it becomes difficult to handle a large amount of data as a common man fails to understand the nature of data. In order to make its presentable, a frequency table is constructed.

Groups	Frequencies

Since these concepts they have already done in grade 8 , for some students it will be a refresher and for others an opportunity to learn from peers.

- The grouped frequency table is a statistic method to organize and simplify a large set of data in to smaller "groups."
- The main purpose of the grouped frequency table is to find out how often each value occurred within each group of the entire data.
- Class intervals are non overlapping intervals selected in such a way that every value can be placed in one and only one of the intervals.

How To Construct Grouped Frequency Table :

1. Collect the data by writing it on a piece of paper.
2. Rearrange the data in ascending order.
3. Find Range: Deduct lowest value from the highest value.
4. Determine the number of groups. Most of the data has between five to 15 groups. It is your decision to choose the number of groups for your data.
5. Determine the width (number of values per group) of group interval by dividing step3 (Range) by step 4 value.
6. Create two columns titled "Groups." And "Frequency."
7. Determine the frequencies for all five groups by tallying the data.

Note: chose the first class interval such that the smallest observation is included.

Duration/Number of Period

40 mins / 1 Period

Material/Resources required

Black/white board, marker, chalk, handout photocopies as per number of groups.

Introduction

Activity

- Ask to the students that if we are to classify the group of 14-year old secondary pupil according to their heights, then we will get a frequency distribution that is representative of all the pupils in the group with a minority that are extremely tall or short while the majority are of average height. Will the frequency distribution of their weights also follow this pattern?
- Help them to recall the steps to create a grouped frequency table.
- Now divide the class in two groups and ask them to construct grouped frequency tables of heights and weights.
- Ask the group I and group II to gather data about the height and weight of the students of your class respectively.

Development

Activity

Exercise the following with the whole class. **"Getting Ready for School"** / "Reaching School"

1. Gather your Data:

   How much time (minutes) do you take in getting ready for your school? Ask them to write on a notebook like this Mohammad Ali 30 minutes. Collect responses from approximately 20 students in the form of minutes only. Like 30 min, 10 min, 12 min, 15 min. and so on.

2. Decide how many classes will be in your frequency table:

   A class is a range of numbers within which data falls. For example , students getting ready for school may be divided into 5-groups.

   5 to 15 minutes, 16 to 26 minutes, 27 to 37 minutes, 38 to 48 minutes and 49 minutes or more. Class width: 10

3. Count the number of data points which falls into each class. You may use tally method

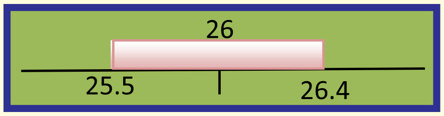
5 to 15 m inutes	6
16 to 26 minutes	3
27 to 37 minutes	7
38 to 48 minutes	3
49 minutes or more	1

4. Construct your table. List your classes in a column on the left side of a piece of paper.
   For each class, write the total number of data points that fell within that class to the right of the class name.
5. Create a title for your frequency table that accurately describes the data as written above.

Number of minutes spent in 'getting ready for school'	
Minutes	Number of students
5 to 15 minutes	6
16 to 26 minutes	3
27 to 37 minutes	7
38 to 48 minutes	3
49 minutes or more	1
Total	20

- Divide the class into mixed ability groups and assign questions to survey within the class/sections and represent on frequency tables. For example conduct a survey about **how many have cell phones and how many don't have, or what brand of cell phone do they use?**

  Or even about their favorite food.....
- Ask them to record their frequency tables so that further applications can be done.
- Now ask the students to OBSERVE the table they got after survey and check if the class interval they have is of inclusive (11-14, 16-19, 21-24, 26-29) or exclusive type (10-15, 15-20, 20-25, 25-30).



- Most of them will find the first type.
- Refer to the example done on board about getting ready for school and ask **"I take 15 and a half minute to get ready for school, where will you put me? ". Also tell that If a fellow says that I spent 26 minutes ; there are chances that he actually takes time between (25.5 to 26.4) minutes.**
- Draw the lines as shown to help them visualize the range.
- Now introduce class boundaries and their meanings.
- For example: class boundary (15.5-26.5) means **15.5 x < 26.5. Where x is any value.**
- Ask them to refer to the books distributed by you and read for them and have the groups understanding developed on the topic. Let them read other examples from the book or text book.
- Have group discussion on how to improve their tables for proper class boundaries.
- Encourage them to finalize their frequency tables with an additional column of boundaries.

Conclusion/Sum up

Explain the importance of construction of frequency table. Also explain some other types of frequency table.

Assessment

Ask few questions to know whether students have learned about frequency table.

- What is frequency?
- What is class / groups?
- What is mid point?
- How much number of classes are suitable to handle the data?

Follow-up

- Ask students to prepare frequency table after having collection of daily sale from shopkeepers or from your school canteen.
- Using the data below, construct the frequency table

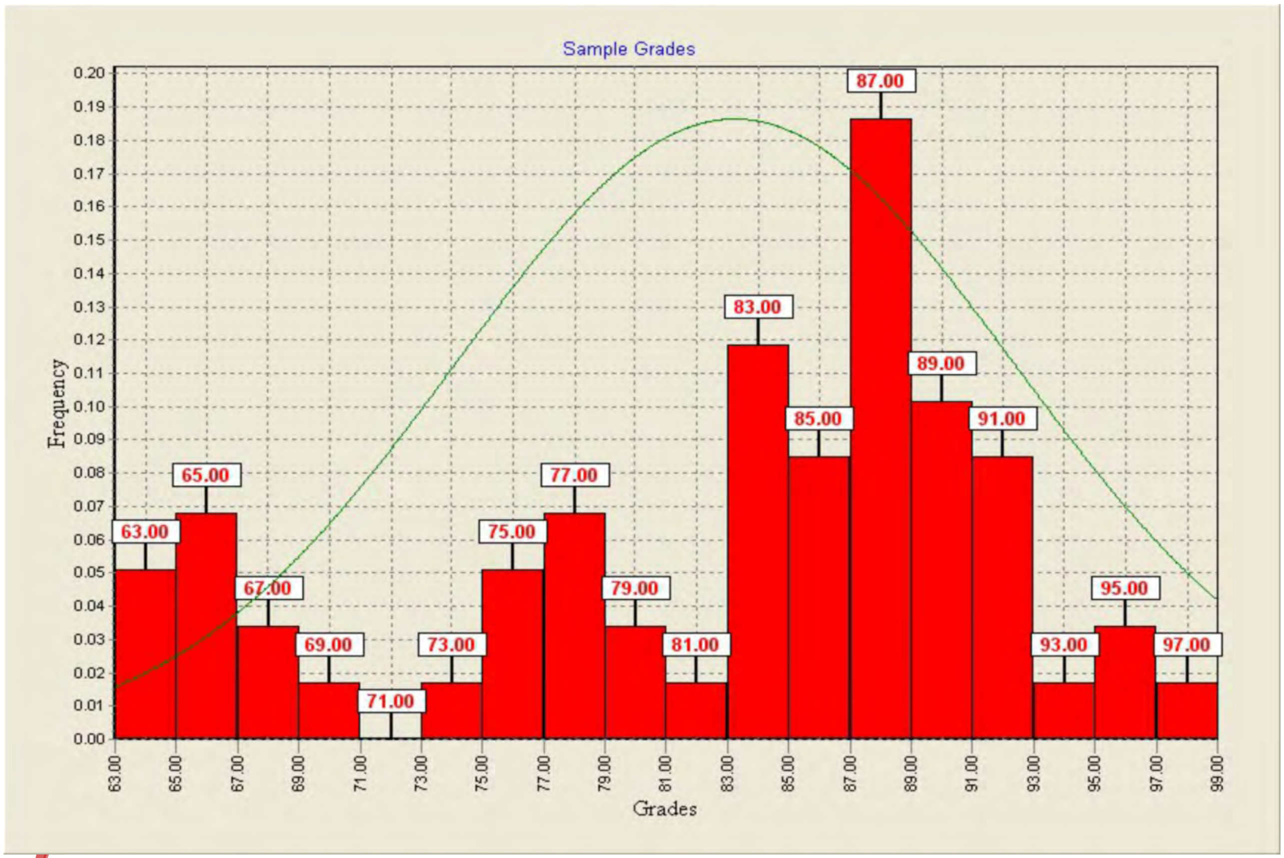
  3, 6,14, 27, 37, 11, 39, 15, 28, 12, 38, 19, 13, 33, 12, 26, 16, 37, 40, 11, 30, 32, 44, 40, 39, 38, 37, 16, 13, 36, 28.
- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

Lesson Plan 2

TOPIC

Frequency Distribution - II

Grade X



Students' Learning Outcomes

- Construct Histograms with equal and unequal class intervals.

Information for Teacher

- A two dimensional frequency density diagram is called a histogram. A histogram is a diagram which represents the class interval and frequency in the form of a rectangle. There will be as many adjoining rectangles as there are class intervals.
- Unequal class intervals are used when there are one or two extremely small or extremely large values.
- Height of rectangle x class width = class frequency [proportional to area of rectangle]?
- Frequency density = class frequency/ class width
- Steps to Draw Histogram of Unequal Class

Interval:

- Choose a suitable scale on the x-axis and represent the class-limits on it
- Determine a class interval which has the minimum class size. Let the minimum class size be h
- Find the adjusted frequency of each class by using the formula :
- Choose suitable scale on the y-axis and represent the corresponding adjusted frequencies on it
- Draw the rectangles , the width of rectangles will be according to class limit

If you're creating a histogram from unequal intervals, you should probably normalize the results. Divide the number by the size of the interval. That way, each group is appropriately weighted.

Duration/Number of Periods

80 mins/2period

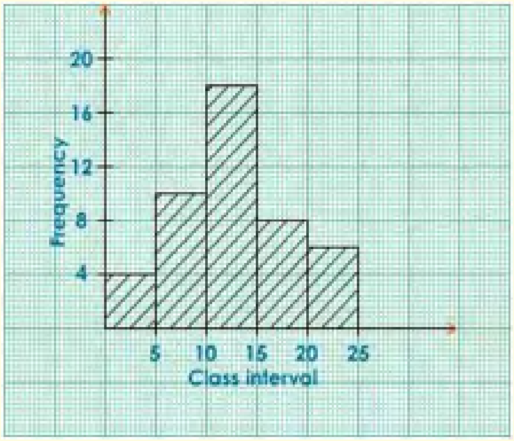
Material/Resources Required

Black/white board, chalk, marker, strips of equal size, handout photocopies as per groups, chart paper made histogram on it. It is better if this lesson is done with the help of I.T Teacher and with the help of computer, library books.

Introduction

Activity 1

- Show them the chart-paper having an example of Histogram on it. Without much explanation.
- Ask them relevant questions.



- Refer to the home work given and collect verbal feedback. Give away the comparison after receiving their responses.

Comparison of Histogram and Bar Graph

Histogram	Bar Graph
1. It consists of rectangles touching each other.	1. It consists of rectangles, normally separated from each other with equal space.
2. The frequency is represented by the area of each rectangle.	2. The frequency is represented by height. The width has no significance.
3. It is two dimensional (width and height are considered)	3. It is one dimensional (only height is considered)
4. It is used as a visual to represent the relationship between class intervals as well as frequency.	4. It is used as a visual aid to represent data relationship w.r.t data kept on width.

Activity 2

- A frequency table / distribution represents the data in a compressed and tabular form Although it is fairly good technique of compressing the data yet the necessity of representing them graphically is felt so as to give a good visual aids. There are two types of frequency table i.e discrete and continuous. The graph drawn for the discrete table is in the shape of bar frequency diagram and continuous table is in the shape of curves.
- Students will work in groups and with the same frequency table they created in the last lesson.
- Ask them to read from the library books provided and draw Histogram for their survey question. Produce it on a bigger graph paper or even chart paper for a gallery display.
- Visit each group and track their understanding as per the group's need.
- Display the work and ask students to take a round and see how every group has attempted it.
- Teacher may give remarks to appreciate the efforts and CORRECT the mistakes then and there to make it a permanent in students mind. Do not rely on just pointing the mistakes. We are to correct not criticise!
- Now they know how to draw a histogram

Development

Activity

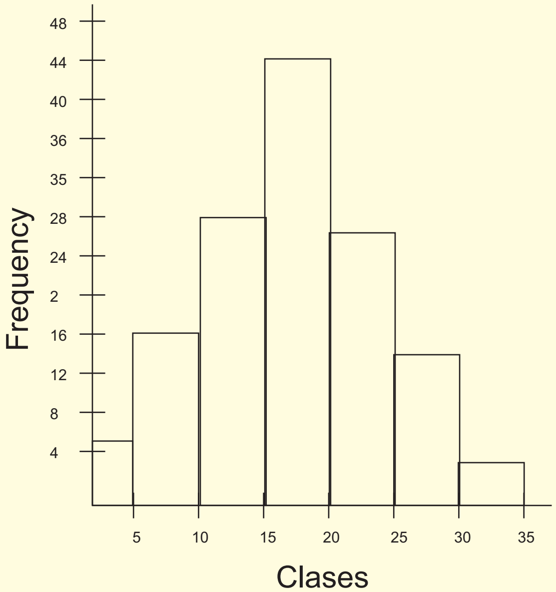
Divide the class into 4 or 5 groups and give them a frequency table and ask them to draw Histogram. When students complete their task check their work then itself draw **Histogram** for both types of data (one-by-one) and explain the process as follows.

<u>Histogram for equal class intervals</u>

Taking the class interval on the horizontal scale and frequencies along vertical scale, plot the data and resulting diagram will be histogram

Data:

Marks	No of students
0-5	5
5-10	10
10-15	30
15-20	45
20-25	27
25-30	14
30-35	3

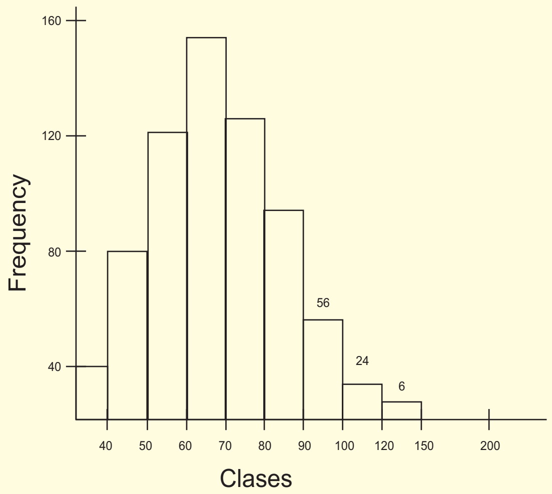


<u>Histogram for unequal class intervals</u>

Taking the class intervals on horizontal scale and frequency on particle scale, plot the data on vertical scale. Class interval for class 7, 8 and 9 are unequal and their corresponding frequencies will be adjusted as ½ x 112 = 56, 1/3 x 72 = 24 and 1/5x30 – 6 because the basis of rectangle corresponding them are ½, 1/3, 1/5 times those equal intervals of magnitude thus (proportional to the width).

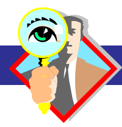
Data:

Marks	No of Students	Reset Frequency
40-50	36	
50-60	87	
60-70	121	
70-80	154	
80-90	133	
90-100	95	
100-120	112	
120-150	72	
150-200	32	



Conclusion/Sum up

Recap the definition and steps of construction of histogram of equal and unequal class intervals.



Assessment

1) Using the data below, complete the

frequency table.
Data: 30, 32, 11, 14, 40, 37, 16, 26, 12, 33, 19, 38, 12, 28, 15, 39, 11, 37, 17, 27, 14, 36

Number	Tally	Frequency
11-15		
16-20		
21-25		
26-30		
31-35		
36-40		

2) The test scores for 10 students in mathematics were 61, 67, 81, 83, 87, 88, 89, 90, 98 and 100. Which frequency table is accurate for this set of data?

A)

Interval	Frequency
61-70	2
71-80	2
81-90	8
91-100	10

B)

Interval	Frequency
61-70	2
71-80	0
81-90	6
91-100	2

C)

Interval	Frequency
61-70	2
71-80	2
81-90	7
91-100	10

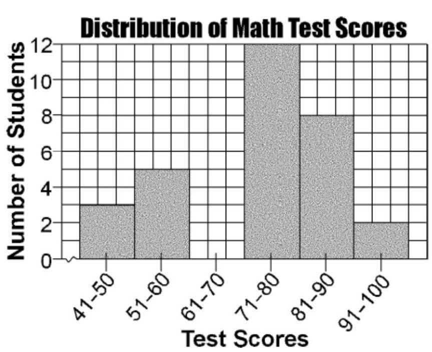
D)

Interval	Frequency
61-70	2
71-80	0
81-90	8
91-100	10

Follow-up

Solve the following questions.

1) The graph below shows the distribution of scores of 30 students on a mathematics test.

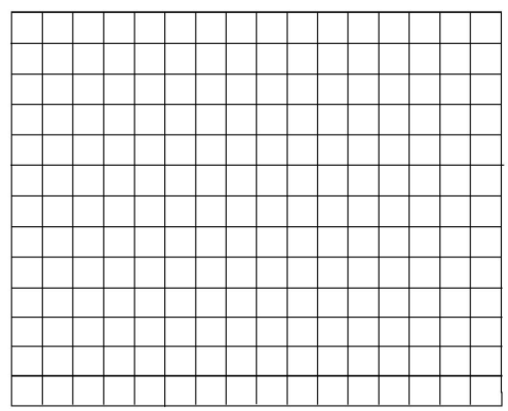


Complete the frequency table below using the data in the frequency histogram shown.

Test Scores	Frequency
91-100	
81-90	
71-80	
61-70	
51-60	
41-50	

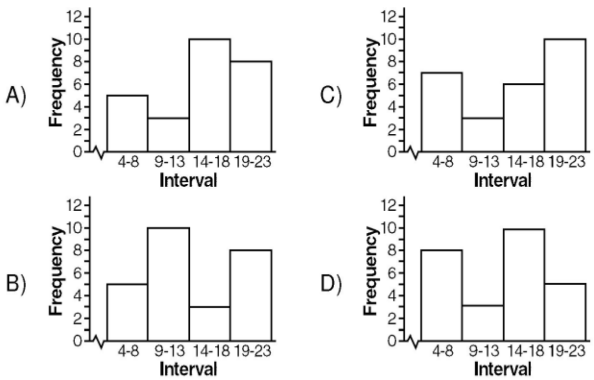
2) The scores on a mathematics test were 70, 55, 61, 80, 85, 72, 65, 40, 74, 68 and 84. Complete the accompanying table, and use the table to construct a frequency histogram for these scores.

Score	Tally	Frequency
40-49		
50-59		
60-69		
70-79		
80-89		



3) Which one of the following histograms represents the data in the table below?

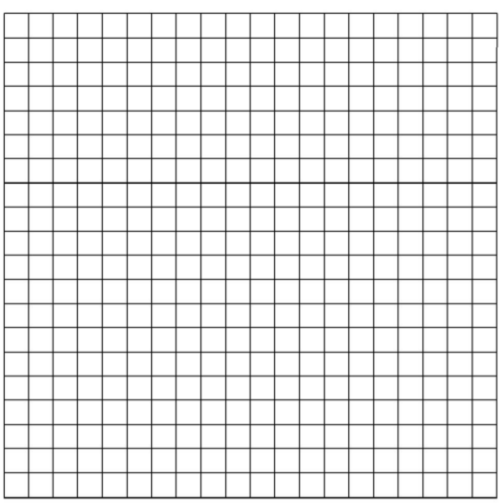
Interval	Frequency
4-8	8
9-13	3
14-18	10
19-23	5



4) The following data consists of the weights, in pounds, of 24 high school students: 195, 206, 100, 98, 150, 210, 195, 106, 195, 108, 180, 212, 104, 195, 100, 216, 99, 206, 116, 142, 100, 135, 98, 160.

Using this data, complete the accompanying cumulative frequency table and construct a cumulative frequency histogram on the grid below.

