Sahar

Business MATHEMATICS

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Chapter - 1

SOME PRELIMINARIES

1- Define the following terms with an example.

i) Identity

ii) Conditional equation

ii) false statement

iv) equivalent equation.

Ans:

i) Identity:

An identity is an equation which is true for all values of the variables. An example of an identity is the equation.

$$6x+12=\frac{12x+24}{2}$$

ii) Conditional equation:

A Conditional equation is true for only a limited number of values of the variables. For example, the equation.

$$x+3=5$$

is true only when x equals 2.

iii) False statement:

A false statement, or contradiction is an equation which is never true. That is, there are no values of the variables which make the two sides of the equation equal. An example is the equation.

$$x=x+5$$

iv) Equivalent equations:

Equivalent equations are equations which have the same roots.

Solved Section 1.1

1. x - 5 = 2x - 8

Solution:

$$x-5 = 2x-8$$
$$x-2x = 5-8$$
$$-x = -3$$
$$\Rightarrow x = 3 \quad \text{Ans.}$$

2. 10 - 2x = 8 - 3x

Solution:

$$10-2x = 8-3x$$
$$3x-2x = -10+8$$
$$x = -2 \quad \text{Ans.}$$

3. 2x + 4 = 16 - x

Solution:

$$2x+4 = 16-x$$
$$2x+x = 16-4$$
$$3x = 12$$
$$x = \frac{12}{3}$$
$$x = 4 \quad \text{Ans.}$$

4. -5x + 2 = 8 - 3x

Solution:

$$-5x+2 = 8-3x$$
$$-5x+3x = 8-2$$
$$-2x = 6$$
$$x = \frac{6}{-2}$$
$$x = -3 \quad \text{Ans.}$$

5. 2(x-3) = 3(x+4)

Solution:

$$2(x-3) = 3(x+4)$$

$$2x - 6 = 3x + 12$$
$$2x - 3x = 6 + 12$$
$$-x = 18$$
$$\Rightarrow x = -18 \quad \text{Ans.}$$

6. $5(3 - x) = 3(5 + x)$

Solution:

$$5(3 - x) = 3(x + 5)$$
$$15 - 5x = 15 + 3x$$
$$-5x - 3x = 15 - 15$$
$$-8x = 0$$
$$x = \frac{0}{-8}$$
$$x = 0 \quad \text{Ans.}$$

7. $6 - 2t = 4t + 12$

Solution:

$$6 - 2t = 4t + 12$$
$$-4t - 2t = 12 - 6$$
$$-6t = 6$$
$$t = -\frac{6}{6}$$
$$t = -1 \quad \text{Ans.}$$

8. $3y - 10 = 6y + 20$

Solution:

$$3y - 10 = 6y + 20$$
$$3y - 6y = 20 + 10$$
$$-3y = 30$$
$$y = -\frac{30}{3}$$
$$y = -10 \quad \text{Ans.}$$

9. $3 - 5t = 3t - 5$

Solution:

$$3 - 5t = 3t - 5$$
$$-3t - 5t = -3 -5$$
$$-8t = -8$$
$$t = \frac{-8}{-8}$$
$$t = 1 \quad \text{Ans.}$$

10. $10y - 20 = 6y + 4$

Solution:

$$10y - 20 = 6y + 4$$
$$10y - 6y = 20 + 4$$
$$4y = 24$$
$$y = \frac{24}{4}$$
$$y = 6 \quad \text{Ans.}$$

11. $3t + 10 = 4t - 6$

Solution:

$$3t + 10 = 4t - 6$$
$$3t - 4t = -6 - 10$$
$$-t = -16$$
$$\Rightarrow t = 16 \quad \text{Ans.}$$

12. $3(2t - 8) = 4(7 + t)$

Solution:

$$3(2t - 8) = 4(7 + t)$$
$$6t - 24 = 28 + 4t$$
$$6t - 4t = 28 + 24$$
$$2t = 52$$
$$t = \frac{52}{2}$$
$$t = 26 \quad \text{Ans.}$$