Interval	Frequency	Cumulative Frequency
51-100		
101-150		
151-200		
201-250		



- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

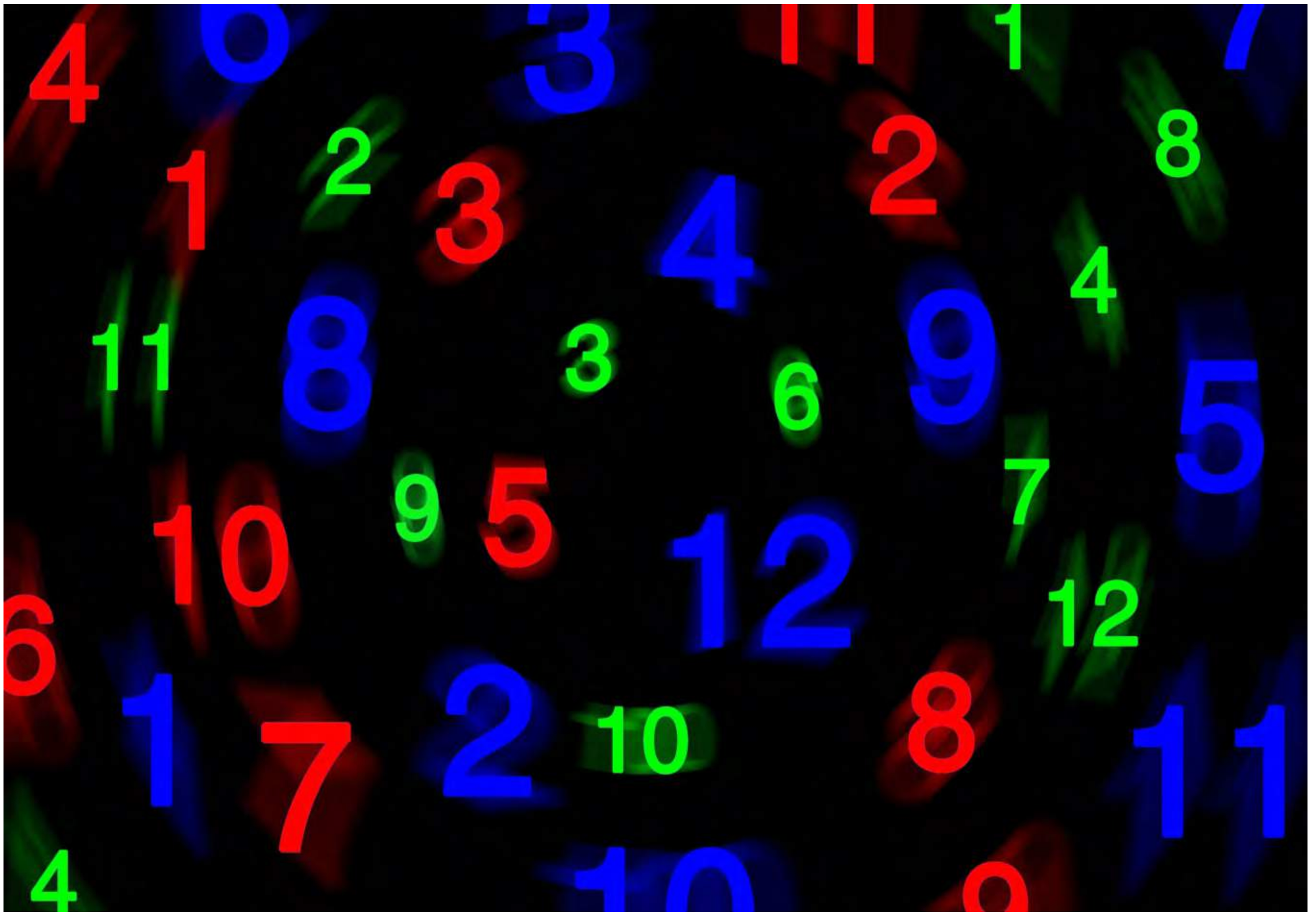
Lesson Plan 3

TOPIC

Measures of Central Tendency - I

Basic Statistics

Grade x



Students' Learning Outcomes

- Calculate (for ungrouped and grouped data):
  - Arithmetic mean by definition and using deviations from assumed mean.
  - Median, mode, geometric mean, harmonic mean.
- Recognize properties of arithmetic mean.

Information for Teacher

- **Average:** Average is a single value which represent the data as a whole. It is also called measure of central tendency as it measures center of the data. It is better to avoid this because sometimes it is vague term. It usually refers to the (arithmetic) mean, but it can also signify the median, the mode, the geometric mean, and weighted means, among other things

- **Arithmetic mean (AM):** The sum of a list of numbers, divided by the total number of numbers in the list is called Average. Also

$$n\,\overline{)\,\textit{sum of n numbers}}^{\;\textit{arithmetic mean}}$$

called **arithmetic mean**.

- Merits of arithmetic mean
  1.) It is rigidly defined.
  2.) It is based on all the values.
  3.) It is more stable than any other average.
- Demerits of arithmetic mean
  1.) It is highly affected by abnormal values.
  2.) The loss of even a single observation makes it impossible to compute the arithmetic mean correctly.

- **Median:**

"Middle value" of a list. If the list has an odd number of entries, the median is the middle entry in the list after sorting the list into increasing order. If the list has an even number of entries, the median is equal to the sum of the two middle (after sorting) numbers divided by two.

- **Mode:**

For lists, the mode is the most common (frequent) value. A list can have more than one mode.

- **Properties of AM:**

- If $\overline{x}_1$ and $\overline{x}_2$ are the means of the two groups computed from the values n_1 and n_2 then the mean $\overline{x}$ is given by the formula $\overline{x} = n_1\overline{x}_1+n_2\overline{x}_2/\ n_1+n_2$
- If each observation in the data is replaced by x, the sum of all the observations remains unchanged.

$x = x_1,x_2,x_3,x_4........,x_n/n$

So $x_1,x_2,x_3,x_4........,x_n = nx$

Replacing each observation by x, we get

$x+x+x........+x = nx$

- If every value of the variable x is either increased, decreased, divided or multiplied by a constant, the observations so obtained also increases, decreases, gets multiplied or gets divided respectively by the same constant.
- Algebraic sum of the deviation of a set of values from their arithmetic mean is **0**

- **For a histogram:**

- The median can be estimated from a histogram by finding the smallest number such that the area under the histogram to the left of that number is 50%.
- For histograms, a mode is a relative maximum ("bump"). A data set has no mode when all the numbers appear in the data with the same frequency. A data set has multiple modes when two or more values appear with the same frequency.

- **Objective of a statistical average:**

- The main objective of the average is to give a bird's eye view of the data. The averages removes all the unnecessary details of the data and give a concise picture of the huge data under investigation. Average is also of great use for the purpose of comparison and for the further analysis of data.

Duration/Number of Period

40 mins/2 period

Material/Resources required

board, chalk/marker, weight machine, inch tape, worksheet, PEC exams marks sheet of 8th class

Introduction

- **Discuss some applications of statistics from real life:**
  - Sports fans compare batting averages, wins and losses of various teams.
  - We try to predict the outcome of elections by analyzing the results of previous political polls that have been taken.
  - School children are always concerned about their socialistic averages
  - The conclusions like for example 30% of people are not issued 'Awami cards' etc. or the disease has struck for example in 45% of new born children.
- Besides being used by **scientists and biologists,** geometric means are also used in many other fields, most notably financial reporting.

Activity 1

- Give the attached work sheet to the groups for ice breaking. Ask their prior knowledge about mean, median and mode.

Activity 2

- Measure the weight of each student in the class. Find the average weight of the students in the class.
- The average (AM) of your last term-end exams.

Development

Activity 1

students will analyze the class survey results.

- Ask the students to take out the record of Measure the height of each student in the class. Each group must have 20 to 25 students' heights.
- Ask them to arrange the data in ascending (increasing) order.
- Now observe the most common/ frequent height. The height possessed by most of the students for example 10 students have 5" height.(there may be more than one heights which are repeated at equal number of students or there may be no height repeated at all). Then introduce MODE.
- Ask them the number of students of which they recorded the heights. Ask them if the number of students you recorded is even or odd (e.g., 21 , 23, 22 etc.)?

Activity 2

- Now ask the students to look for the middle value in the arranged list of heights. For odds list the middle and for even list average of two middle values. e.g, sum of 10^{th} and 11^{th} student height /2. Let them find and then announce that we have just calculated "MEDIAN".
- Now ask what would be your answer if I ask you to find the average. They would do sum of all divided by number of students and give you answers. Then tell them that they just did was Arithmetic mean. Also tell that median and mode are also Averages.

Activity 3

- Give away Activity sheet: **Mean, median mode Puzzle time**

Activity 3

Discuss with class: Which one is better: mean, median or mode?

Teacher: It depends on your goals. I can give you some examples to show you why.

Consider a company that has nine employees with salaries of 35,000 a year, and their supervisor makes 150,000 a year. If you want to describe the typical salary in the company, which statistics will you use?

Student: I will use mode (35,000), because it tells what salary **most** people get.

Teacher: What if you are a recruiting officer for the company that wants to make a good impression on a prospective employee?

Student: The mean is (35,000x9 + 150,000)/10 = 46,500 I would probably say: "The average salary in our company is 46,500" using mean.

Teacher: In each case, you have to decide for yourself which statistics to use.

Student: It also helps to know which ones other people are using!

Now solve the question with the help of students.

A student has gotten the following grades in his tests: 87, 95, 76, and 88. He wants an 85 or better overall. What is the minimum grade he must get on the last test in order to achieve that average?

Solution: The unknown score is "x". Then the desired average is:

$(87+95+76+88+x)\div 5=85$ Multiplying through by 5 and simplifying, I get:

$87+95+76+88+x=425$

$346+x=425$

$x=79$

He needs to get at least a 79 on the last test.

Conclusion/sum up

Averages are of great importance and the choice of an average is usually determined by the purpose of our investigation. It is an important and difficult problem, Since each average has its own merits and demerits, therefore proper average can be determined only after studying the nature, type and objective to the investigation.

Assessment

- Alina allows himself an average of Rs250 a day for lunch at work. If she spent Rs200 on Monday, Rs275 on Tuesday, Rs225 on Wednesday, and Rs250 on Thursday, how much may he spend for lunch on Friday?
- The average marks of three batches of students having 70, 50 and 30 students respectively are 50, 55 and 45. Find the average marks of all the 150 students, taken together.

Follow-up

Ask students to collect following data in groups then calculate averages.

- 1st group Record height of 7th class students.
- 2nd group Record weight of 7th class students.
- 3rd group Record marks of each student in 8th PEC examination.

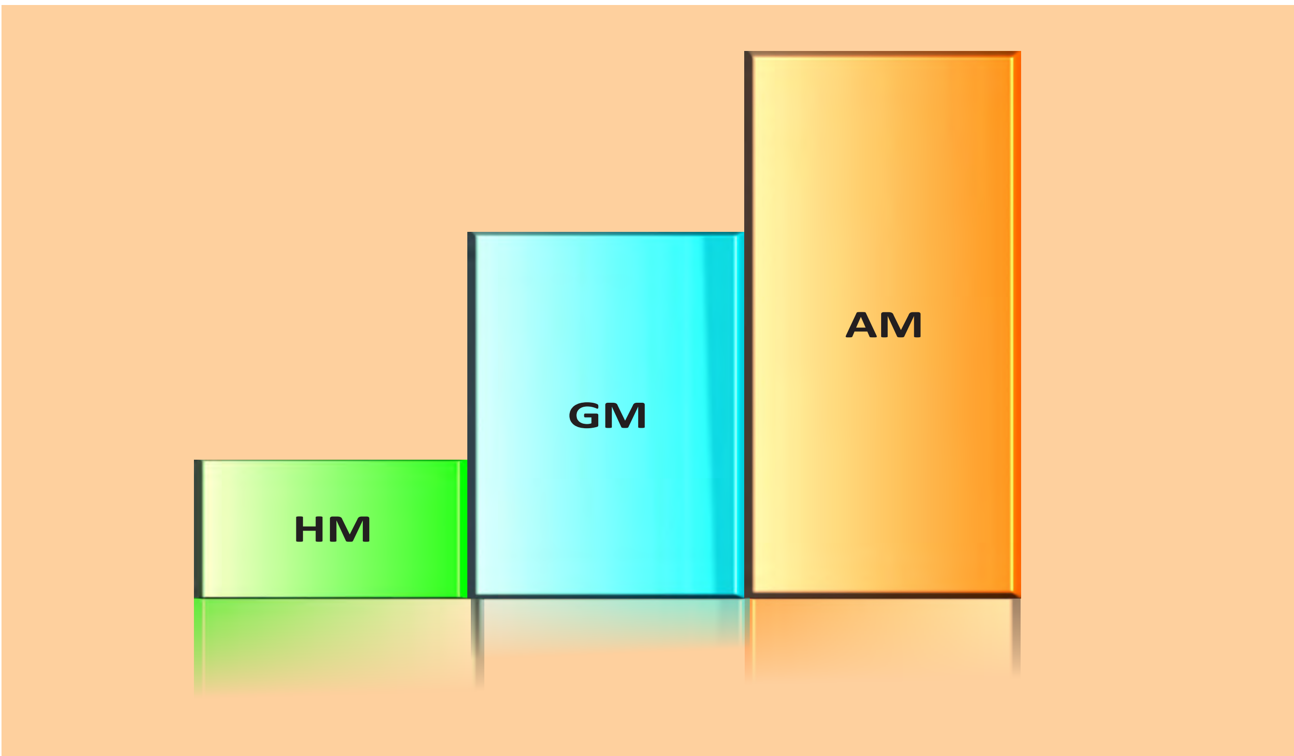
- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

Lesson Plan 4

TOPIC

Measures of Central Tendency - II

Grade - x



Students' Learning Outcomes

- Calculate geometric and harmonic mean.

Information for Teacher

- Geometric mean: Geometric mean is a kind of average of a set of numbers that is different from the arithmetic average. The geometric mean is well defined only for sets of positive real numbers. "the 'n'th root of product of 'n' numbers".
- **Geometric Mean :**

  Geometric Mean = $((X_1)(X_2)(X_3)........(X_N))^{1/N}$
  where
  X = Individual score
  N = Sample size (Number of scores)
  The geometric mean must be used when working with percentages (which are derived from values), whereas the standard arithmetic mean will work with the values themselves.
- **Harmonic mean(HM)** = $N/(1/a_1+1/a_2+1/a_3+1/a_4+.......+1/a_N)$
  Arithmetic mean is usually larger than HM. HM gives more detailed analysis.

- **Solved Example on Harmonic Mean**

Find the harmonic mean of 15, 30, and 45.

Choices:

A. 55.24

B. 10.22

C. 24.55

D. 16.81

Correct Answer: C

Solution:

Step 1: Harmonic Mean of a set of number is the number of items divided by the sum of the reciprocals of the numbers. Harmonic Mean of a set of *n* numbers i.e. a_1, a_2, a_3, ... a_n, is given as

$$\frac{n}{\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+ \text{..........}+\frac{1}{a_n}}$$

Step 2: Harmonic mean =

$$\frac{3}{\frac{1}{15}+\frac{1}{30}+\frac{1}{45}} \quad \frac{3}{\frac{11}{90}}$$

[Add the reciprocals of the numbers 15, 30, and 45 and divide the sum by 3, as there are three numbers.]

Step 3: $= \frac{270}{11} = 24.55$

Step 4: So, the harmonic mean of the numbers 15, 30, and 45 is 24.55.

Duration/Number of Periods

80 mins/2 period

Material/Resources required

Black/white board, marker, chalk, handout photocopies as per number of groups

Introduction

Activity

- With the students recall the concepts done up till now and then introduce two other types of average i.e., GM and HM.
- A common example of where the geometric mean is the correct choice is when averaging growth rates. For example: the population of this city has an average growth rate of 20% per year.
- **GM and HM are used for an even deeper analysis of data.**
- **Share with them some facts from history:** The Greeks were into means and the harmonic mean in particular. Here is the Greek definition from Porphyry in the "Commentary on Ptolemy's
- Harmonics": The sub contrary mean, which we call harmonic.
- The Greeks (Pythagoreans specifically) used these means in music: holding strings in certain ratios and plucking them.
- It was named harmonic by the circle of Archytas and Hippasus, because it seemed to furnish harmonious and tuneful ratios.

Development

Activity 1

- Introduce the formula for geometric mean with simple link from known to unknown that Geometric mean of A and B is the square root of (A*B). The geometric mean of A, B, and C is the cube root of (A*B*C). And so forth.

- Now write the formula on board for GM without any explaination.

GM Mental Math Problem:

- Can you calculate the geometric mean of these 5 numbers, in your head? $2^3, 2^5, 2^8, 2^3, 2^1$

  (These values of course equal 8, 32, 256, 8, and 2) (Hint: The 5 exponents add up to 20.)
  Answer: The exponents add up to 20, 20 divided by 5 is 4, so the geometric mean is 2^4 or 16.

- Refer back to the formula written on board and then explain.

Second problem:

- If Geometric mean of (8,a)=12, what is a?

  The question can be most easily be rephrased using the nth root definition of geometric mean. That is:

  square root of (8 x a)=12

  solve first by squaring both sides:

  (8xa)=144

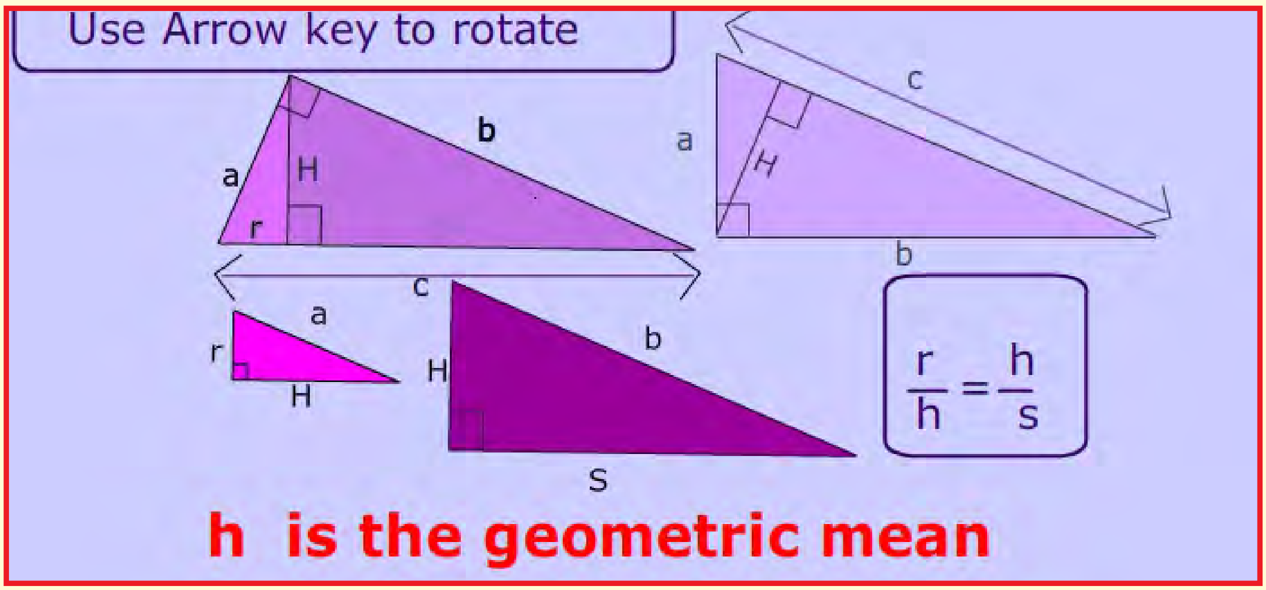
  a=144/8 = 18

- You can explain GM with the help of similar triangles as well.

Understanding GM with similar Triangles:

The geometric mean is any 'b' that can be expressed $\boxed{\frac{a}{b} = \frac{b}{c}}$ in a proportion.

To better understand how the **altitude** of a right triangle acts as a geometric mean in **similar** triangles, look at the **triangle** below with sides a, b and c and altitude H.



- **Assign questions from the book to groups and then individuals to attempt exercise.**

Harmonic mean:

Finding HM: recall with students that To find the mean or the "arithmetic" mean, you add up all the numbers and divide by how many numbers there are. They would reply 'yes'.

For the **harmonic mean**, you first take the reciprocal of each number, then take the usual average, then take the reciprocal again (because reciprocal is the same as "un-reciprocal"
... the operation is its own inverse).

- Explain with a simple example: HM is such that by whatever part of itself the first term exceeds the second, the middle term exceeds the third by the same part of the third.
- That is, b is the harmonic mean between a and c if (a-b)/a = (b-c)/c.
- Can you get b = 2ac/(a+c) out of this? (let them try in pairs)
- Along with the students go through the steps with an example question done in "Information for Teacher".
- Emphasize on 'STEP BY STEP' approach.
- Now divide the class into groups of fours and give away the "Discussion sheet" let them develop the understanding at their own pace.
- Join every group and collect their issues and then explain each of the statements to the whole class collectively.
- Assign questions to the groups to solve and share the answers.

Which one is better: AM/ HM/ GM? (Class discussion)

AM, GM, and HM satisfy these inequalities: $AM \geq GM \geq HM$. Equality holds only when all the elements of the given sample are equal.

- Explain the scenario to the students on board. If x are the MARKS in theory and y are the marks in practical. Calculate the aggregate by using (AM, GM, HM) for the following marks. What conclusion do you derive?
- Write the values for X and y on board and let them solve it in pairs. Also ask them to write their observation for AM, GM, and HM. (assign time-slot)

You may write the formulae for all the three:**Mean**	**Simplest formula**
arithmetic mean of x and y	(x + y) / 2
geometric mean of x and y	v(xy)
harmonic mean of x and y	2xy / (x + y)

x	y	arithmetic mean	geometric mean	harmonic mean
50	50	50	49	48
40	60	50	46	42
30	70	50	40	32

Notice that the arithmetic mean is in all cases 50. Clearly the geometric and harmonic means penalize uneven performances, but the harmonic mean penalizes them more heavily.

Activity 2

- Teacher divide students into suitable number of groups preferably consisting of 5-6 students and give following data to calculate geometric mean (GM) and harmonic mean and ask them to find out the relationship between geometric mean and Harmonic Mean

  X = 4, 5, 6, 8, 10

Conclusion/Sum up

Recap the definition and formula of COM, HM and AM. Also tell them importance, comparison and use of three types of means. (i.e GM, AM, HM)

Assessment

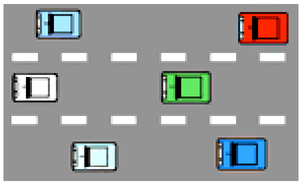
Project: Geometric Mean and Pythagoras Theorem (attached in projects)

Questions assigned for independent work.

First-Worksheet

Example 1: Find the mean driving speed for 6 different cars on the same highway.

66 mph, 57 mph, 71 mph, 54 mph, 69 mph, 58 mph

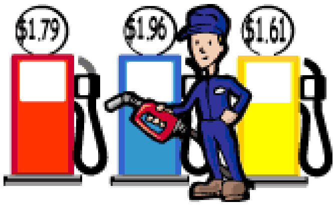


Solution:

Answer:

Example 2: The Sheikh family drove through 4 cities on their summer vacation. Gasoline prices varied from state to state. What is the mean gasoline price?

Rs1.79, Rs 1.61, Rs 1.96, Rs 2.08



Solution:

Answer:

Example 3: A marathon race was completed by 5 participants in the times given below. What is the mean race time for this marathon?

2.7 hr, 8.3 hr, 3.5 hr, 5.1 hr, 4.9 hr

Solution:

Answer:

Follow-up

- Give different questions and ask students to calculate arithmetic mean, geometric mean and Harmonic mean.
- Find the mode, median, mean of the following data

a) 4, 1, 7, 3, 1

b) 10, 15, 9, 1, 5, 5, 15

c) 23, 19, 19, 16, 18, 18, 8, 5, 8

d) 2, 7, 1, 9, 3, 7, 2, 4, 7, 10

- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

UNIT

14

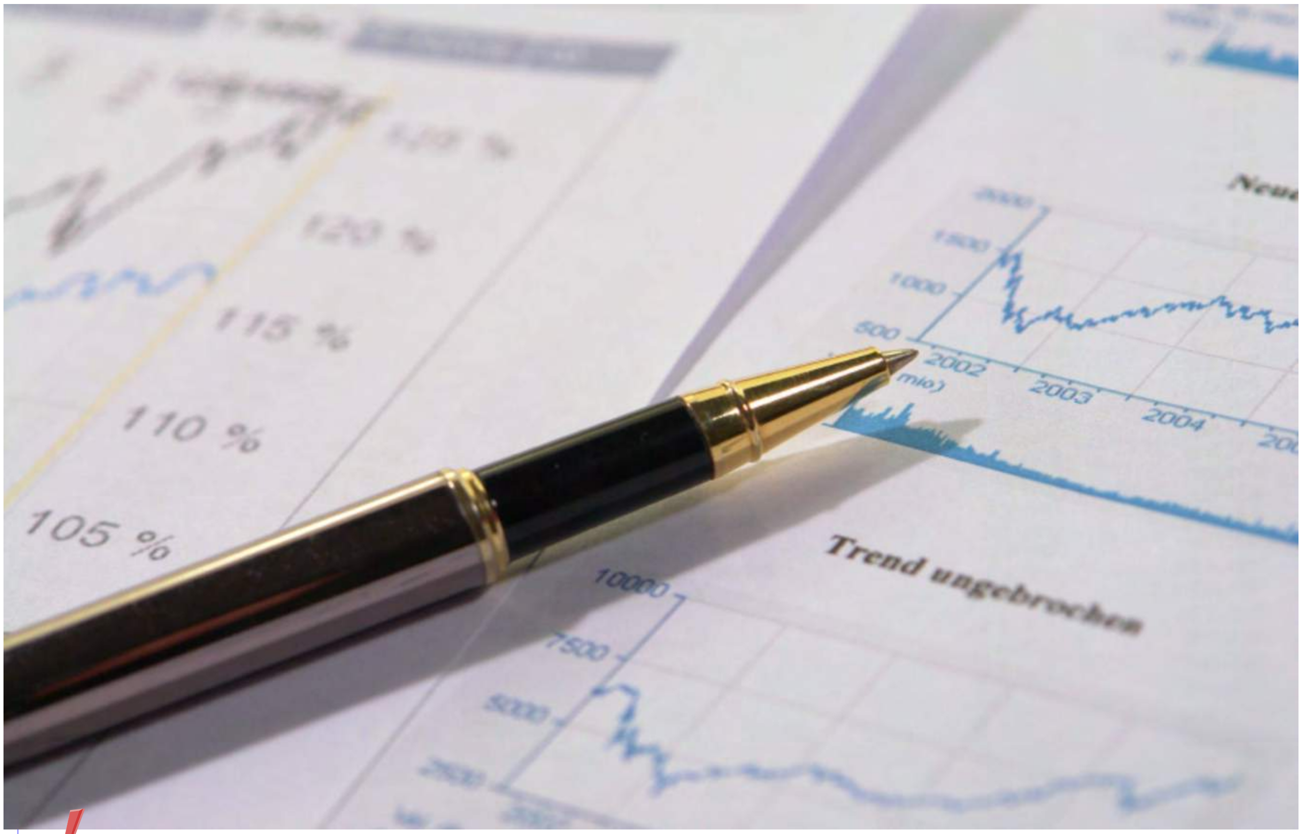
Linear Graphs and Their Application

Lesson Plan 1

TOPIC

Cartesian Plane and Linear Graphs-I

Grade X

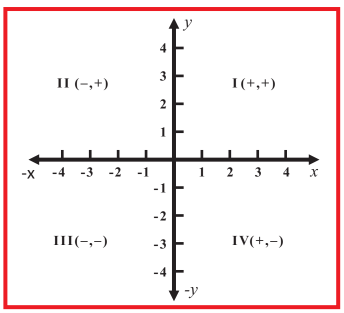


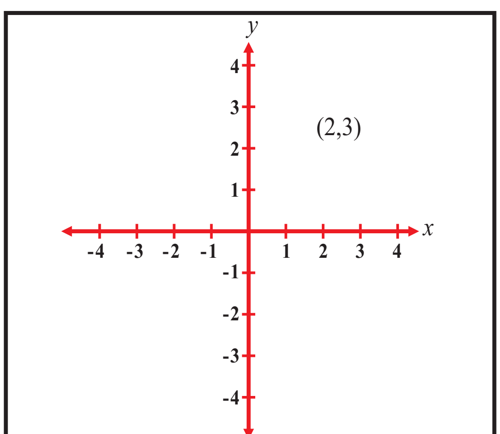
Students' Learning Outcomes

- Identify pair of real numbers as an ordered pair.
- Recognize an ordered pair through different examples; for instance an ordered pair (2,3) to represent a seat, located in an examination held at the 2nd row and 3rd column.
- Describe rectangular or Cartesian plane consisting of two number lines intersecting at right angles at the point 'o'.
- Identify origin (o)and coordinate axes (horizontal and vertical axes or x-axis and y-axis) in the rectangular plane.
- Locate an ordered pair (a,b) as a point in the rectangular plane and recognize:
  - a as the x-coordinate (or abscissa),
  - b as the y-coordinate (or ordinate).
- Draw different geometrical shapes (e.g., line segment, triangle and rectangle etc) by joining a set of given points.

Information for Teacher

- **Coordinates of a point:** Each point on a number line is assigned a number. In the same way, each point in a plane is assigned a pair of numbers.
- **x-axis and y-axis:** To locate points in a plane, two perpendicular lines are used—a horizontal line called the x-axis and a vertical line called the y-axis.
- **Origin:** The point of intersection of the x-axis and y-axis.
- **Coordinate plane:** The x-axis, y-axis, and all the points in the plane they determine.
- **Ordered pairs:** Every point in a coordinate plane is named by a pair of numbers whose order is important; these numbers are written in parentheses and separated by a comma.





Ordered Pair

- **x-coordinate:** The number to the left of the comma in an ordered pair is the x-coordinate of the point and indicates the amount of movement along the x-axis from the origin. The movement is to the right if the number is positive and to the left if the number is negative.
- **y-coordinate:** The number to the right of the comma in an ordered pair is the y-coordinate of the point and indicates the amount of movement perpendicular to the x-axis. The movement is above the x-axis if the number is positive and below the x-axis if the number is negative.

Note: The coordinates [ordered pair] for the origin are (0, 0).

Duration/Number of Periods

80 mins/2 period

Material/Resources required

worksheet 'who am I?, colour pencils, scales

Introduction

Activity

- Call the students not by name rather like "3rd from the left 2nd row, please tell me your name then on the board write ALI (3,2) , repeat the same with two or three other students and ask them to write Alina (4,5) , ahmed (8,4), respectively etc.
- Now ask the students "WHAT HAVE I WRITTEN ON THE BOARD? ", can I say that these are the addresses of Ali, Alina and Ahmed in the classroom? [let them think and answer yes!]

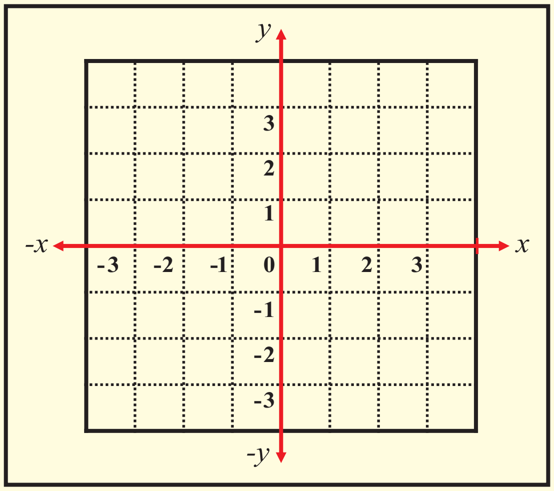
- We have used a pair of numbers to represent the location in the classroom. Similarly the same idea is used when naming units in a block of flats.
- Tell them that each location is addressed by a Pair called 'ordered pair'. As we have said for example (3,4) that is 3rd in 4th row, we would defiantly have a 'starting point' like (0,0) in our mind as well ? let them think and say "yes". A system of representing points in term of their distance from origin is known as **"cartesian coordinates"** and a plane having all points is known as **"cartesian Plane"**.
- Each wall of the classroom, the floor and roof is a plane.

Today we are going to discuss about coordinate plane and ordered pairs in detail.

Development

Activity 1

- Ask the students to sit in pair.
- Ask the students to draw number line on their notebooks. (give them and then draw one on the board).
- Now ask can you draw another number line perpendicular to it. [give them and then draw one on the board]
- Explain the coordinate plane and axes in detail. The key points mentioned above will be discussed.
- Demonstrate, that the first number in an ordered pair represents a horizontal direction from 0. The second number represents a vertical direction from 0. Also, tell the students that the first number, x, is found first and the second number, y, is found second.



Activity 2

- Ask the students to sit in groups.
- Give out the **What Am I?** Sheet.
- Ask them to connect given points and identify what shape is it?
- They will draw straight line between the following points to reveal a picture.

1. (8, 6) to (4, 10)
2. (4, 2) to (8, 2)
3. (4, 4) to (10, 4)
4. (2, 4) to (4, 4)
5. (4,6) to (4,10)
6. (8,6) to (4,6)
7. (4,2) to (2,4)
8. (8,2) to (10,4)
9. (4,4) to (4,6)

- Ask students to share 'who am I worksheet with each other'.
- Discuss the findings, of the students in the class.

Activity 3

- **Take out the coordinate plane you drew with two number lines and do as I say, lets see who do it first.**

- **Locate a point (5,2) then locate (2,2) join both what do you get? [they will answer 'a line']. Then tell them that similarly different segment, geometrical shapes are also drawn if ordered pair of each point is known.**
- On the same plane can you locate (-4,0) mark it as A, now locate (-4 , 4) mark as B [give them time to attempt]. Ask if my third point C is at (0,0) what is the shape I get by joining A,B and C [right angle triangle].
- Now ask the students to locate four points on the Plane and try to draw rectangle.
- After taking their response demonstrate it on the board.
- Ask about quadrant and also help them if they had any difficulty.

Conclusion/sum up

Revise all the key points by giving examples and involve the students to conclude the answers.

Assessment

- Write the following ordered pairs on the board:

1.	(4, 2)	6.	(8,12)
2.	(6, 1)	7.	(1,0)
3.	(9, 8)	8.	(7,2)
4.	(0, 8)	9.	(3,6)
5.	(8, 4)	10.	(7, 7)

Ask the students to plot these points on their coordinate plane or graph paper sheet.

Follow-up

Locate the points to make a shape of square in the plane and also draw the shape.

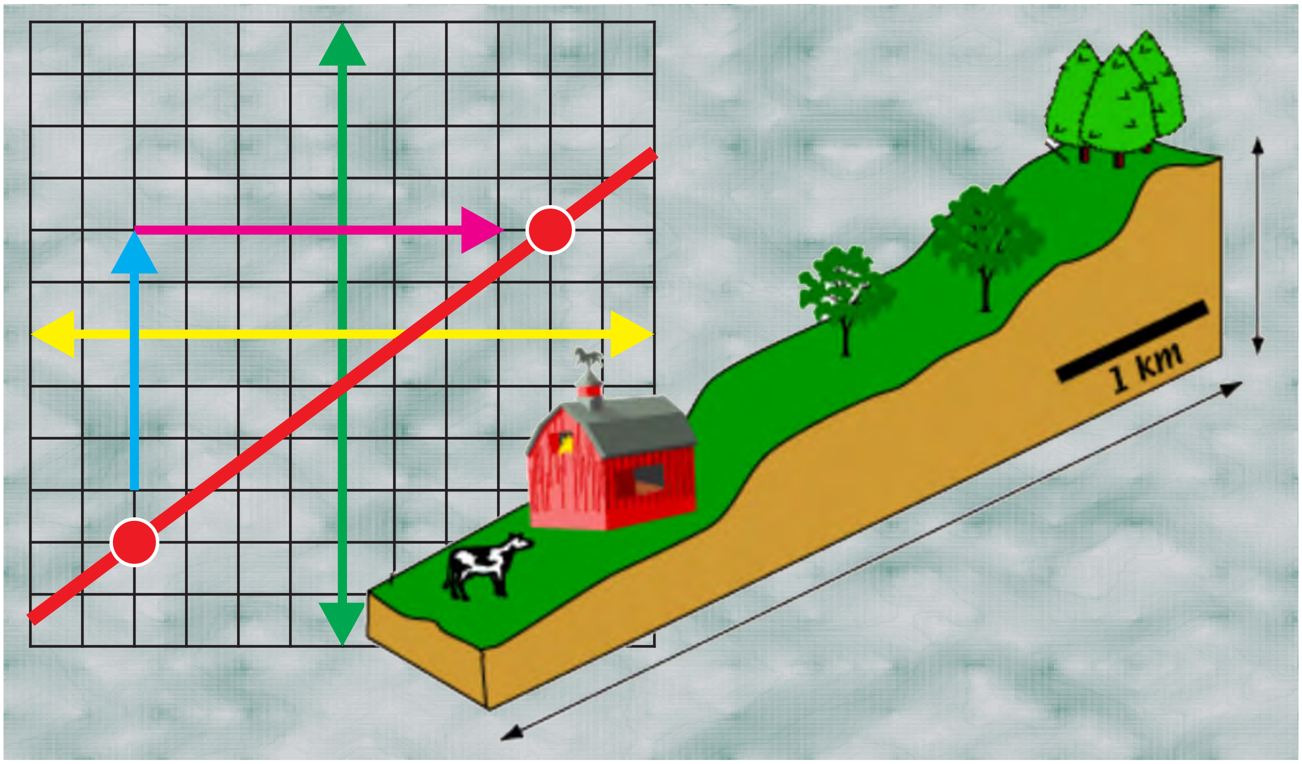
- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

Lesson Plan 2

TOPIC

Cartesian Plane and Linear Graphs

Grade X

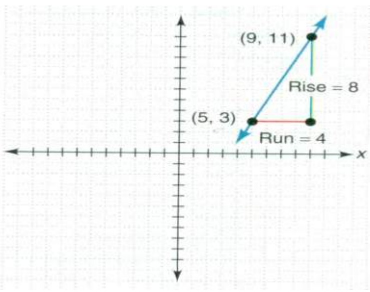


Students' Learning Outcomes

Draw the graph of an equation of the form $y = mx + c$.

Information for Teachers

- The graph of equation of the type $y=mx+c$ is a straight line. Infact every line except those parallel to vertical axis has an equation of the form $y=mx+c$, where m is the slope/gradient or steepness of the line and c is the y-intercept.
- The concept of slope occurs in many applications of Mathematics. Highway engineers measures the slope of a road by comparing the vertical rise to each 100 feet of horizontal distance.
- To measure the slope of a line, two points on a line are selected and the slope of line connecting these points is the difference



between two

y-coordinates (the rise) divided by the difference of two coordinates (the run).

Duration/Number of Periods

80 mins/2period

Material/Resources Required

pencils, scales, white sheet and graph paper

Introduction

Activity

Ask the students to copy and complete the table below for the equation y=2x+3

X	1	2	3	4	5
Y					

Make a graph and observe the line

Development

Let us recall what we have learnt about the graph of straight line. The graph of an equation of the form ax+ by+c = 0 is a straight line. If b ≅ 0 this equation can be written as $\frac{a}{b}x + y + \frac{c}{b} = 0$ by dividing each term by b. In fact we can reduce each equation of the form $ax + by + c = 0$ to $y = mx + l'$

Activity 1

Arrange the students to sit in pairs and ask them to reduce each of the equation

I. $5x + 3y - 3 = 0$

ii $7x - y = 0$

iii $y = mx + c$

Once they have done ask them to make graphs of these equations and discuss with each other.

Ask them the following questions:

i. Do these graphs are straight lines?

ii. Does the value of c has any connections with y-axis?

iii. What does m represents?

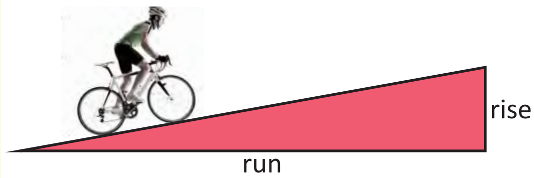
After discussion, introduce that m is the slope or steepness of the line and c is the point where the line cuts y-axis.

Activity 2

Demonstrate the concept of slope of line by following example

Slope or gradient of a line: When we walk on an inclined plane, we cover horizontal distance **(run)** as well as vertical distacne **(rise)** at the same time.

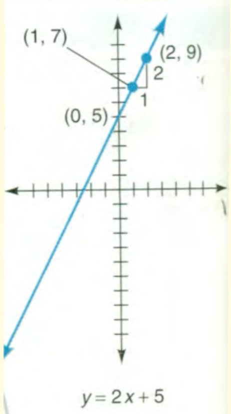
It is harder to climb a steeper inclined plane. The measure of steepness (ratio of rise to the run) is termed as slope or gradient of the inclined path and is denoted by m.



$m \frac{rise}{run} = \frac{y}{x} = \tan$

In analytical geometry, slope or gradient m of a non-vertical straight line with θ its inclinatiuon is defined by: m=tanθ

Now draw the graph of $y = 2x + 5$ on board by selecting two points. (ordered pairs)



Discuss with the students that the slope of line is the difference in two y-coordinates divided by the difference in two x-coordinates.

$$m = \frac{\textit{the difference in y-corrdinates between 2 points}}{\textit{the difference in the corresponding x-cordinates}}$$

Conclusion/Sum up

- Recall with the students that first the values for x and y are found and recorded and later we used it to plot graphs.
- By drawing a graph for the given relation we get a visual aid to understand it. It also helps in finding value of coordinate against given (x or y) along the line.
- The graph of an equation of the type $y = mx + l'$ is a straight line where m is the slope of line and c is the y intercept.

Assessment

Draw the graphs of equations

i. $y = 3x + 3$

ii. $y = 2x + 3$

Follow-up

Ask them to take any two linear equations and plot graph of both on same paper and observe the relation in these equations.

- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

UNIT 15

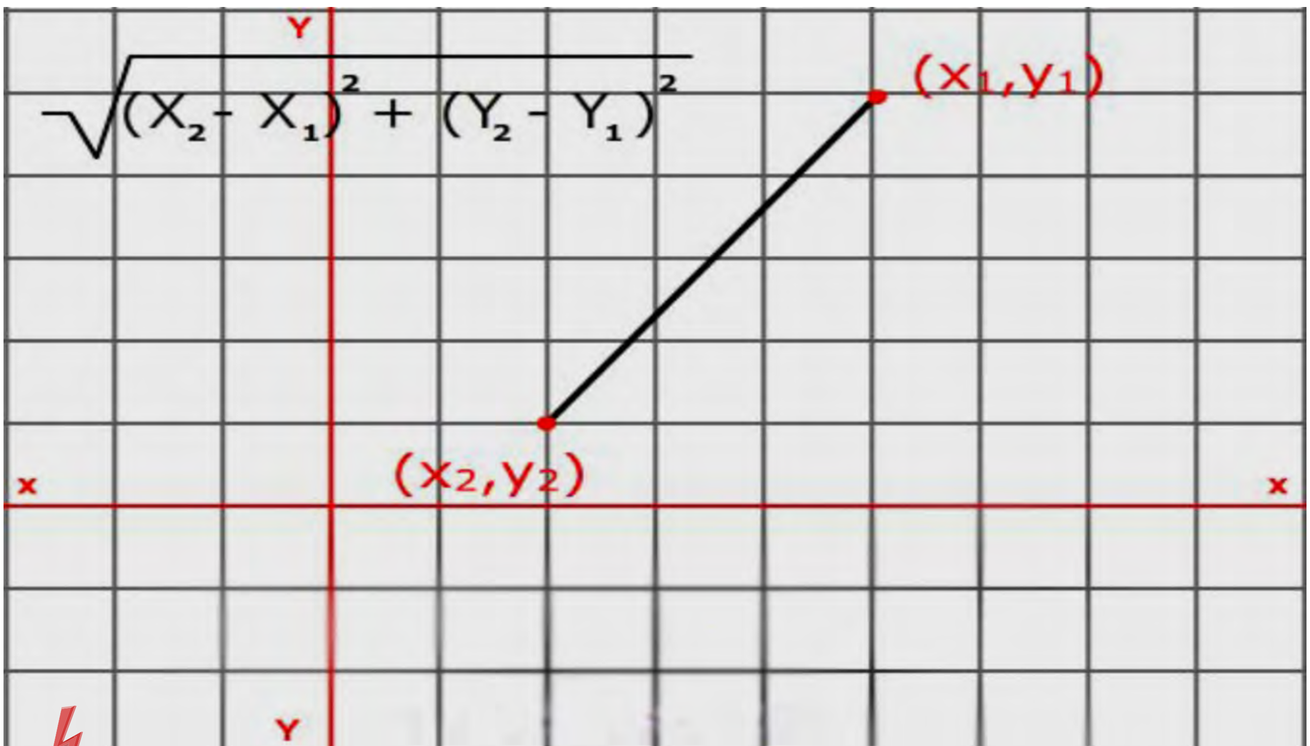
TOPIC

Lesson Plan 1

Distance Formula - I

Introduction to Coordinate Geometry

Grade X

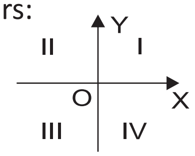


Students' Learning Outcomes

Define coordinate geometry.