13. $(x + 6) - (5 - 2x) + 2 = 0$

Solution:

$$(x - 6) - (5 - 2x) + 2 = 0$$
$$x + 6 - 5 + 2x + 2 = 0$$
$$x + 2x + 6 - 5 + 2 = 0$$
$$3x + 3 = 0$$
$$3x = -3$$
$$x = \frac{-3}{3}$$
$$x = -1 \quad \text{Ans.}$$

14. $\frac{x}{6} - 5 = \frac{x}{2} - 7$

Solution:

$$\frac{x}{6} - 5 = \frac{x}{2} - 7$$

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$$\frac{x-5\times 6}{6} = \frac{x-2\times 7}{2}$$
$$\frac{x-30}{6} = \frac{x-14}{2}$$
$$6(x - 14) = 2(x - 30)$$
$$6x - 84 = 2x - 60$$
$$6x - 2x = 84 - 60$$
$$4x = 24$$
$$x = \frac{24}{4}$$
$$x = 6 \quad \text{Ans.}$$

15. $\frac{t-3}{2}+\frac{t+3}{4}=\frac{8-t}{3}+2$

Solution:

$$\frac{t-3}{2}+\frac{t+3}{4} = \frac{8-t}{3}+2$$
$$\frac{2(t-3)+(t+3)}{4} = \frac{8-t+(2\times 3)}{3}$$
$$\frac{2t-6+t+3}{4} = \frac{8-t+6}{3}$$
$$\frac{3t-3}{4} = \frac{14-t}{3}$$
$$3(3t - 3) = 4(14 - t)$$
$$9t - 9 = 56 - 4t$$
$$9t + 4t = 56 + 9$$
$$13t = 65$$
$$t = \frac{65}{13}$$
$$t = 5 \quad \text{Ans.}$$

16. $3-\frac{x}{2}=\frac{x}{3}-2$

Solution:

$$3-\frac{x}{2} = \frac{x}{3}-2$$
$$\frac{6-x}{2} = \frac{x-6}{3}$$
$$2(x - 6) = 3(6 - x)$$
$$2x - 12 = 18 - 3x$$
$$2x + 3x = 18 + 12$$
$$5x = 130$$
$$x = \frac{30}{5}$$
$$x = 6 \quad \text{Ans.}$$

17. $\frac{v}{2}-3=5+\frac{v}{2}$

Solution:

$$\frac{v}{2}-3 = 5+\frac{v}{2}$$
$$\frac{v-6}{2} = \frac{10+v}{2}$$
$$2(v - 6) = 2(10 + v)$$
$$2v - 12 = 20 + 2v$$
$$2v - 2v = 20 + 12$$
$$0 = 32$$

No Solutuion.

18. $4 + x = 3 + \frac{x}{2}$

Solution:

$$4 + x = 3 + \frac{x}{2}$$
$$4 + x = \frac{6+x}{2}$$
$$2(4 + x) = 6 + x$$
$$8 + 2x = 6 + x$$
$$2x - x = 6 - 8$$
$$x = -2 \quad \text{Ans.}$$

19. $3(x - 2) = (x + 3)/2$

Solution:

$$3(x - 2) = (x + 3)/2$$
$$2 \times 3(x - 2) = x + 3$$
$$6(x - 2) = x + 3$$
$$6x - 12 = x + 3$$
$$6x - x = 12 + 3$$
$$5x = 15$$

$$x = \frac{15}{3}$$

$x = 3$ Ans.

20. (t - 3)/2 = (4 - 3t)/4

Solution:

$$(t - 3)/2 = (4 - 3t)/4$$
$$4\times(t - 3)/2 = 4\times(4 - 3t)/4$$
$$2(t - 3) = 4 - 3t$$
$$2t - 6 = 4 - 3t$$
$$2t + 3t = 4 + 6$$
$$5t = 10$$
$$t = \frac{10}{5}$$

$t = 2$ Ans.

21. 3(12 - x) = 16 = 2

Solution:

$$3(12 - x) - 16 = 2$$
$$36 - 3x - 16 = 2$$
$$-3x = 2 + 16 - 36$$
$$-3x = 18 - 36$$
$$-3x = -18$$
$$x = \frac{-18}{-3}$$

$x = 6$ Ans.

22. 2(y + 1) - 3 (y - 1) = 5 - y

Solution:

$$2(y + 1) - 3(y - 1) = 5 - y$$
$$2y + 2 - 3y + 3 = 5 - y$$
$$2y - 3y + y = 5 - 2 - 3$$
$$3y - 3y = 5 - 5$$
$$0 = 0$$

No Solution.

23. 3x + 1 = 2 - (x - 4) + 3x

Solution:

$$3x + 1 = 2 - (x - 4) + 3x$$
$$3x + 1 = 2 - x + 4 + 3x$$
$$3x + 1 = 3x - x + 6$$
$$3x + 1 = 2x + 6$$
$$3x - 2x = 6 - 1$$

$x = 5$ Ans.

24. 3(x - 2) + 4(2 - x) = x + 2 (x + 1)

Solution:

$$3(x-2) + 4(2-x) = x + 2(x + 1)$$
$$3x - 6 + 8 - 4x = x + 2x + 2$$
$$-x + 2 = 3x + 2$$
$$-x - 3x = 2 - 2$$
$$-4x = 0$$
$$x = \frac{0}{-4}$$

$x = 0$ Ans.

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Solved Section 1.2

Solve the following quadratic equations using factoring.

1. $x^2 + x - 6 = 0$

Solution:

$$x^2 + x - 6 = 0$$
$$x^2 + 3x - 2x - 6 = 0$$
$$x(x+3) - 2(x+3) = 0$$
$$(x-2)(x+3) = 0$$
$$\Rightarrow x - 2 = 0 \quad \text{or} \quad x + 3 = 0$$
$$x = 2 \qquad x = -3$$

Ans.

2. $x^2 - 25 = 0$

Solution:

$$x^2 - 25 = 0$$
$$(x)^2 - (5)^2 = 0$$
$$(x - 5)(x + 5) = 0$$
$$\Rightarrow x - 5 = 0 \quad \text{or} \quad x + 5 = 0$$
$$x = 5 \qquad x = -5$$

Ans.

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3. $x^2 + 2x + 1 = 0$

Solution:

$$x^2 + 2x + 1 = 0$$
$$x^2 + x + x + 1 = 0$$
$$x(x+1)+1(x+1) = 0$$
$$(x+1)(x+1) = 0$$
$$\Rightarrow x + 1 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = -1 \qquad x = -1 \text{ Ans.}$$

4. $x^2 + 3x - 4 = 0$

Solution:

$$x^2 + 3x - 4 = 0$$
$$x^2 + 4x - x - 4 = 0$$
$$x(x+4)-1(x+4) = 0$$
$$(x-1)(x+4) = 0$$
$$\Rightarrow x - 1 = 0 \quad \text{or} \quad x + 4 = 0$$
$$x = 1 \qquad x = -4 \text{ Ans.}$$

5. $x^2 - 3x - 10 = 0$

Solution:

$$x^2 - 3x - 10 = 0$$
$$x^2 - 3x - 10 = 0$$
$$x^2 - 5x + 2x - 10 = 0$$
$$x(x-5)+2(x-5) = 0$$
$$(x+2)(x-5) = 0$$
$$\Rightarrow x + 2 = 0 \quad \text{or} \quad x - 5 = 0$$
$$x = -2 \qquad x = 5 \text{ Ans.}$$