Information for Teacher

- History corner: Rene Descartes, a French philosopher in early 17^{th} century invented the coordinate system. He was the first person to use Algebraic methods to study geometry. He was also the first person who declared the word *"I think therefore I am"*
- A single line that separates the plane into two pieces (each of which is a half-plane), so the two axes separate the plane into four parts, called ***quadrants***. The four quadrants are identified by numbers:

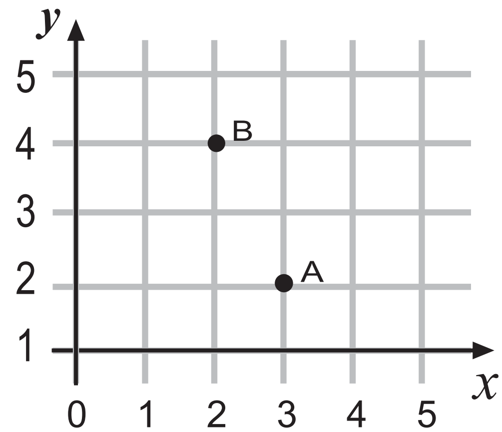


- **Co-ordinates**

  In order to fix a point in two dimensions we give its position in relation to a point called **origin**. We draw two axes at right angles to each other. The horizontal axis is called the x-axis while the vertical axis is called the y-axis. The x-axis is numbered from left to right.

They y-axis is numbered from bottom to top.

- The position of point A is given by two co-ordinates, the x co-ordinate and they y co-ordinate. So the co-ordinates of point A are (3, 2). Similarly, the co-ordinates of point B are (2, 4).

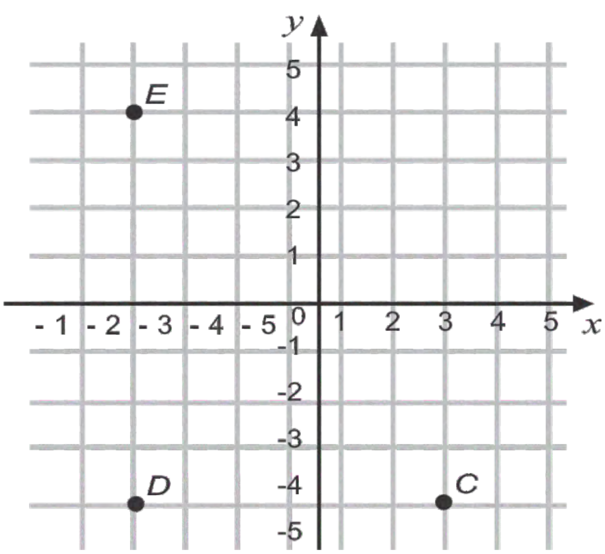


- The axes can be extended in both directions. By extending the x and y axes below zero this grid is produced.
- We can describe points C, D and E by their co-ordinates.

Point C is at (3, -3)

Point D is at (-4, -3)

Point E is at (-4, 3)



- The x-coordinate is also known as the **abscissa** and y-coordinate known as **ordinate**.

Material/Resources required

visit the website to find Cartesian plan points and graphing.

http://mathforum.org/cgraph/cplane/plane.html

Title: What Am I? sheet

Annotation: Students will use this sheet to reveal a hidden picture using coordinate pairs.

Battleship Game sheet (if needed), Coordinate Graphing Assessment (or graph paper)(Small, coloured foam / Chart paper squares, ½ inch x ½ inch (20 per pair/group), markers/ colour pencils, circle stickers containing coordinates/ chart paper slips, large map of city (where students live)

Duration/Number of Period

80 mins/2 period

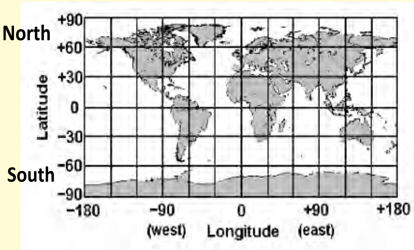
Introduction

Activity 1

Have you ever wondered how convenient it is to locate when in the examination hall we have to find our seat in the rows and columns arranged? We use the idea of coordinate geometry in locating our seats.

Activity 2

Show any picture like this on computers/ from library books / draw roughly on board and ask students give their comment about it.



Let them conclude that Latitude and Longitude are how a site location is defined on the surface of the earth. They have some other logic to follow and are measured in degrees. However we can just bring their attention to HOW HORIZONTAL and VERTICAL point's conjunction shows the location address of any region on earth.

Development

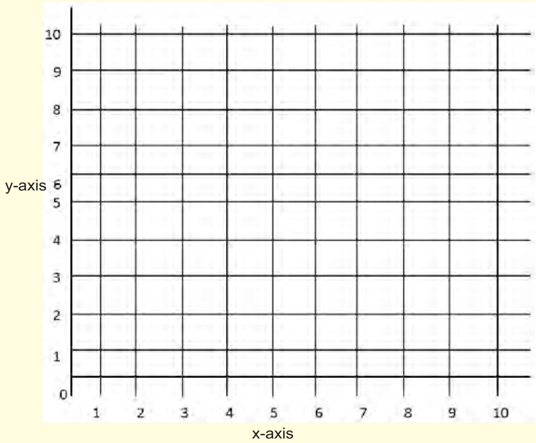
[individual/ pair/ group as per students level]

Activity 1

(Locating school) Ask the students to sit in groups. ask the students to explain how they would go about finding the location of the school. Allow students to discuss ways to find the school by telling landmarks, following street signs, memorizing the roads, using a road map, etc.

Step 1: Tell students they will be using coordinate points to help locate areas on a grid or map. Present a large coordinate graph on the whiteboard.

Example:



Review the terms ordered pair, x-axis, and y-axis by writing the following on the board:

Ordered pair - two coordinates that are used to find a point on a map or grid

x-axis – the horizontal axis

y-axis – the vertical axis

Step 2: Begin by writing the ordered pairs (3, 4) and (4, 3) on the whiteboard.

Step 3: Demonstrate by plotting the point (3, 4).

Step 4: While demonstrating, emphasize that the first number in an ordered pair represents a horizontal direction from 0. The second number represents a vertical direction from 0. Also, tell students that the first number, x, is found first and the second number, y, is found second.

Step 5: Then plot the point (4, 3) by placing a sticker/colour chart paper slip on this point. Use a marker to write the point (4, 3) on the sticker/slip.

Step 6: Discuss with the students what was different about the directions moved to get to each point. Also ask students to brainstorm whether or not the final point would be different if we changed one of the numbers in the ordered pair.

Activity 2

Step 1: Ask the students in groups, pass out the circle stickers/ chart paper slips

Step 2: Call out the following ordered pairs to student. The student who is holding the correct ordered pair should place their sticker correctly on the graph.

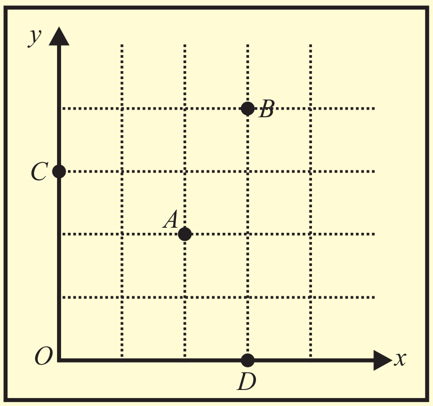
- (8, 3)
- (12, 5)
- (5, 3)
- (9, 9)
- (3,8)

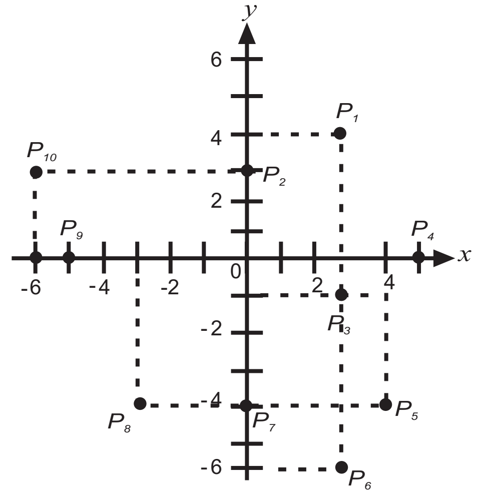
Step 3: The teacher and students will discuss the placement of the stickers. The teacher should ask questions such as: What does each ordered pair represent? How could we change the ordered pairs to have them mark a different point?, and What ordered pairs would allow us to land directly above each of these points we have marked on the graph?

Activity 3

Ask the students to sit in groups. Provide them the work sheet/graph paper with marked points A,B,C and D.

Ask them to write coordinate points through discussion. After completion ask them to present in front of the class.

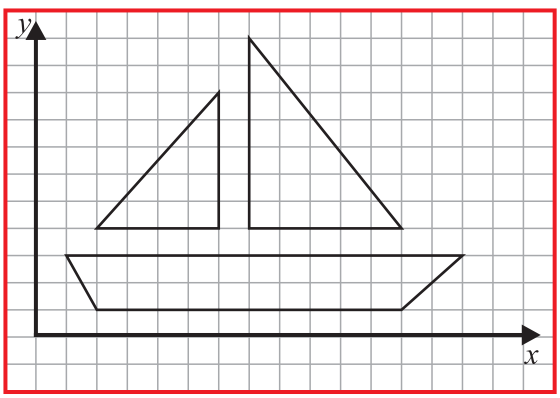




2. What are the coordinates of the origin?
3. What is the *y*-coordinate of the point (3,-5)? of the point (5,-3)? of the point (-5,3)?

Follow-up

- Distribute the attached worksheet copies as per number of groups and ask to write the answer of it. (first do not give any explanation and let them do)



What are the coordinates that define this sailboat?

- Ask them to draw more pictures on grid paper and write their coordinate.
- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

Conclusion/sum up

Using the coordinate system on the plane we are able to drive many elegant geometrical results about lines, polygons, circles and so on. We can prove the properties of geometrical shapes by using these concepts.

Assessment

1. (a) Give the coordinates of each point *P* of the figure as an ordered pair of numbers.

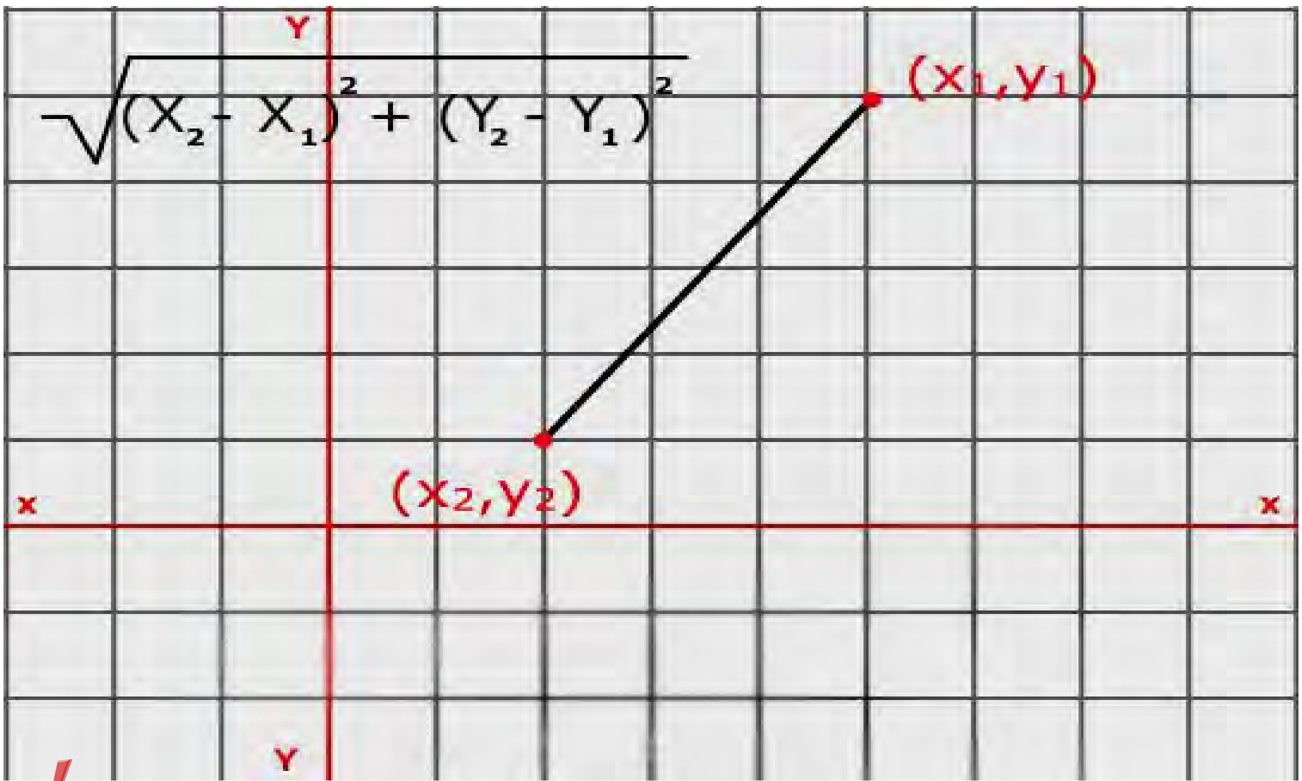
   (b) Name the points in the quadrant **1**; in quadrant **IV**.

Lesson Plan 2

TOPIC

Distance Formula - II

Grade X

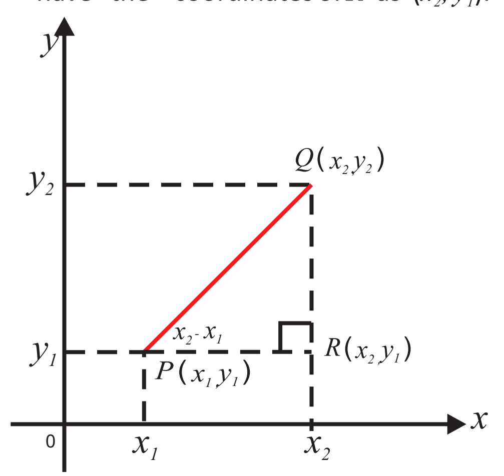


Students Learning Outcomes

- Derive distance formula to calculate distance between two points given in Cartesian plane.
- Use distance formula to find distance between two given points.

Information for Teacher

- **Proof of Distance Formula:** In general, consider any two points P and Q with coordinates (x_1, y_1) and (x_2, y_2) respectively. By completing the right-angled triangle PQR, we have the coordinates of R as (x_2, y_1).



Hence, $PR = x_2 - x_1$

And $QR = y_2 - y_1$

Using Pythagoras' theorem,

$$PQ^2 = PR^2 + QR^2$$
$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- **Distance Formula:** The general formula for the length of any line segment PQ, where the coordinates of P and Q are (x_1, y_1) and (x_2, y_2) respectively, is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Material/Resources required

board, chalk / marker, work sheet, graph paper, rubber, pencil

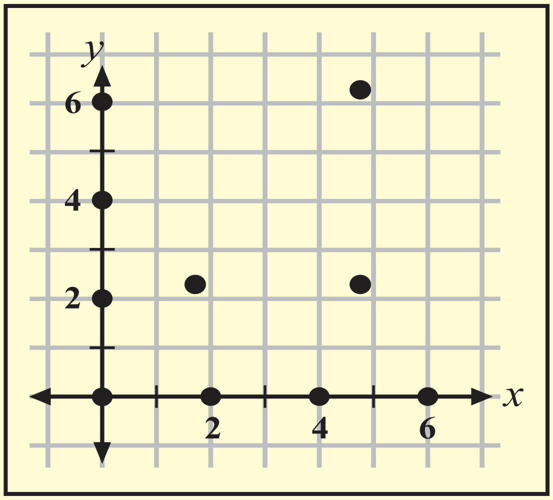
Duration/Number of Period

80 mins/2 period

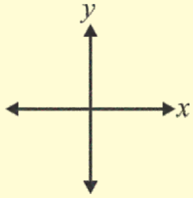
Introduction

Activity 1

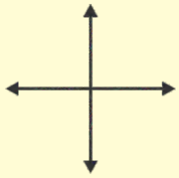
- Draw on board and ask what is it?
- Label them? and label it.



- We can label the axes to make them easier to tell apart. The axis that goes from side to side is the x-axis, and the axis that goes straight up and down is the y-axis.



- Axes cross has a special name: it is called the origin. And has address (0,0)



Activity 2

- Just mention point B on the board and tell that Ali is standing at point B, ask student its ordered pair, (5, 2) and you are standing at point C (5, 6) (ask to label its ordered pair address).
- How far are you from Ali? (6-2 =4). Similarly ask that Nadeem is standing at A (2, 2), let the students tell its address. Ask what the distance between Nadeem and YOU. Let They answer (5-2 = 3). Now ask CAN WE KNOW THE DISTANCE BETWEEN ALI AND NADEEM?
- Let them think and give their ideas. Do not discourage even a wrong answer. Record it in your mind to clear while further discussion.

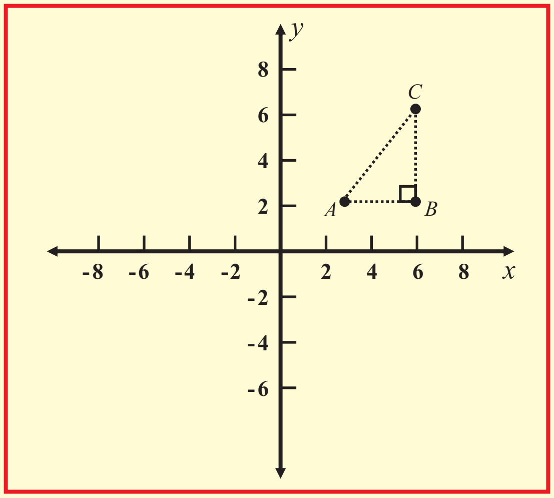
Development

Activity 1

- Ask the students to sit in groups and ask them to draw the lines AB, BC and

CA and ask what is it? They will reply you the RIGHT-angle triangle.

- Recall Pythagoras theorem with them. i.e., $hyp^2 = base^2 + per^2$. Now ask since we know the base and perpendicular, can we find hypotenuse?



- Let them do the arithmetic themselves individually.
- Collect their answers and then tell that WE HAVE JUST APPLIED DISTANCE FORMULA TO FIND DISTANCE BETWEEN ANY TWO POINTS.
- Show these steps on board for everyone: Triangle *ABC* is a right triangle with AC the hypotenuse. Therefore, by the *Pythagorean Theorem*,
- write the formula on board :

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- There are two points A(x_1, y_1) and B (x_2, y_2)and'd as above formula ' be the distance. Keeping in mind the discussion we have just done. Prove this formula. (Pair work / four per group).

$$\overline{AC}^2 = AB^2 + BC^2$$
$$\overline{AC} = \sqrt{AB^2 + BC^2}$$
$$\overline{AC} = \sqrt{3^2 + 4^2}$$
$$\overline{AC} = \sqrt{9 + 16}$$
$$\overline{AC} = \sqrt{25}$$
$$\overline{AC} = 5$$

- Announce the time and after that from as the time allows, invite students on board to show the proof. ANNOUNCE the correct one. Appreciation is must!
- Finally let them do the same on their notebooks.
- As an illustration of the fact that this formula applies even when the points P_1 and P_2 do not lie in the same quadrant, ask them to find the distance between P_1 (3,4) and P_2 (-5,-2)

Activity 2

- Assign questions to the same groups to discuss and apply the distance formula.
- Assign questions for independent practice at home.
- Group assignment: A triangle has vertices *A* (12, 5), *B* (5, 3), and *C* (12, 1). Show that the triangle is isosceles. **Solution: By the *Distance Formula*,**

$$AB = \sqrt{(5^2 - 12) + (3 - 5)^2} \qquad BC = \sqrt{(12 - 5) + (1\ \ 3)^2}$$
$$AB = \sqrt{(-7^2) + (-2^2)} \qquad BC = \sqrt{(7^2) + (-2)^2}$$
$$AB = \sqrt{49 + 4} \qquad BC = \sqrt{49 + 4}$$
$$AB = \sqrt{53} \qquad BC = \sqrt{53}$$

Because $AB=BC$, triangle ABC is isosceles.