6. $t^2 - 2t - 8 = 0$

Solution:

$$t^2 - 2t - 8 = 0$$
$$t^2 - 4t + 2t - 8 = 0$$
$$t(t - 4) + 2(t - 4) = 0$$
$$(t + 2)(t - 4) = 0$$
$$\Rightarrow t + 2 = 0 \quad \text{or} \quad t - 4 = 0$$
$$t = -2 \qquad t = 4 \text{ Ans.}$$

7. $2t^2 + 9t + 4 = 0$

Solution:

$$2t^2 + 9t + 4 = 0$$
$$2t^2 + 8t + t + 4 = 0 \qquad \text{As } 2 \times 4 = 8$$
$$2t(t + 4) + 1(t + 4) = 0 \qquad \text{So } 9t = 8t + t$$
$$(2t + 1)(t + 4) = 0$$
$$\Rightarrow 2t + 1 = 0 \quad \text{or} \quad t + 4 = 0$$
$$2t = -1 \quad ,$$
$$t = -\frac{1}{2} \quad , \quad t = -4 \text{ Ans.}$$

8. $5r^2 + 2r - 3 = 0$

Solution:

$$5r^2 + 2r - 3 = 0$$
$$5r^2 + 5r - 3r - 3 = 0 \qquad \text{As } 5\times(-3) = -15$$
$$5r(r + 1) - 3(r + 1) = 0 \qquad \text{So } 2r = 5r - 3r$$
$$(5r - 3)(r + 1) = 0$$
$$\Rightarrow 5r - 3 = 0 \quad \text{or} \quad r + 1 = 0$$
$$5r = 3 \quad ,$$
$$r = \frac{3}{5} \quad , \quad r = -1 \text{ Ans.}$$

9. $6y^2 - 9y - 6 = 0$

Solution:

$$6y^2 - 9y - 6 = 0$$
$$6y^2 - 12y + 3y - 6 = 0 \qquad \text{As } 6\times(-6) = -36$$
$$6y(y-2)+3(y-2) = 0 \qquad \text{So } -9y = -12y+3y$$
$$(6y+3)(y-2) = 0$$
$$\Rightarrow 6y + 2 = 0 \quad \text{or} \quad y - 2 = 0$$
$$6y = -2 \quad , \quad y = 2$$
$$y = -\frac{2}{6} \quad ,$$
$$y = -\frac{1}{3} \quad , \quad y = 2 \text{ Ans.}$$

10. $x^2 + 10x + 25 = 0$

Solution:

$$x^2 + 10x + 25 = 0$$
$$x^2 + 5x + 5x + 25 = 0$$
$$x(x+5)+5(x+5) = 0$$
$$(x+5)(x+5) = 0$$
$$\Rightarrow x + 5 = 0 \quad \text{or} \quad x + 5 = 0$$
$$x = -5 \qquad x = -5 \text{ Ans.}$$

11. $r^2 - 16 = 0$

Solution:

$$r^2 - 16 = 0$$
$$(r)^2 - (4)^2 = 0$$
$$(r-4)(r+4) = 0$$
$$\Rightarrow \quad r - 4 = 0 \quad \text{or} \quad r + 4 = 0$$
$$r = 4 \qquad r = -4 \text{ Ans.}$$

12. $3t^2 + 9t + 6 = 0$

Solution:

$$3t^2 + 9t + 6 = 0$$
$$3t^2 + 6t + 3t + 6 = 0 \quad \text{As } 3\times6 = 18$$
$$3t(t+2)+3(t+2) = 0 \quad \text{So } 9t = 6t + 3t$$
$$(3t+3)(t+2) = 0$$
$$\Rightarrow \quad 3t + 3 = 0$$
$$3t = -3 \quad \text{or} \quad t + 2 = 0$$
$$t = -1 \quad \text{or} \quad t = -2 \text{ Ans.}$$

13. $x^2 - 2x + 15 = 0$

Solution:

$$x^2 - 2x + 15 = 0$$

cannot be factorized.

14. $2x^2 - 17x - 1 = 0$

Solution:

$$2x^2 - 17x - 1 = 0$$

cannot be factorized.

15. $4y^2 + 18y - 10 = 0$

Solution:

$$4y^2 + 18y - 10 = 0$$
$$4y^2 + 20y - 2y - 10 = 0 \quad \text{As } 4\times(-10) = -40$$
$$4y(y+5)-2(y+5) = 0 \quad \text{So } 18y = 20y - 2y$$
$$(4y-2)(y+5) = 0$$
$$\Rightarrow \quad 4y - 2 = 0 \quad \text{or} \quad y + 5 = 0$$
$$4y = 2 \quad \text{or} \quad y = -5$$
$$y = \frac{2}{4}$$
$$y = \frac{1}{2} \quad \text{or} \quad y = -5 \text{ Ans.}$$

16. $x^2 + 4x - 21 = 0$

Solution:

$$x^2 + 4x - 21 = 0$$
$$x^2 + 7x - 3x - 21 = 0$$
$$x(x+7)-3(x+7) = 0$$
$$(x-3)(x+7) = 0$$
$$\Rightarrow \quad x - 3 = 0 \quad \text{or} \quad x + 7 = 0$$
$$x = 3 \qquad x = -7 \text{ Ans.}$$

17. $x^2 + 8x + 12 = 0$

Compare it with

$ax^2 + bx + c = 0$

$a = 1, b = 8, c = 12$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-8 \pm \sqrt{(8)^2 - 4(1)(12)}}{2(1)}$$
$$= \frac{-8 \pm \sqrt{64 - 48}}{2}$$
$$= \frac{-8 \pm \sqrt{16}}{2}$$
$$= \frac{-8 \pm 4}{2}$$
$$x = \frac{-8-4}{2} \quad \text{or} \quad x = \frac{-8+4}{2}$$
$$x = -\frac{12}{2} \qquad x = -\frac{4}{2}$$
$$x = -6 \qquad x = -2$$

18. $x^2 + 12x + 36 = 0$

Solution:

$ax^2 + 12x + 36 = 0$

Compare it with

$ax^2 + bx + c = 0$

$a = 1, b = 12, c = 36$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-12 \pm \sqrt{(12)^2 - 4(1)(36)}}{2(1)}$$
$$= \frac{-12 \pm \sqrt{144 - 144}}{2}$$

$$= \frac{-12 \pm 0}{2}$$

$$= \frac{-12}{2}$$

$$x = -6 \quad \text{Ans.}$$

19. $x^2 + 2r + 1 = 0$

Solution:

$r^2 + 2r + 1 = 0$

Compare it with

$ar^2 + br + c = 0$

$a = 1, b = 2, c = 1$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4-4}}{2}$$

$$= \frac{-2 \pm 0}{2}$$

$$= \frac{-2}{2}$$

$$x = -1 \quad \text{Ans.}$$

20. $t^2 - 2t + 1 = 0$

Solution:

$t^2 - 2t + 1 = 0$

Compare it with

$at^2 + bt + c = 0$

$a = 1, b = -2, c = 1$

We know that

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4-4}}{2}$$

$$= \frac{-2 \pm 0}{2}$$

$$= \frac{2}{2}$$

$$t = 1 \quad \text{Ans.}$$

21. $x^2 + x + 10 = 0$

Solution:

$x^2 + x + 10 = 0$

Compare it with

$ax^2 + bx + c = 0$

$a = 1, b = 1, c = 10$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{(1)^2 - 4(1)(10)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1-40}}{2}$$

$$= \frac{-1 \pm -\sqrt{39}}{2}$$

Solution is not possible

22. $x^2 + 3x + 5 = 0$

Solution:

$x^2 + 3x + 5 = 0$

Compare it with

$ax^2 + bx + c = 0$

$a = 1, b = 2, c = 1$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{(3)^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{9-20}}{2}$$

Solution is not possible

23. $x^2 + 3x - 4 = 0$

Solution:

$x^2 + 3x - 4 = 0$

Compare it with

$ax^2 + bx + c = 0$

a = 1, b = 3, c = -4

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-4)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{9+16}}{2}$$

$$= \frac{-3 \pm \sqrt{25}}{2}$$

$$= \frac{-3 \pm 5}{2}$$

$x = \frac{-3-5}{2}$ or $x = \frac{-3+5}{2}$

$x = -\frac{8}{2}$ $\quad x = \frac{2}{2}$

$x = -4$ $\quad x = 1$ Ans.