Conclusion/sum up

- The distance between two points A (x_1,y_1) and B (x_2, y_2) is $AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
- This is known as distance formula.
- The distance between two points is always positive value.
- This distance formula is applied to all the points regardless their positions in any quadrant.

Assessment

- If A (6, 2), B (2, 2). Find distance AB
- If A (0, 0), B (4, 0), C (4, 3) prove that it is right angle triangle.

Follow-up

A quadrilateral has vertices, A(-5,2), B(8,2), C(8,-4) and D (-5,-4), Using distance formula show whether the figure is a rectangle or square.

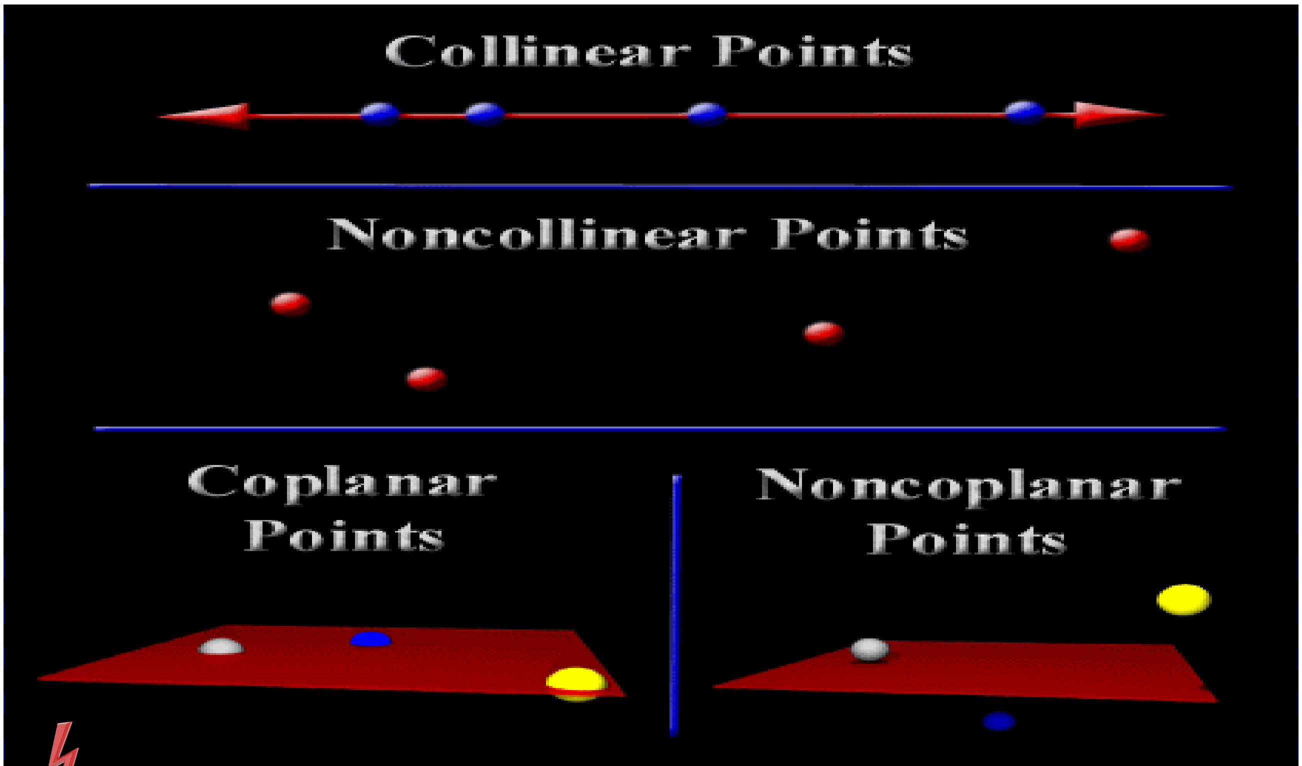
- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

Lesson Plan 3

TOPIC

Collinear Points

Grade X

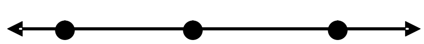


Students Learning Outcomes

- Define collinear points. Distinguish between collinear and non-collinear points.
- Use distance formula to show that given three (or more) points are collinear.

Information for Teacher

- Points that lie on the same line are called as collinear points. Two points are always said to be collinear since a straight line can be drawn through two points. Three or more points A, B, C, are said to be collinear if they lie on a single straight line.



- If the addition of the distances between two pairs of points is equal to the distance between the third pair of points, then the points are said to be collinear.
- If the points do not lie on a straight line then the points are said to be Non-collinear points. Three Non-collinear points can form a triangle. Here, in the figure 1, A, B, and C are collinear as they lie in a straight line, whereas points A, B, and D are non collinear.

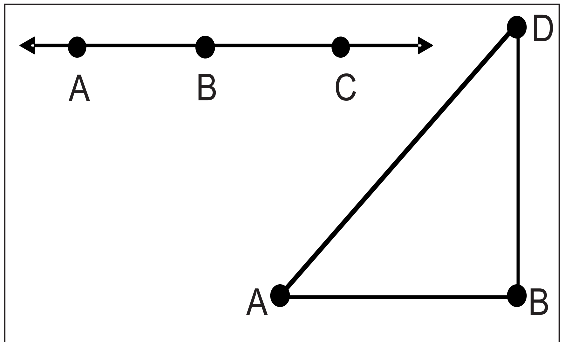
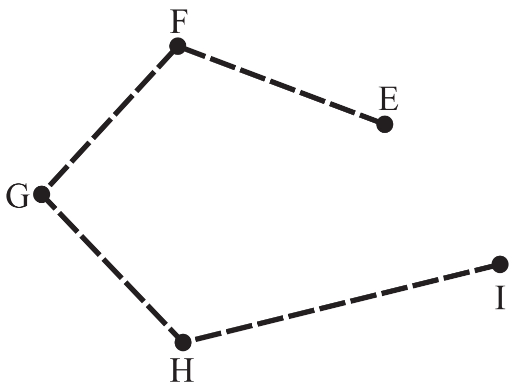


Fig (1)

- The example for non collinear point is given in the following diagram,



The diagram shows the non collinear points.

Duration/Number of Period

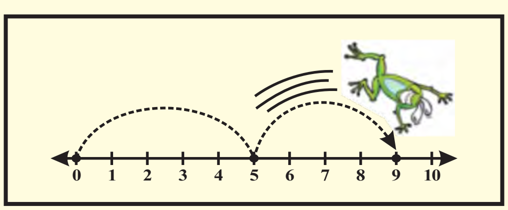
80 mins/2 period

Material/Resources required

(grid) graph paper, board, chalk / marker, work sheet, rubber, pencil

Introduction

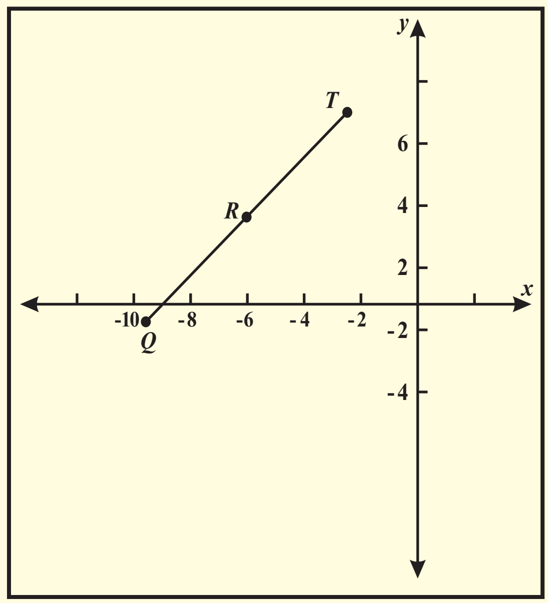
Activity 1



Draw a number line and take their acceptance on that steps between 0 to 9 are equal to steps between 0 to 5 'PLUS' 5 to 9.

We know in Geometry, the distance between two points is minimum when connected by a straight line. This concept is used here to find if the points are collinear.

- Ask students their idea about being collinear or coplanar. Let them brainstorm.
- Collect their points on board without any comments.
- Then introduce the term collinear.



Activity 2

- Draw a plane on board and show any line on it. Take three points on the line drawn.
- Ask students to write ordered pair for each of the points. (whole class discussion)
- Recall the distance formula done in previous lesson.

Development

Activity 1

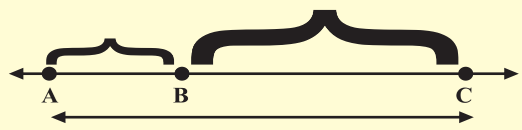
- Divide the class into mixed ability groups and ask them to follow given steps.
- Find distance between R and Q
- Find the distance between R and T
- Find the distance between Q and T
- Analyze if you find any relation between the three.
- Visit every group and help them in understanding however do not solve it for them.
- Have plenary session on board by every group.

Record your assessment about the group's findings to be discussed during your closure session.

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Activity 2

- Let's have another example and do it together on board.
- There are three points about which we don't know if collinear or not. (1,2),(2,3),(4,5)
- Distance between first two points = root of (2-1)2+(3-2)2 = $\sqrt{2}$
- Distance between second and third points = root of (4-2)2+(5-3)2 = $\sqrt{(4+4)}$ = $\sqrt{8}$ = $2\sqrt{2}$
- Distance between first and third points = root of(4-1)2 +(5-2)2 = $\sqrt{(9+9)}$ = $\sqrt{18}$ = $3\sqrt{2}$
- So we get by adding the first two distances equal the third distance.
- Hence the three points are collinear.



- If mAB + mBC = m AC
- Then the points are collinear
- Assign questions for further group work.
- Once they have practiced enough about 'how to prove the points are collinear or not'.

Activity 3

- Rearrange them for pair work. Pick people from different groups. Or this time you can do 'friendly-pairs' of their own choice.
- Ask them to get a new page and make a cartesian Plane with all four grids.
- Ask them to take any three points which are collinear and write their ordered-pairs.
- Ask them to take another three points which they think are not collinear.
- Write their ordered pairs. (allocate time for it).
- Ask : How can you prove that they are not collinear. Let them think unless they reply by applying the distance formula as we did in group work.
- Ask them to play with these points and make any shape out of it.
- They would answer yes we can make triangles.
- Recall with them all types of triangles

they know and draw on board.

- Now ask them to identify their TRIANGLE's type. (isosceles, equilateral, right angled, or scalene).

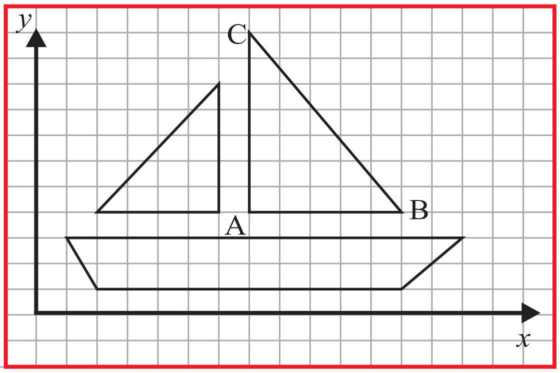
  (you may share the TRIANGLES TYPES classroom display made earlier.)
- Can you prove that these points are NON_COLLINEAR by using distance formula.
- Visit every group and record their assessment.
- Refer them to the book content and let them compare their work and do 'self-assessment.'

Conclusion/sum up

Using the coordinate system on the plane we are able to derive many elegant geometrical results about lines, polygons, circles and so on. We can prove the properties of geometrical shapes by using these concepts.

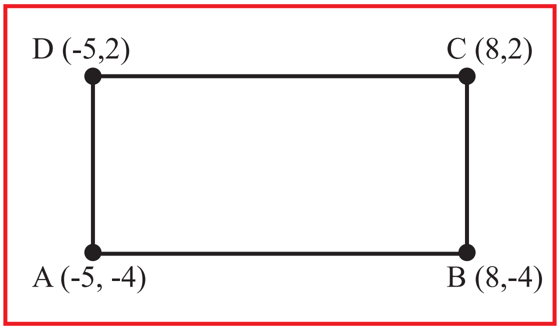
- Three point A, B and C are said to be collinear if $\overline{AC} = \overline{AB} + \overline{BC}$
- If $\overline{AC} \neq \overline{AB} + \overline{BC}$ then the three points are non collinear.

Assessment



- Find out the collinear and non-collinear points and write their coordinates
- Find distance AB, BC and AC

Follow-up



- Find $\overline{AC}$ and $\overline{BD}$
- What is the relation between $\overline{AC}$ and $\overline{BD}$
- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

Lesson Plan 4

TOPIC

Mid-point Formula

Grade X

Midpoint Formula

Developing the mid-point formula

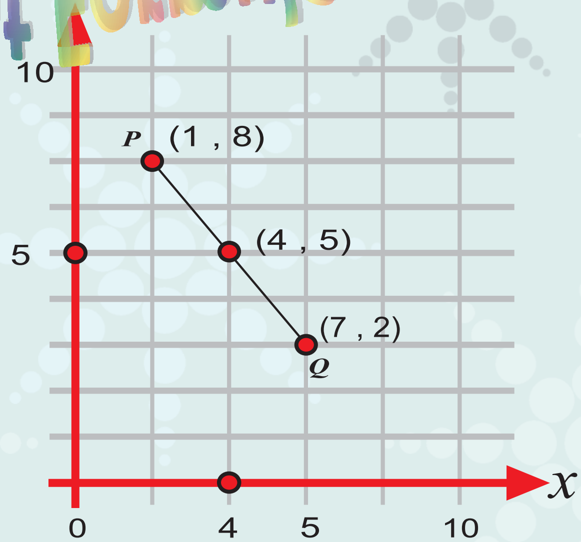
Find the mid-point of line PQ with endpoint P (1,8) and Q (7,2).

The mid-point is the middle of the line.

Find the sum of the y co-ordinates and divide by 2.
1 + 7 = 8 8 / 2 = 4

Find the sum of the y co-ordinates and divide by 2.
8 + 2 = 10 10 / 2 = 5

The midpoint of PQ is (4,5).



Students' Learning Outcome

Apply the distance and mid-point formulae to solve / verify different standard results related to geometry.

Information for Teacher

Distance formula: $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Mid-point formula: $M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

Duration/Number of Periods

40 mins/1 period

Material/Resources required

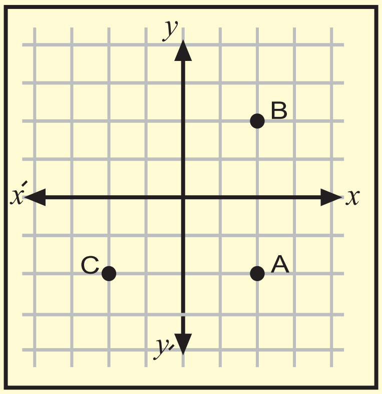
board, chalk/marker, work sheet, graph paper, rubber, pencil

Introduction

Activity

- Ask the students to sit in pairs.

- Provide them graph paper.
- Ask them to draw the Cartesian coordinates.
- Instruct them to mark any three or four points.
- Ask them to interchange their graph sheet.
- Ask them to find the coordinates of points.
- Then ask them to join the points:



- Ask them to discuss their findings.
- Also ask them to find distances AB , BC and CA.
- Ask them to re-check the work of each other.

Development

Activity 1

- Ask the students to sit in the groups.
- Write a question on the board. e.g. show that the point A (-1,2), B (7, 5) C(2,-6) are the vertices of a right angle triangle.
- Ask the students to draw, solve and discuss in their groups.

Activity 2

- Write the two points $A(x_1, y_1)$, $B(x_2, y_2)$ on the board.
- The mid points of $\overline{AB}$ can be found as

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

e.g. if A (4, 1) , B (-2, -5)

Then the mid points of line AB are:

$$\left(\frac{4+(-2)}{2}, \frac{1+(-5}{2}\right)$$

$$\left(\frac{2}{2}, \frac{-4}{2}\right)$$

$$(1, -2)$$

Activity 3

- Ask the class to sit in groups.
- Write the problem on the board.

  e.g. A(9, 3), B(-7, 7), C(-3, -7), D(-5, 5) are the vertices of quadrilateral.
- Find the mid points of its sides.
- Ask them to discuss their findings between them and in front of the class.
- The group presenting its will be winner.

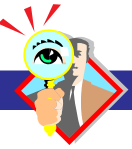
Conclusion/sum up

if $A(x_1, y_1)$, $B(x_2, y_2)$ are two points the distance $\overline{AB}$ can be found by the formula:

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

and mid points of AB can be found by the formula:

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$



Assessment

- Find x such that ($\sqrt{3}$, -1), B (0, 2) and C (x, -2) are the vertices of a right angle triangle with right angle at vertex A.

- Also find mid points of it sides.

Follow-up

- Ask them take three points on graph paper.
- Draw a triangle.
- Find the coordinates of all the three points.
- Find the length of all three sides.
- Also find the mid points.
- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

UNIT 16

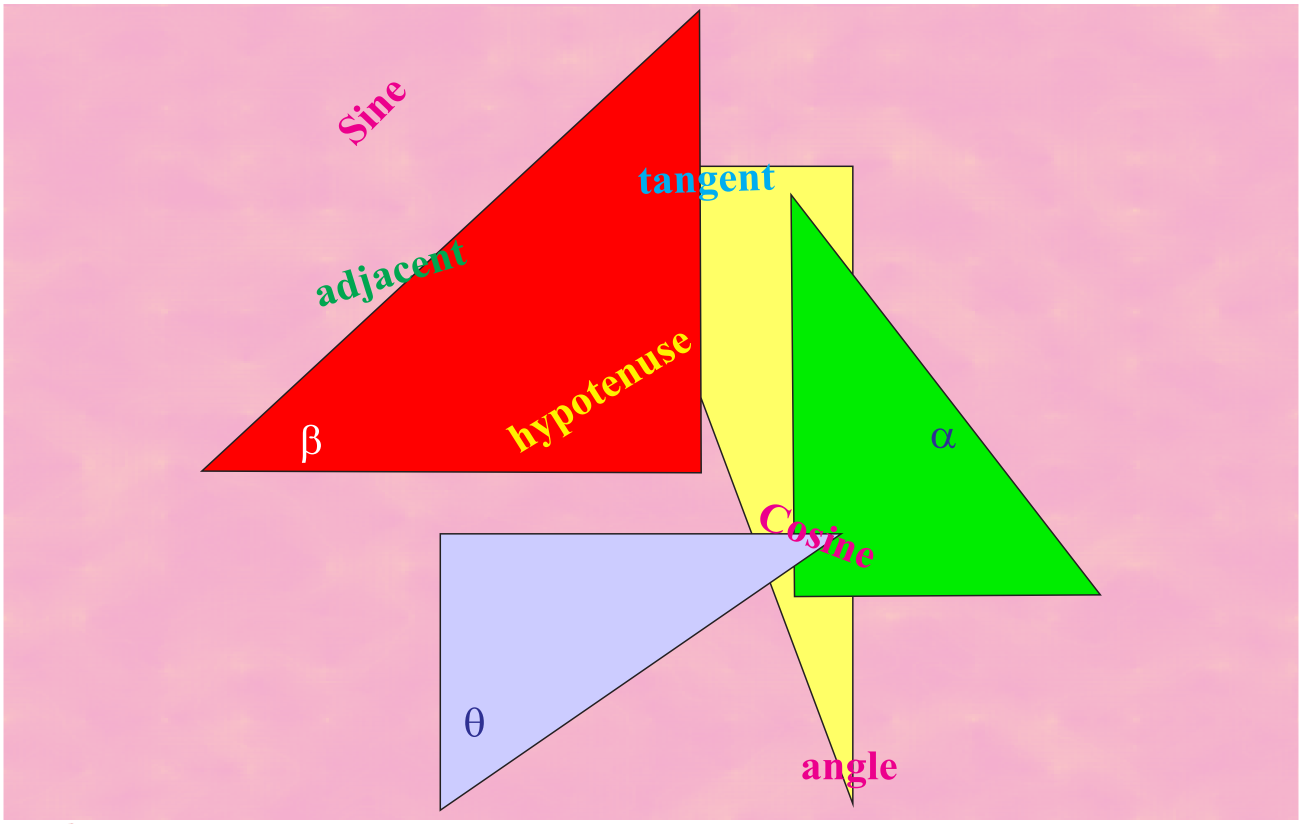
Lesson Plan 1

TOPIC

Angle of Elevation and Depression

Introduction to Trigonometry

Grade IX-X



Students' Learning Outcomes

- Find angle of elevation and depression.
- Solve real life problems involving angle of elevation and depression.

Information for Teachers

- **Trigonometry** (from Greek trigōnon "triangle" + metron "measure") is a branch of mathematics that studies triangles and the relationships between their sides and the angles between these sides.
- **Introduction of Trigonometry** Sumerian astronomers introduced angle measure, using a division of circles into 360 degrees. They and their successors the Babylonians studied the ratios of the similar triangles and discovered some properties of these ratios, but did not turn that into a systematic method for finding sides and angles of triangles. The ancient Nubians used a similar methodology. The ancient Greeks transformed trigonometry into an ordered science.

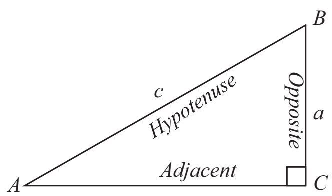
The first trigonometric table was apparently compiled by Hipparchus, who is now consequently known as "the father of trigonometry."

- **Trigonometric ratios;** If one angle of a triangle is 90 degrees and one of the other angles is known, the third is thereby fixed, because the three angles of any triangle add up to 180 degrees.
- Once the angles are known, the ratios of the sides are determined, regardless of the overall size of the triangle.
- If the length of one of the sides is known, the other two are determined.
- **Trigonometric functions**: These ratios are given by the following **trigonometric functions** of the known angle *A*, where *a*, *b* and *c* refer to the lengths of the sides in the accompanying figure:
  - **Sine function (sin),** defined as the ratio of the side opposite the angle to the hypotenuse.

$$\sin A = \frac{opposite}{hypotenuse} = \frac{a}{l'}$$

  - **Cosine function (cos), defined as the ratio of the adjacent leg to the hypotenuse.**

$$\tan A = \frac{opposite}{adjacent} = \frac{a}{c} = \frac{\sin A}{\cos !}$$

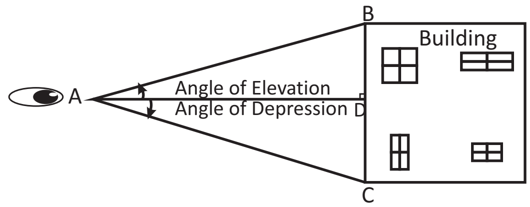


Note: The terms **perpendicular and base** are sometimes usede for the opposite and adjacent sides respectively. It easy to remember what sides of the right triangle are equal to sine, cosine, or tangent, by memorizing the word SOH-CAH-TOA

Sin = Opposite ÷ Hypotenuse
Cosine = Adjacent ÷ Hypotenuse
Tangent = Opposite ÷ Adjacent

One way to remember the letters is to sound them out phonetically (i.e "SOH-CAH-TOA" which is pronounces 'so-ke-tow-uh) Another method is to expand the letters into a sentence, such as "some Old Hippy Caught Another Hippy Trippin On Acid"

- **Angle of Elevation and Depression**



Duration/Number of Periods

80 mins/2period

Material/Resources Required

a clear plastic ruler or straw, a clear plastic protractor, clear tape, cotton and a small weight

Introduction

Activity 1

Discuss applications or uses of trigonometry.

Application/Uses of trigonometry

Sextants are used to measure the angle of the sun or stars with respect to the horizon. Using trigonometry and **amarine chronometer,** the position of the ship can be determined from such measurements.

There are an enormous number of uses of trigonometry and trigonometric functions. For instance, the technique of **triangulation** is used in **astronomy** to measure the distance to nearby stars, in geography to measure distances between landmarks, and in **satellite navigation systems.** The sine and cosine functions are **fundamental** to the theory of **periodic functions** such as those that describe sound and light waves.

Fields that use trigonometry or trigonometric functions include astronomy music theory, optics, analysis of financial markets, electronics, probability theory, statistics, biology, medical imaging (CAT scans and ultrasound), pharmacy, chemistry, number theory many physical sciences, land surveying architecture, economics, electrical engineering, mechanical engineering, civil engineering, computer graphics, game development etc.

Activity 2

Discuss the role of trigonometry and geosciences.

Mass wasting

- Calculating slope steepness, gradients and forces on a hill slope, determining slope stability.

Topographic maps

- Determining compass direction and orienteering, calculating slopes, locating objects and landmarks, calculating the distance to objects far - away.

Structural geology

- Measuring the tilting of rocks, determining the location of rocks in the subsurface, marking geologic maps.

Oceanography

- Describing ocean waves, calculating wind and current direction.

Seismology

- Measuring seismic waves, determining the nature of the interior of the Earth through refraction.

Clinometers are instruments that measure the angles and then the height of objects

Activity 3

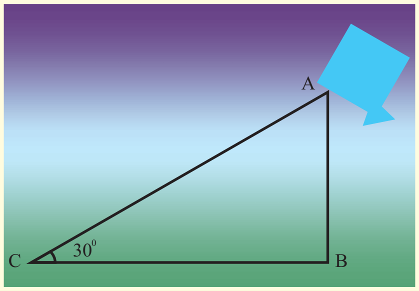
Divide the class in groups and give following questions or more to solve.

A little kid is flying a kite. The string of the kite makes an angle of 30 degrease with the ground. If the height of the kite is h = 12 ft, find the length of the string that the boy has used?

Solution:

1) sin (30) = 12ft/x (This follows our SOH-CAH-TOA rule, in this we are using the SOH, sin opposite over hypotenuse)
2) Then we need isolate x on one side, so the x is canceled out and goes on the other side. The sin (30) is placed on the other side on top of the 12ft

   X = 12ft/sin (30)
3) 24ft (Answer)



Q2. You are at a live performance and you are in the balcony. You are looking down at the stage at an angle of 40 degrees, If you were to be at floor level you would be 20 ft away from the stage. How high up is the balcony?

Solution:

Tan 40 = 20/x

20 = x tan 40

20/ tan 40 = X

X = 23.84

Development

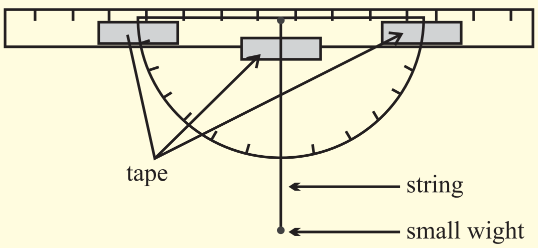
Activity 1

Demonstrate and describe how trigonometry can be used to find the height of a tall building or tree height of a high hill, or other high object where one cannot stand directly beneath the highest part.

Clinometers are instruments that measure the angles and then the height of objects

- Now first students will construct clinometers in group of three, and then they will use it to measure angles and to find heights.
- Draw pictures on board and then explain about angle of elevation and angle of depression.
- Ask students for the examples come into their minds.
- Divide the class into groups and instruct them to make their clinometers to measure angle of elevation.
- Follow the given steps: (write on chart paper and paste or write on board).

1. Ask students to drill a small hole into their protractor.
2. The straw and protractor will be perpendicular to each other. Put the string through the hole and the zero degree point on the protractor and tie it off or tape it to the protractor.
3. Tape or glue the protractor to the straw. If you plan on using the rulers and protractors again as separate tools, use tape instead of glue.
4. Attach weight to end of string.

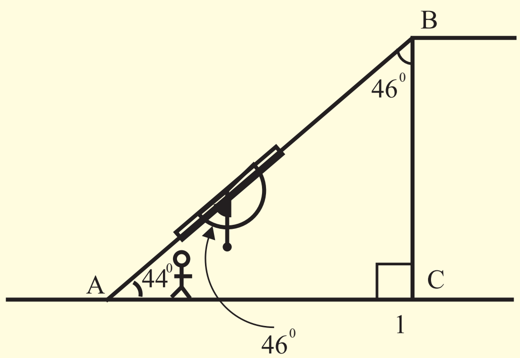


- Explain in groups about how to use clinometers to measure angle. Using your clinometers:

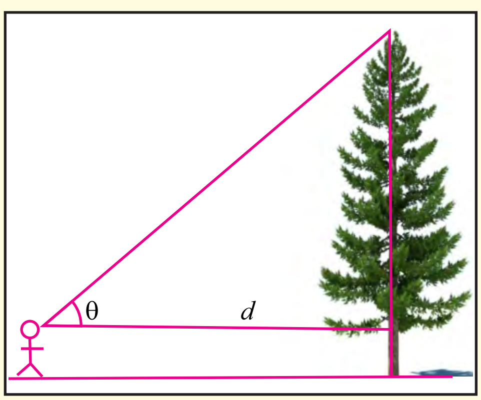
1. Looking through the straw or down the ruler, point the clinometers at the top of the object. As you move the clinometers up, the string and weight will start to move down the protractor.
2. When the clinometers is lined up with the top of the object, use your other hand to grab the string so that it stays where it is along the clinometers and read the angle that the string is lying against or, have another person read the angle that the string is lined up with.
3. Measure how far you are from the object you are measuring.
4. Apply the formula and calculate the height.

Activity 2

Divide the class into groups then with the help of clinometers, allow the students to measure the angle ABC. They are then able to calculate the angle BAC. The students can then be sent to a few places around the school to practice measuring such angles. Usually it is best for one person to hold the clinometers in such a way that his or her eye looks along the ruler to the top of the object concerned, and a team member reads off the angle. Students should take turns doing this.



Carry out the activity, and write up a poster explaining how this procedure is carried out. Each student will need to measure the



Distance between his or her feet and eyes. Imagine the tall object to be measured is a tree. The diagram above indicates how this is done. Students measure the distance d from the base of the tree. They use the clinometers to find the angle θ. Using tan they can find h (because h = d tan θ). And of course, they need to add the distance between their feet and eyes at the end.

Each group should make tree measurements of the angle concerned (each at a different distance); one for each member of the group. One their return to class, lest the activity be quickly forgotten, get each student to make a small poster that fully describes how clinometers is make and how it was used for making the relevant measurements just completed.

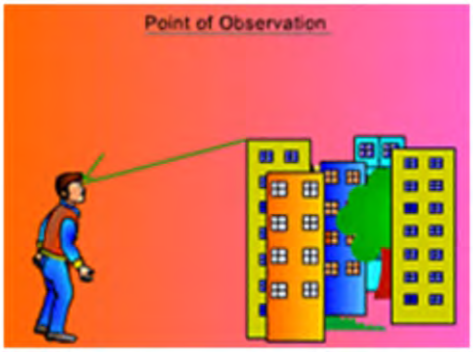
Distance between his or her feet and eyes. Imagine the tall object t be measured is a tree. The diagram above indicates how this is done. Students measure the distance d from the base of the tree. They use the clinometers to find the angle θ. Using tan
they can find h (because h = d tan θ). And of course, they need to add the distance between their feet and eyes at the end.

Each group should make tree measurements of the angle concerned (each at a different distance); one for each member of the group. One their return to class, lest the activity be quickly forgotten, get each student to make a small poster that fully describes how clinometers is make and how it was used for making the relevant measurements just completed.

Activity 3

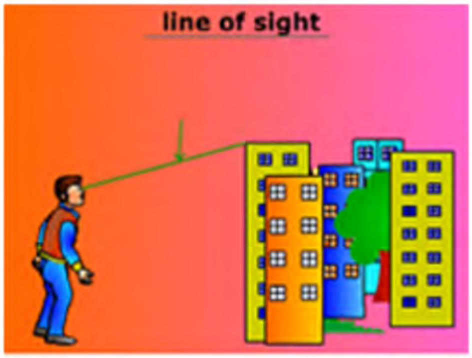
Following terms, are essential to know because these terms help you to apply trigonometric ratios, in the problems related with height and distance. Point of **Observation:**

Whenever you see any object, your eye works as a point of observations. Though human beings can see an object with both eyes but we, for sake of convenience, consider it as we are watching the object with the single eye. The eye or the point from where you are watching the object is known as point of observation. In the pictorial representation of a height and distance problem this is represented by a single dot (point). There can be more than one point of observation you see an object from two or more than two points.



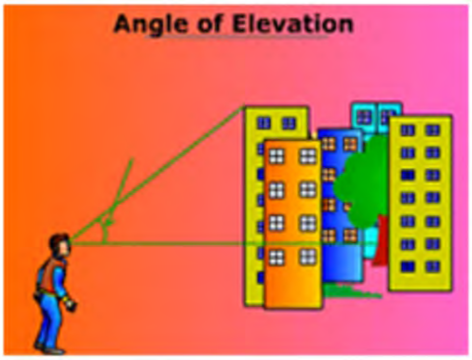
Line of Sight:

From the point of observation we see the object, it is supposed that we are not watching the object but only a particular point on it. It can be at the top of the object or at the bottom or anywhere else in the body of that object (according to the given problem). An imaginary line segment joining that point with the point of observation is known as line of sight or alternatively "line of vision".



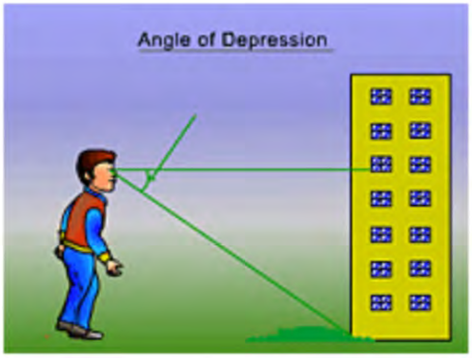
Angle of Elevation:

The object which is in the consideration (in the sight) can be above or below the level of the eye (point of observation). At the level of point of observation we will draw a horizontal line. When the object is above the level of eye, then the angle which line of sight is making with this horizontal line is known as angle of elevation.



Angle of Depression:

When the object is below the level of point of observation, then the angle made by the line of sight with the horizontal line (drawn at the level of eye) is known as angle of depression.



Conclusion/Sum up

The trigonometry is everywhere, in order to good with number; you have to be good in trigonometry. The trigonometry is art of science. It can be used to measure the heights of mountains easily. This is the basic information to aircraft designing and navigation. Carpenters need basic trigonometry. You can explore more of it!

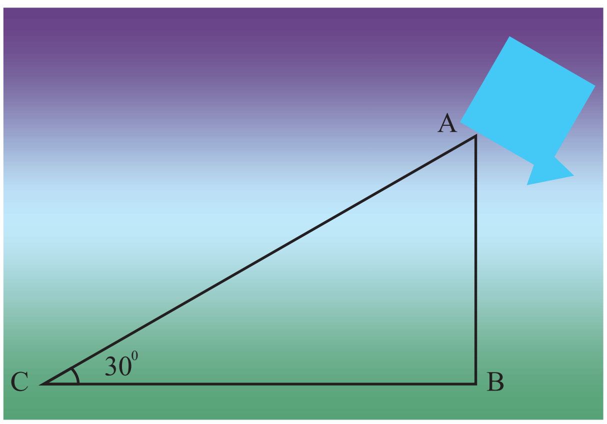
Also recap concepts of angle of elevation and depression.

Assessment

Divide the class in groups and provide worksheet or make pictures on board with hints and ask them to solve the questions.

Problem (I)

A kite is flying at a height of 30m, from a point on the ground its angle of elevation is 30°. Find the length of string.



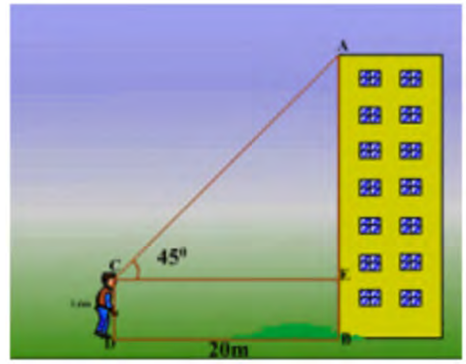
The pictorial representation of this problem is the ΔABC.

Problem (II)

A man is 1.6m tall. He watches at the top of a tower which is 20m away from him and finds the angle of elevation 45°. Find the height of the tower.

Hint:

The pictorial representation of this problem is the adjacent figure. In the figure AB represents the height of the tower CD represent the height of man CE and DB represents the distance of man from the tower. Here AC represents the line of sight and $\angle ACE = 45^\circ$ represent the angle of elevation.



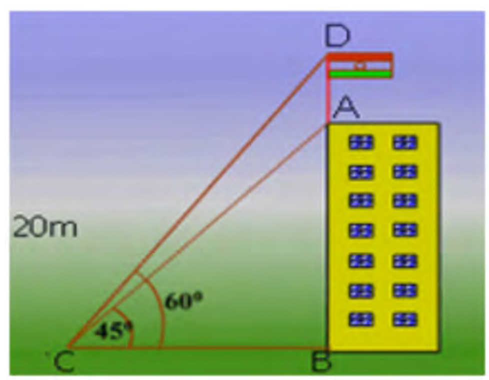
Follow-up

Problem (I)

A flag is mounted at the top of a building. From a point on the ground the angle of elevation of the top and bottom of the flag are 60° and 45°. If the building is 20m high then find the height of flag.

Hint:

The pictorial representation of this problem is the adjacent figure. In the figure AB represents the height of building. AD represents the flag staff. C represents the point of observation on the ground AC is the first line of sight and CD is another. $\angle ACB = 45^\circ$ and $\angle DCB = 60^\circ$ are the two angles of elevations.



Problem (II)

Two cars are heading towards a tower through a straight road. From the top of the tower the angles of depression of these cars are 30° and 60°. If the height of the tower is 30m. Then find the distance between these cars.

Hint:

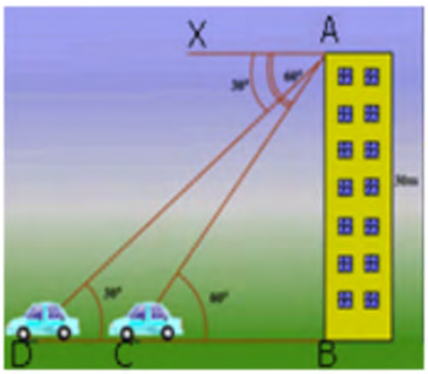
The pictorial representation of this problem is the adjacent figure AB represents the height of tower.

Here A is point of observation C and D represents each car.

AX is the imaginary horizontal line. AC and AD are the two lines of sight. $\angle CAX$ and $\angle DAX$ are the two angels of depression $\angle DAX = 30^\circ$ and $\angle CAX = 60^\circ$ AX and BD are the two parallel lines

$\angle CAX = \angle ACB = 60^\circ$

and $\angle XAD = \angle ADB = 30^\circ$[alternative angles



Problem (III)

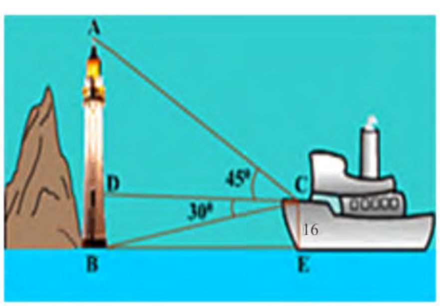
From the deck of a ship the angle of elevation of the top of a light house is 45°. While the angle of depression of the bottom of the light house is 30°. If the deck of the ship is 16m high then find the height of the light house.

Hint:

The pictorial representation of this problem is the adjacent figure. In the figure AB represents the light house. CE represent the height of the deck of the ship C is the point of observation. CD the horizontal line which is situated at the level of point observation.

BE is the ground level (level of sea).

AC and CB are the two lines of sight



$\angle ACD = 45^\circ$ is the angle of elevation

$\angle DCB = 30^\circ$ is the angle of depression

Since CD | | BE

$\angle DCB = \angle CBE = 30^\circ$

UNIT 17

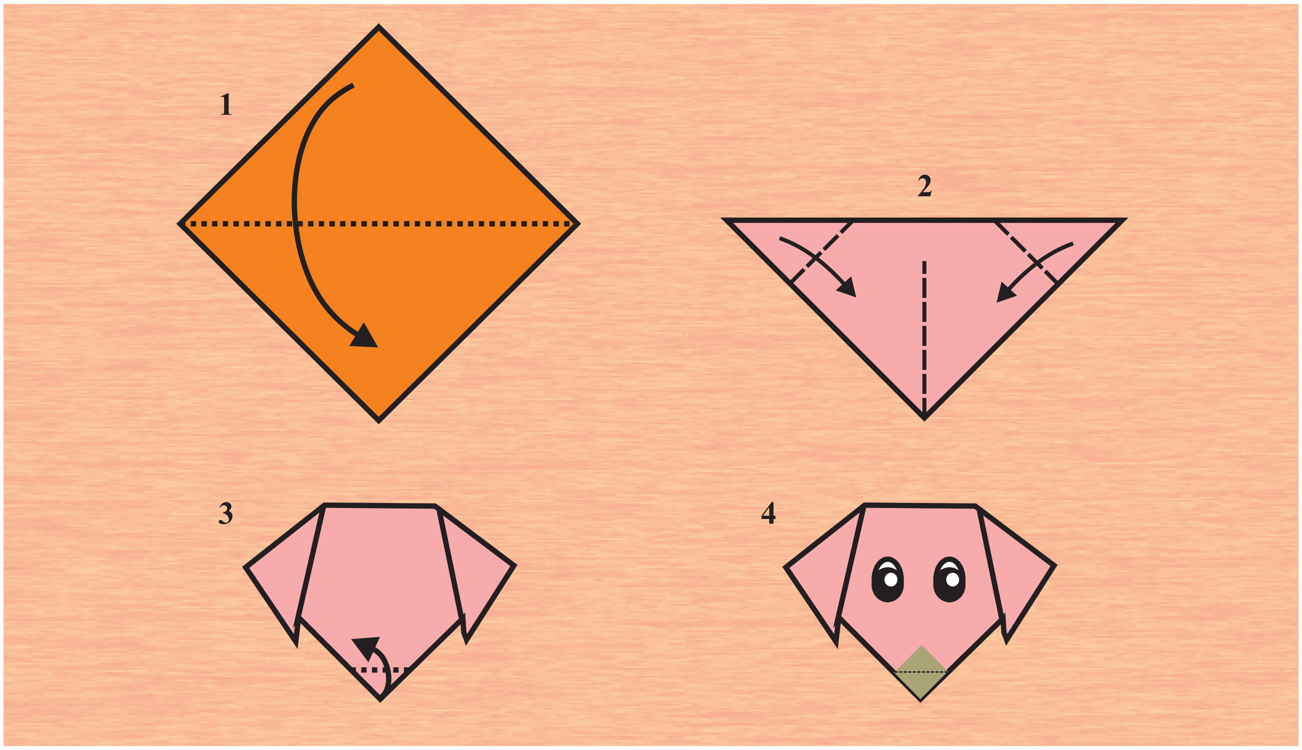
Lesson Plan 1

TOPIC

Congruent Triangles

Congruent Triangles

Grade IX



Students' Learning Outcomes

Prove the following theorems along with corollaries and apply them to solve appropriate problems.

- In any correspondence of two triangles, if one side and any two angles of one triangle are congruent to the corresponding side and angles of the other, the two triangles are congruent. (ASA)
- In a correspondence of two triangles, if three sides of one triangle are congruent to the corresponding three sides of the other, the two triangles are congruent.(SSS)

Information for Teachers

- A figure and its image under translation, rotation or reflection are congruent.
- There are five ways to test that two triangles are congruent.
- C P C T C means corresponding parts of congruent triangle are congruent.
- SSS when 3 corresponding sides are equal.

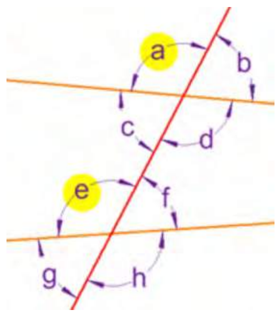
Congruent Shapes in the floor cushion

- SSA or SAS or ASS when two corresponding sides and one corresponding angle is equal.
- ASA or AAS or SAA when two corresponding angles and one corresponding side are equal.
- **Corresponding Angles**

When two lines are crossed by another line (called the Transversal) The angles in matching corners are called **Corresponding Angles**.

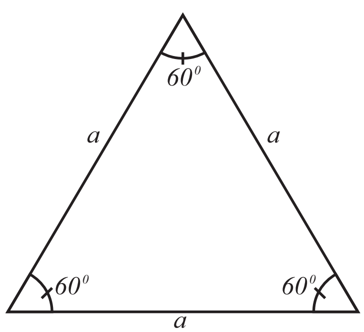
In this example, these are corresponding angles:

- a and e
- b and f
- c and g
- d and h



- **Equilateral triangle**

An equilateral triangle is a triangle with all three sides of equal length, corresponding to what could also be known as a "regular" triangle. An equilateral triangle is therefore a special case of an isosceles triangle having not just two, but all three sides equal. An equilateral triangle also has three equal 60^0 angles



Duration/Number of Periods

80 mins/2period

Material/Resources Required

Colour papers, tooth picks, charts scales of different colours

Introduction

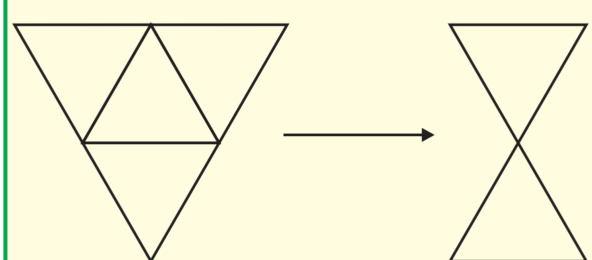
Activity 1

- Share **congruency**-examples from daily life:

  "We are able to buy a **refill** for a pen, we are able to **replace** any part of our cycle. The medium size socks of a brand are alike. In each of these examples the **"idea of sameness of size and shape comes in, mathematically we call this notion Congruence"**
- Let's identify and list down congruent objects in our school, home or classroom. (On-board activity, spend 3-min. on this)

Activity 2

- Show them two scales of different colours but same size and talk about congruency.
- Brain-storming about terms like vertex, corresponding angles and sides, types of triangles etc. to trigger their thinking about triangles.
- Then introduce corresponding angles and sides concept.
- The diagram below show four equilateral triangles formed by using 9-toothpics. By removing three toothpicks and rearranging the figure, can you form congruent triangles?

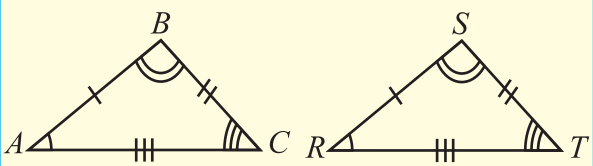


Development

Activity 1

- Have students draw a triangle and cut it out. Use it as a pattern to draw a second triangle and cut that triangle out. If one triangle is placed on top of the other, the two coincide or match exactly. This means that each part of the first triangle matches exactly the corresponding part of the second triangle. You have made a pair of congruent triangles.
- Discuss corresponding parts of congruent triangles on the board

If Δ ABC is congruent to ΔRST (ΔABC Δ RST). The vertex labeled A corresponds to the vertex labeled R. Vertex B corresponds to S. and vertex C corresponds to T. This correspondence can be described in terms of angles and sides as follows.



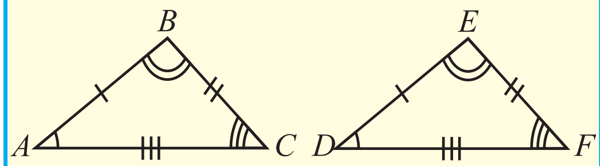
∠A corresponds to ∠R	$\overline{AB}$ corresponds to $\overline{RS}$
∠B corresponds to ∠S	$\overline{BC}$ corresponds to $\overline{ST}$
∠C corresponds to ∠T	$\overline{AC}$ corresponds to $\overline{RT}$

Since the two triangles match exactly, the corresponding parts are congruent.

- Conclude the discussion on the abbreviation CPCTC means corresponding part of congruent triangles are congruent.
- Assign an example question

Example: The Adams family is having their game room renovated. This room will have two triangular windows.

a. Draw two identical windows and label the vertices ABC on one window and DEF on the other so that ΔABC Δ DEF.



a. What able in ΔABC is congruent to F in ΔDEF? - F

b. Which side of ΔDEF is congruent to AC in ΔABC? DF

Activity 2

- Divide the class into groups of five and ask each member of a group to make any triangle by paper cutting. (no angles and no sides are given)
- After the triangles are made, ask them to write their names on the other side of triangle. Let them compare within the group and later on with other groups to find their congruent pair.
- Now every student has got their pair and their triangles are also congruent.
- In their pairs ask them to identify the triangle's are also congruent.
- In their pairs ask them to identify the triangle's corresponding sides and angles.
- Now present the postulates SSS, ASA and their meanings.
- Ask the pairs if they can find these postulates in the congruent triangles they have. For example: *mAB=mLM* and list all six of them.

- Ask students to paste their pair of congruent triangles on the soft-board or anywhere in the class so that work can be seen. Appreciate the correct labeling.
- Invite students on board to explain the postulates once again.

Activity 3

If in any correspondence of two triangles, two angles and one side of a triangle are congruent to the corresponding two angles and one side of the other, the triangles are congruent.

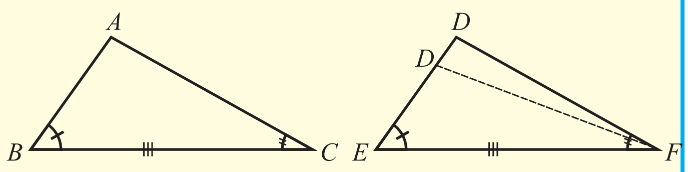
- On the board first write the statement, draw the figure and the table as given below. However do not fill the table.

Given : In $\Delta ABC \longleftrightarrow \Delta DEF$

$\angle B \cong \angle E$

$\angle C \cong \angle F$

$\overline{BC} \cong \overline{EF}$



To Prove : $\Delta ABC \quad \Delta DEF$

Construction Suppose $\overline{AB} \cong \overline{DE}$ and there is a point $D^{/}$ on such that $\overline{AB} \quad \overline{D^{/}E}$. Join $D^{/}$ to F.

Proof :

Statements		Reasons
In $\Delta ABC \longleftrightarrow \Delta D^{/}EF$		
$\overline{AB} \cong \overline{D^{/}E}$	(i)	Construction / Supposition
$\overline{BC} \cong \overline{EF}$	(ii)	Given
$\angle B \cong \angle E$	(iii)	Given
$\therefore \Delta ABC \cong \Delta D^{/}EF$		S.A.S. Postulate
So, $\angle C \cong \angle D^{/}EF$		Corresponding angles of congruent triables)
But $\angle C \cong \angle DEF$		Given
$\therefore \Delta DEF \cong \Delta D^{/}EF$		Both congruent to $\quad /$
This is possible only if D and $D^{/}$ are the same points.		
So, $\overline{AB} \cong \overline{DE}$		Proved that D and D' are the same points
Thus from (ii), (iii) and (iv), we have		
$\Delta ABC \quad \Delta DEF$		S.A.S. Postulate

- Discuss what is given and what we have to prove. Help students to think and come up to the statements which you will write in the left column.
- Let the groups discuss and fill the right side of the column. These are the points which they have already discussed enough. [give time]

Corollary:

If in any correspondence of two triangles, two angles and the included side of one triangle are congruent to the corresponding two angles and the included side of the other, the triangles will be congruent. (A.S.A ≅ A.S.A)

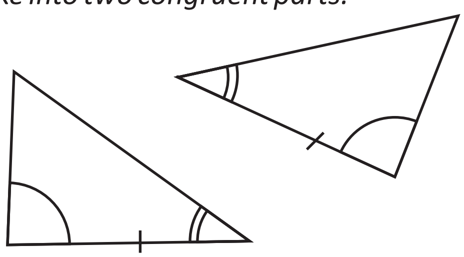
Conclusion/Sum up

Recap the definition of geometric terms used to prove theorems and triangle postulates.

Assessment

a. Draw two triangles on board which are similar but not congruent. Ask students to analyze tell ***why***, *they are not called congruent, By reflecting on points discussed.*

b. *How many ways can you cut a square piece of cake into two congruent parts.*



Q2. **Prove the following:**

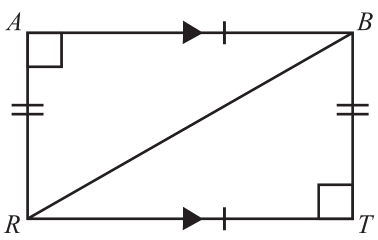
Given: $\overline{AB} \prod \overline{RT}$

$\overline{AR} \perp \overline{AB}$

$\overline{BT} \perp \overline{RT}$

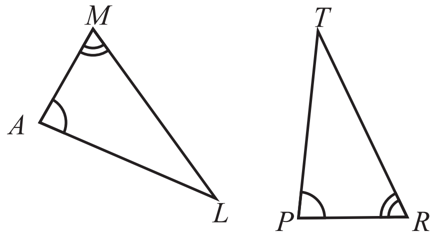
$\overline{AB} \cong \overline{RT}$

$\overline{AR} \cong \overline{Ç}$



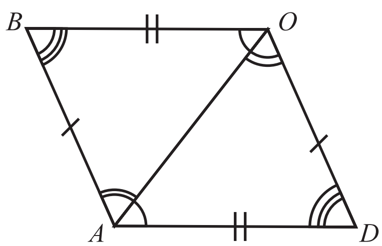
Prove: ΔABR ≅ ΔTRB

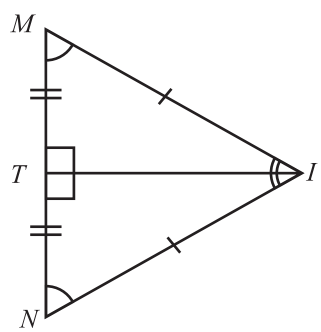
Refer to ΔALM and ΔPRT. Name one additional pair of corresponding parts that need to be congruent in order to prove ΔALM ≅ ΔPRT. What postulate would you use to prove the triangles are congruent?



Q3. In a correspondence of two triangles, if three sides of one triangle are congruent to the corresponding three sides of the other, the two triangles are congruent.(SSS)

Q4. Write a congruence statement for the congruent triangles in each diagram.





Follow-up

- **Project:**

Student's conduct this project at home in the form of group to understand congruent triangle postulates and theorems using inductive reasoning, write their findings and discuss in class. Display the model in class with cards.

Materials needed: straws, protractor, ruler, and construction paper or cardstock

Groups: small groups from 2 to 4 students

Have students cut straws into the following lengths:

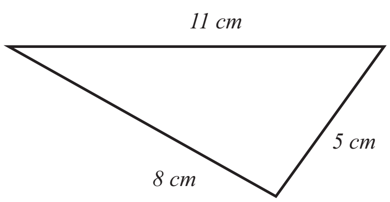
2 straws 8 centimeters in length

2 straws 11 centimeters in length

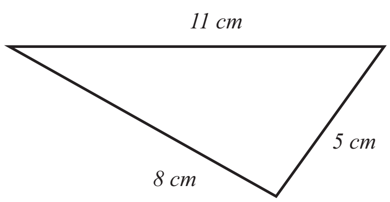
2 straws 5 centimeters in length

Part 1

1. Have Put the 3 straws of different lengths together to form a triangle as shown.



2. Form another triangle with the other set of straws.



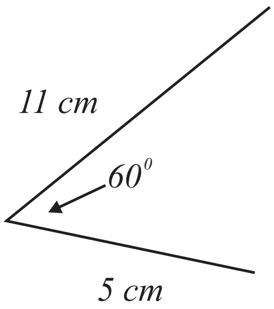
3. Measure the angles of both triangles using a protractor.

Questions:

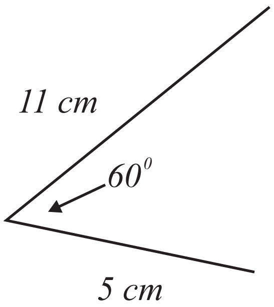
1. What are the measures of the 3 angles in the first triangle?
2. What are the measures of the 3 angles in the second triangle?
3. What is the relationship between the angles of each triangle?
4. Are the triangles congruent?
5. Can the straws be rearranged to form a triangle with different angles?

Part 2

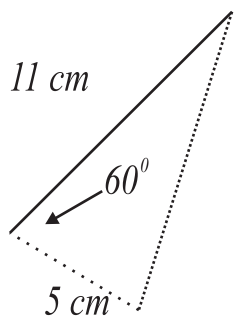
1. Take 2 of the straws, place them on a piece of paper, and form a 60 degree angle between them.

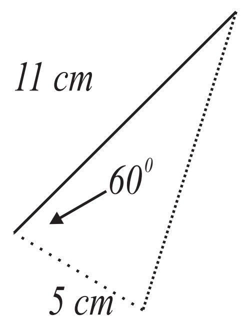


2. Take the 2 straws of the same length and also form a 60 degree angle between them.



3. Draw a line to represent the 3rd side. Repeat the process for the 2nd triangle.





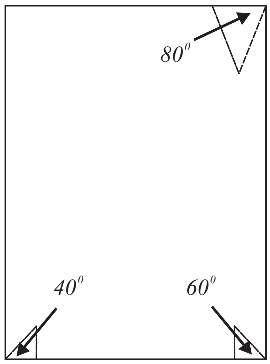
4. Measure the length of the 3rd side and the two remaining angles for each triangle.

Questions:

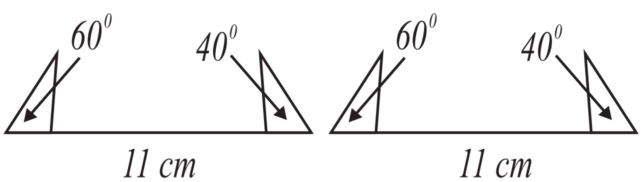
1. What is the length of the 3rd side?
2. What are the measures of the remaining angles?
3. Are the two triangles congruent?
4. Use any two straws and any angle of your choice. Do you get the same result?
5. Will you always get the same result?

Part 3

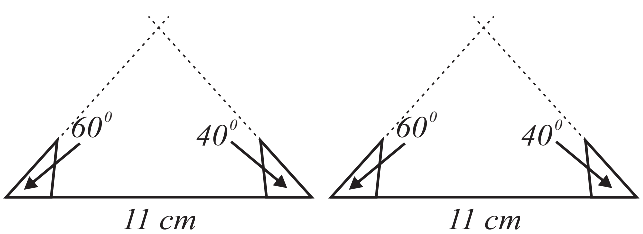
1. Measure three angles measuring 80, 60, and 40 degrees on the corners of 2 pieces of construction paper or cardstock, cut them out, and label them.



2. On a piece of paper, take one of the straws, and place two of the cut-out angles on each end as shown. Repeat the process for the 2^{nd} triangle.



3. Using a ruler, draw a segment along each of the angle. The two segments should intersect forming the last angle. Repeat the process for the 2^{nd} triangle.



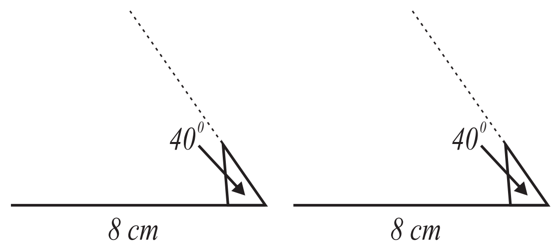
4. Measure the 3^{rd} angle and the lengths of the 2 sides in each triangle.

Questions:

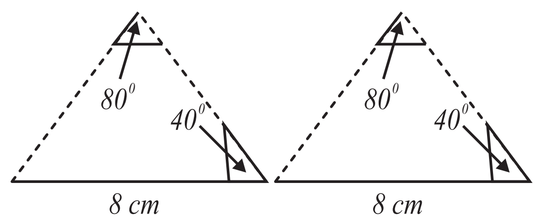
1. What is the measure of the 3^{rd} angle for each triangle?
2. What are the measures of the remaining 2 sides for each triangle?
3. Are the triangles congruent?
4. What if you used the 5cm straw? The 8cm straw? A straw with a different length?

Part 4

1. Use two of the angles used in the example above.
2. Use one of the straws and place one of the angles alongside it as shown. Draw a long segment like the dashed one in the drawing. Repeat the process for the 2^{nd} triangle.



3. Place the second angle along this segment so that when a 2^{nd} segment is drawn, it will connect with the end of the straw.



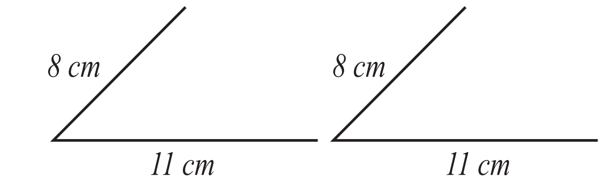
4. Measure the 3^{rd} angle and the two remaining sides.

Questions:

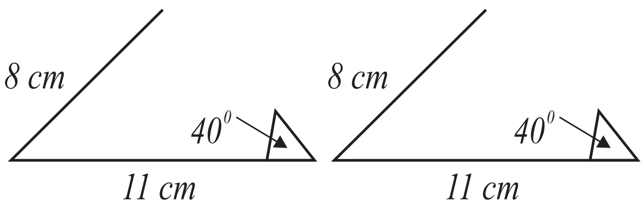
1. What is the measure of the 3^{rd} angle for each triangle?
2. What are the measures of the remaining 2 sides for each triangle?
3. Are the triangles congruent?

Part 5

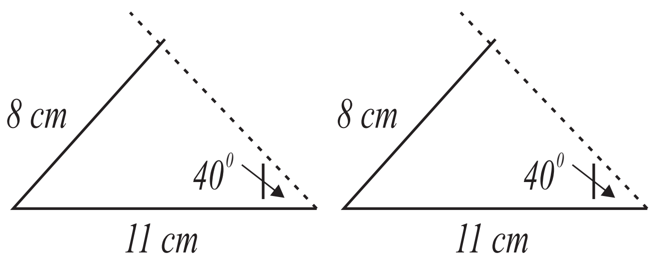
1. Place two of the straws together forming an angle of any degree for one triangle, and repeat the process for the 2^{nd} triangle.



2. Use one of the pre-cut angles and place alongside the longer of the sides but not as the included angle.



3. Draw a segment to connect the 3^{rd} side to the other two sides.



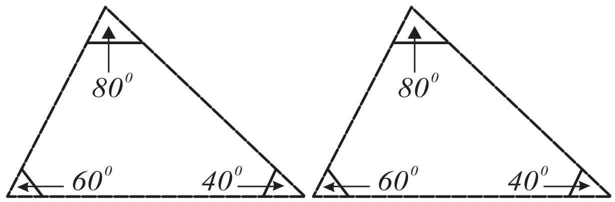
4. Swing the 8cm straw so that it hits the 3^{rd} side at a different spot in the 2^{nd} triangle as in the first.
5. Measure the 3^{rd} side and the remaining 2 angles in each triangle.

Questions

1. What is the measure of the 3^{rd} side for each triangle?
2. What are the measures of the remaining 2 angles for each triangle?
3. Are the two triangles congruent?
4. Do you think that you would get different results if you used a different angle?

Part 6

1. Place the 3 angles so that they can form a triangle without measuring the sides initially. Draw segments connecting the angles. Repeat the process for the second triangle.



2. Measure the 3 sides for each triangle.

Questions

1. What are the measures of the 3 sides for each triangle?
2. Are the two triangles congruent?

Part & (For all Groups)

3. Each group draw two triangles for each part, and using the correct marks, show which sides and angles are congruent. Match the correct shortcut for each set of triangles from the following choices, and tell whether or not the shortcut is valid for proving triangles congruent.

   SSS, SAS, AAS, AAA, ASA, SSA

- S means that the corresponding sides of the triangles are congruent.
- A means that the corresponding angles of the triangles are congruent.

UNIT 18

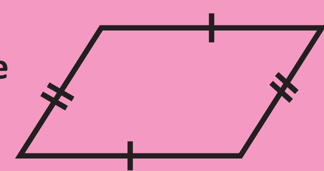
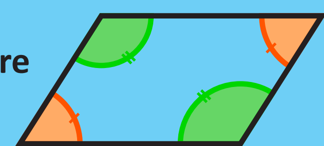
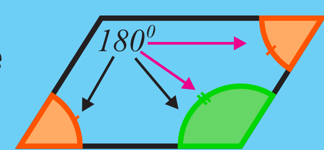
Lesson Plan 1

TOPIC

Parallelograms and Triangles

parallelograms and Triangles

Grade IX

Sides	**...both pairs of opposite sides of a quadrilateral are congruent, then...**	
Angles	**...both pairs of opposite angles of a quadrilateral are congruent, then...**	
	....One angle is supplementary to both consecutive angles in a quadrilateral, then....	
Diagonals	**....the diagonals of a quadrilateral bisect each other, then...**	

Students' Learning Outcomes

Prove the following theorem along with corollaries and apply them to solve appropriate problems.

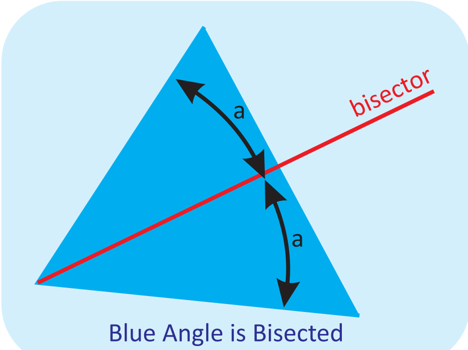
- 1.in a parallelogram :
- Opp. Sides are congruent
- Opp. Angles are congruent
- The diagonals bisect each other.

Information for Teachers

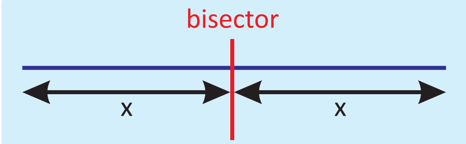
- **A parallelogram**

  is quadrilateral comprised of two pairs of parallel lines. There are several rules involving
- the angles of a parallelogram
- the sides of a parallelogram
- the diagonals of a parallelogram
- **Bisector**

The line that divides something into two equal parts. You can bisect lines, angles, and more.



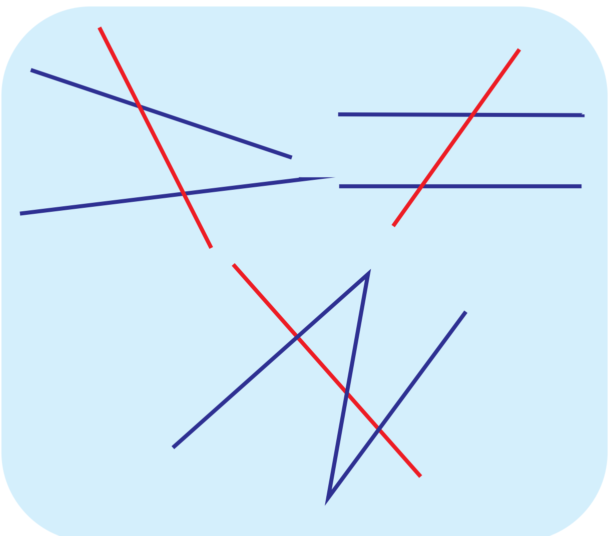
Blue Angle is Bisected



Blue Line is Bisected

- **Transversal**

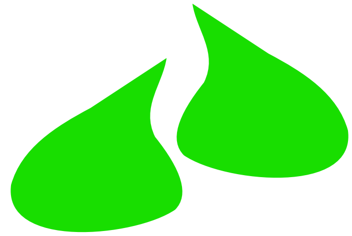
A line that crosses at least two other lines



- **Congruent**

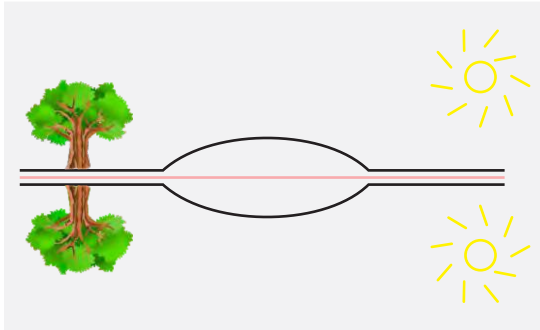
The same shape and size.

Two shapes are congruent if you can Turn, Flip and/or Slide one so it fits exactly on the other. In this example the shapes are congruent (you only need to flip one over and move it a little)



- **Reflection**

An image or shape as it would be seen in a mirror, or in a smooth lake.



- **The Angle Side Angle postulate**

The Angle Side Angle postulate (often abbreviated as ASA) states that if two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then these two triangles are congruent.

Duration/Number of Periods

80 mins/2period

Material/Resources Required

Worksheet, board, marker/chalk, charts etc

Introduction

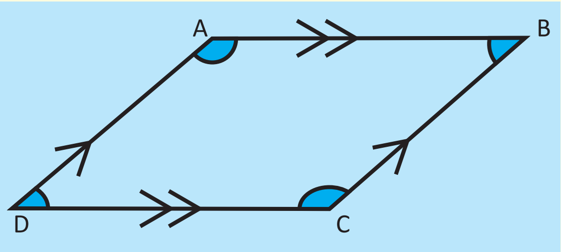
Activity

Divide the class in three groups. Inform the class about following properties of parallelogram by using board or worksheets.

group 1: Inform about the following properties of angles of parallelogram then student solve the questions given below through mutual discussion.

Angles of A Parallelogram

Opposite Angles are Congruent



$$\angle D \cong \angle B$$

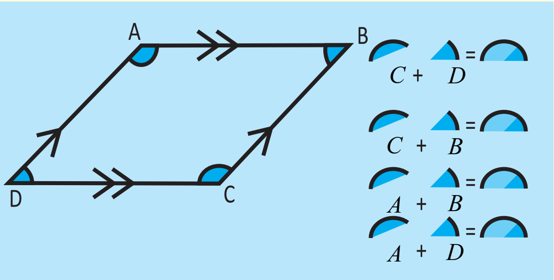
$$\angle A \cong \quad C$$

Triangles can be used to prove this rule about the opposite angle.

Consecutive angles are supplementary.

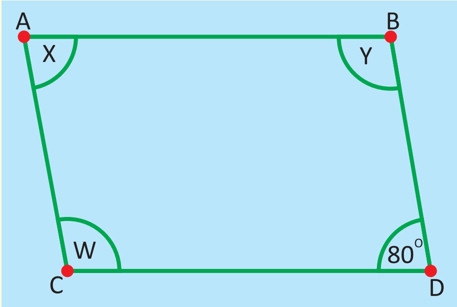
The sum of interior angles of a parallelogram is 360^0 degrees.

C and $\angle$ D
C and $\angle$ B
A and $\angle$ B
A and $\angle$ D

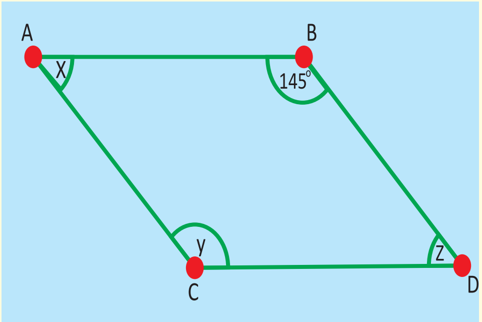


The following pairs of angles are supplementary

What is the measure of angles A, B and C in parallelogram ABCD?



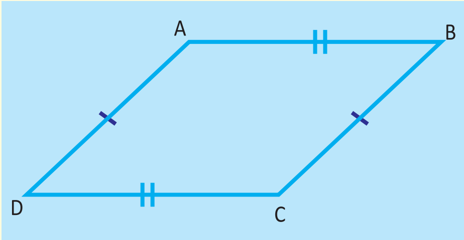
What is the measure of x, y, z in parallelogram below?



In a parallelogram one of the angles measures 25°, what are the measures of the other angles

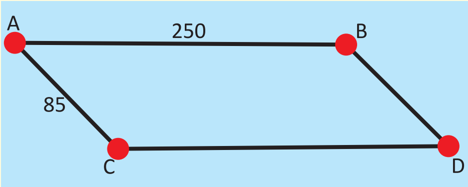
group 2: inform about the following property of sides of parallelogram then student solve the question given below through mutual discussion.

Sides of a Parallelogram



The opposite sides of a parallelogram are congruent.

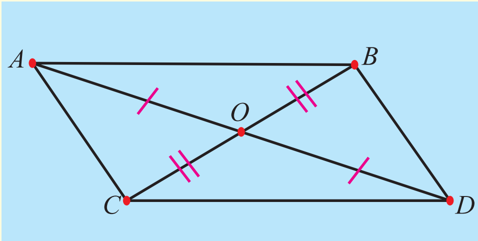
Triangles can be used to prove this rule about the opposite sides.



What is the length of sideBD and side CD in parallelogram ABCD?

Group 3: inform about the following property of diagonals of parallelogram then student solve the question given below through mutual discussion.

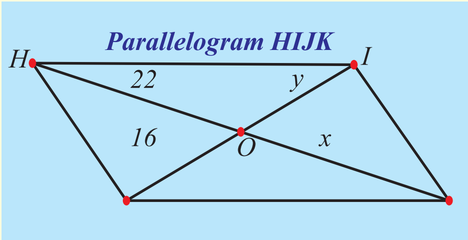
Diagonals of a Parallelogram



The diagonals of a parallelogram bisect each other.

AO=OD, CO=OB

What is x and Y?



At the end whole class discussion to find conclusion.

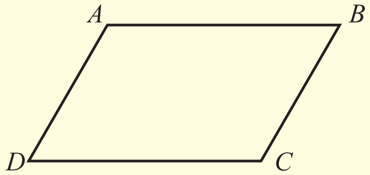
Development

Activity 1

- Write following key-words on the board and ask students to recall and write its meanings on notebooks. (CPCTC, supplementary angles, ASA postulate etc).
- After individual work, divide them into groups and let them share their written work.
- On the board first write the statement **"opposite angles are congruent",** draw the figure and the table as given below. However do not fill the table.

- Opposite angles are congruent:

Write a two - column proof of Theorem



Given: ▭ ABCD

Prove: $\angle A \cong \angle C$, $\angle D \cong \angle B$

Prove:

Statements	Reasons
1. ▱ABCD	1. Given
2. $\overline{AB} \parallel \overline{DC}$, $\overline{AC} \parallel \overline{BC}$	2. Definition of parallelogram
3. $\angle A$ and $\angle D$ are Supplementary. $\angle D$ and $\angle C$ are Supplementary. $\angle C$ and $\angle B$ are Supplementary.	
4. $\angle A \cong \angle C$ $\angle D \cong \angle B$	4. Supplements of the same angles are congruent.

Corollary:

The opposite angles in a rhombus are congruent.

- Discuss what is given and what we have to prove. Help students to think and come up to the statements which you will write in the left column.
- Let the groups discuss and fill the right side of the column. These are the points which they have already discussed enough.[give time]

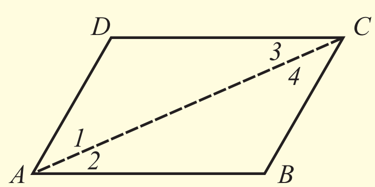
Activity 2

- Split the groups and ask them to prove the theorem individually on their notebooks.
- On the board now write the statement **"opposite sides are congruent",** draw the figure and the table as given below. However do not fill the table.

- Opposite angles are congruent:

Given: Parallelogram ABCD

Prove: A ≅ DC, AD ≅ BC



Statements	Reasons
1. Draw AC	1. Through any 2 pts. There is exactly one line.
2. Parallelogram ABCD	2. Given
3. $\overline{DC} \parallel \overline{AB}$	3. Definition of parallelogram
4. $\angle 3 \cong \angle 2$, $\angle 1 \cong \angle 4$	4. If 2$\parallel$lines are cut by a transversal. A.I.A. alternate interior angles are $\cong$
5. $\overline{AC} \cong \overline{AC}$	5. Reflexive
6. $\Delta ABC \cong \Delta CAD$	6. ASA Postulate
7. $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$	7. C.P.C.T.C

Corollary

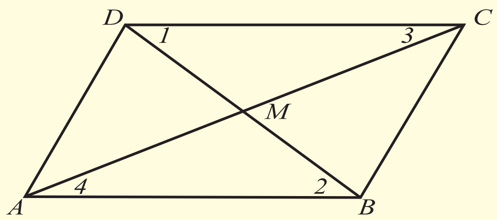
The parallel line from the midpoint of one non-parallel side of a trapezium to the parallel sides bisects the other non-parallel side.

Activity 3

- Follow the same method this will be done quickly as Key-word are already discussed
- Write the statement on board "**Diagonals bisect each other**". Also give the following plan and ask the groups to complete the proof.

Given: Parallelogram with diagonals $\overline{AC}$ and $\overline{DB}$.

Prove: $\overline{AC}$ and $\overline{DB}$ bisect each other.



Plan for proof: You can prove that Δ AMB ≅ ΔCMD using ASA postulate ($\overline{DC} \cong \overline{AB}$ since opposite sides of parallelograms are congruent) and $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$ (since they are alternate interior angles). Then $\overline{DM} \cong \overline{MB}$ and $\overline{AM} \cong \overline{MC}$ by C.P.C.T.C

- Take rounds of the groups and help them in composing their scripts.

Conclusion/Sum up

Recap steps of to prove theorems.

Assessment

Proof: Write the indicated type of proof.

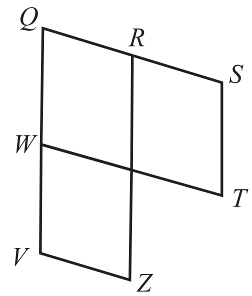
Q1. Two – column

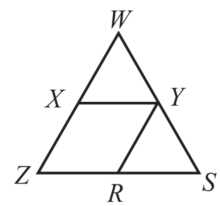
a. **Given:** $\square$ VZRQ and $\square$ WQST

2. Paragraph

b. **Given:** $\square$ XYRZ, $\overline{WZ} \cong \overline{WS}$

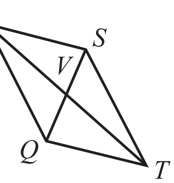
Prove: $\angle Z \cong \angle T$ **Prove:** $\angle XYR \cong \angle S$





Q2. Complete each statement about $\square$ QRST Justify your answer:

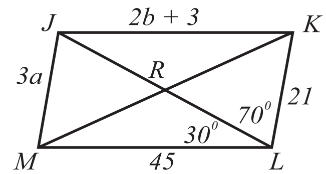
1. SV ≅ ?
2. Δ VSR ≅ ?



3. ∠ TSR is supplementary to?

Q3. Use $\square$ JKLM to find each measure or value.

1. m ∠MJK
2. m ∠ JML
3. m ∠JKL
4. m ∠KJL
5. a
6. b



Follow-up

- Measure sides, diagonals and angles of real shapes of parallelogram around us e.g keyboard buttons etc, draw conclusion and compare result with these theorems.
- Make a parallelogram with paper and measure its diagonals, sides and angles and find result.
- Remember the grometric terms used to prove theorems.
- Guide the students to solve the exercise problems given at the end of unit / chapter of the textbook.

Glossary

Abscissa	The x-coordinate is called abscissa
Addition of matrices	adding corresponding elements of two matrices.
AM (Arithmetic mean)	It is an average which is most commonly used and calculated by diving sum of all values by their numbers
Angle of Depression	The angle made between horizontal line through eye and a line from the eye to the object below the horizontal line is called the angle of depression
Angle of Elevation	The angle between the horizontal line through eye and a line from the eye to the object above the horizontal line is called an angle of elevation
Antilog	Converting the log values back into the real values through consulting the antilog table
Averages	These are values which represent the data
Base / Adjacent	In a right angled triangle, the side facing the common arm of right angle and the angle under consideration is called base/Adjacent
Bijective	Both Injective and Surjective together
Bisector	The line that divides something into two equal parts is called bisector.
Characteristic	The characteristic of a common logarithm shows the position of the decimal point in the associated number.
Class interval	Width of lower and upper class limit
Co – Domain	What may possibly come out of the function is called co-domain
Coefficient	It is a multiplicative factor in some term of an expression (or of a series); it is usually a number, but in any case does not involve any variables of the expression
Collinear	A set of points is collinear if there is a line which contains all the points of the set.
Column of Matrix	Number of vertical categories in any matrix is called column/s of matrix.
Common log	The common logarithm is the logarithm with base 10. It is also known as the decadic logarithm, named after its base
Congruent	Two shapes are congruent if you can Turn, Flip and/or Slide one so it fits exactly on the other.

Constant	Any quantity that has a single value is known as a constant
Corollary	Some results which can be deduced directly from the theorems .
Correspondence angles	When two lines are crossed by another line (called the Transversal), the angles in matching corners are called Corresponding Angles.
Cramer' Rule	By the name of Mathematician who propose the process of solving matrices.
Decibel	The decibel (dB) is a logarithmic unit that indicates the ratio of a physical quantity (usually power or intensity) relative to a specified or implied reference level
Direct variation/Joint variation	A relation between two quantities such that an increase in one quantity causes an increase in the other quantity or a decrease in one quantity causes a decrease in the other quantity in the same ratio is called direct variation or Joint Variation
Domain	What goes into the function is domain
Elements of Matrix	The individual items in a matrix are called its *elements*.
Equilateral triangle	An equilateral triangle is a triangle with all three sides of equal length and also has three equal $60°$ angles
Frequency	The number of observation falls within specific class
Frequency distribution	The distribution of data through which items of data are classified into certain groups or classes and the number of items laying in each group or class is put against that group or class is known as frequency distribution
Frequency table	Arrangement of data into classes / groups along with their frequency
General Function	Shows relations between each member of "A" to a member of "B"
Geometric mean	Positive n^{th} square root of the product of values
Geometrical theorem	The theorem which can be proved with the help of principles of geometry are called geometrical theorem.
Graph	A graph is a drawing which shows the relationship between numbers or quantities
Harmonic mean	Reciprocal of AM and reciprocal of values
Histogram	A graph which represents the class intervals and frequency in the form of adjacent rectangles
Hypotenuse	In a right angled triangle, the side facing right angle is called hypotenuse.
In - equation	An in - equation is a statement that two objects or expressions are not the same, or do not represent the same value

Infinity (symbol ∞)	It is a concept in many fields, most predominantly mathematics and physics, that refers to a quantity without bound or end
Injective	Every member of "A" has **its own unique** matching member in "B"
Integers	Whole numbers with positive and negative signs are integers.
Logarithm	The logarithm of a number is the exponent by which another fixed value, the base, has to be raised to produce that number
Mantissa	The mantissa is the significant and in a common logarithm or floating-point number.
Matrix	A matrix (plural matrices) is a rectangular array of numbers, symbols, or expressions.
Median	Central value after dividing data into two equal halves (ascending or descending order)
Mid point	The central point of lower and upper class limit
Mode	Most frequent value in the data is called mode.
Multiple	Multiple of a number is what you get when you multiply that number by some other whole number
Natural Log	The natural logarithm is the logarithm to the base *e*, where *e* is an irrational and transcendental constant approximately equal to 2.718281828
Number line	A number line is a picture of a straight line on which every point is assumed to correspond to a real number and every real number to a point
Order of Matrix	Number of row/s by number of column/s is called order of matrix which is used to determine size of matrix.
Ordinate	The y-coordinate is called ordinate
Perpendicular / Opposite	In a right angled triangle, the side facing the angle under consideration is called perpendicular/Opposite.
PH value	A method of expressing differences in the acidity or alkalinity of a solution
Polynomial	A polynomial is a mathematical expression involving a sum of powers in one or more variables multiplied by coefficients
Proportion	The statement of equality of two ratios is called proportion.
Quotient	The result of division; the number of times one quantity is contained in another
Range	What actually come out of a function is called range
Ratio	The ratio of two quantities a and b of same kind is denoted as a:b and is defined as: $a:b=a \div b$ (i.e relation between two quantities)

Rational Expression	A rational expression is an algebraic expression of the form $^{P}/_{Q}$, where P and Q are simpler expressions (usually polynomials), and the denominator Q is not zero
Rational Number	A rational number is any number that can be expressed as the quotient or fraction a/b of two integers, with the denominator b not equal to zero
Rows of Matrix	Number of horizontal categories in any matrix is called row/s of matrix.
Set	Well defined collection of distinct elements which may be objects, names, numbers or ideas is called set.
Slide rule	The slide rule, also known colloquially as a slip stick, is a mechanical analog computer
Solution of triangles	Finding the measures of the unknown elements is called the solution of a triangle
Subtraction of matrices	subtraction of corresponding elements of two matrices
Surjective	Every "B" has **at least one** matching "A" (maybe more than one). There won't be a "B" left out.
Transitive property	If a < b and b < c, then a < c , If a > b and b > c, then a > c
Trichotomy property	The property that for natural numbers *a* and *b* , either *a* is less than *b* , *a* equals *b* , or *a* is greater than *b* .
Trigonometry	The study of angles and of the angular relationships of planar and three-dimensional figures is known as trigonometry
Variable	It is a value that may change within the scope of a given problem or set of operations
Venn diagram	Venn diagrams are pictorial representation of sets/subsets and relationship that the sets/subsets have among them. It helps us to analyze relationship and carry out valid set operations in a relatively easier manner which is a symbolic representation.