24. 9x² - 3x = 2

Solution:

$9x^2 - 3x = 2$

$9x^2 - 3x - 2 = 0$

Compare it with

$ax^2 + bx + c = 0$

a = 9, b = -3, c = -2

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(9)(-2)}}{2(9)}$$

$$= \frac{3 \pm \sqrt{9+72}}{18}$$

$$= \frac{3 \pm \sqrt{81}}{18}$$

$$= \frac{3 \pm 9}{18}$$

$x = \frac{3-9}{18}$ or $x = \frac{3+9}{18}$

$x = -\frac{6}{18}$ $\quad x = \frac{12}{18}$

$x = -\frac{1}{3}$ $\quad x = \frac{2}{3}$ Ans.

25. x² + 1 = x

Solution:

$x^2 + 1 = x$

$x^2 + 1 - x = 0$

$x^2 - x + 1 = 0$

Compare it with

$ax^2 + bx + c = 0$

a = 1, b = -1, c = -1

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1-4}}{2}$$

$$= \frac{1 \pm \sqrt{-3}}{2}$$

Solution is not possible.

26. 3r² = 14r - 8

Solution:

$3r^2 = 14r - 8$

$3r^2 - 14r + 8 = 0$

Compare it with

$ar^2 + br + c = 0$

a = 3, b = -14, c = 8

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-14) \pm \sqrt{(-14)^2 - 4(3)(8)}}{2(3)}$$

$$= \frac{14 \pm \sqrt{196-96}}{6}$$

$$= \frac{14 \pm \sqrt{100}}{6}$$

$$= \frac{14 \pm 10}{6}$$

$$x = \frac{14-10}{6} \text{ or } x = \frac{14+10}{6}$$

$$x = \frac{4}{6} \qquad x = \frac{24}{6}$$

$$x = \frac{2}{3} \qquad x = 4 \text{ Ans.}$$

27. $x^2 = 2x - 2$

Solution:

$x^2 = 2x - 2$

$x^2 - 2x + 2 = 0$

Compare it with

$ax^2 + bx + c = 0$

$a = 1, b = -2, c = 2$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4-8}}{2}$$

$$= \frac{2 \pm \sqrt{-6}}{2}$$

Solution is not possible.

28. $4t^2 + 3t = 1$

Solution:

$4t^2 + 3t = 1$

$4t^2 + 3t - 1 = 0$

Compare it with

$at^2 + bt + c = 0$

$a = 4, b = 3, c = -1$

We know that

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{(3)^2 - 4(4)(-1)}}{2(4)}$$

$$= \frac{-3 \pm \sqrt{9+16}}{8}$$

$$= \frac{-3 \pm \sqrt{25}}{8}$$

$$= \frac{-3 \pm 5}{8}$$

$$t = \frac{-3-5}{8} \text{ or } t = \frac{-3+5}{8}$$

$$t = \frac{-8}{8} \qquad t = \frac{2}{8}$$

$$t = -1 \qquad t = \frac{1}{4}$$

29. $y^2 + 2 = 2y$

Solution:

$y^2 + 2 = 2y$

$y^2 - 2y + 2 = 0$

Compare it with

$ay^2 + by + c = 0$

$a = 1, b = -2, c = 2$

We know that

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4-8}}{2}$$

$$= \frac{2 \pm \sqrt{-4}}{2}$$

Solution is not possible.

30. $x^2 + 4x + 5 = 0$

Solution:

$x^2 + 4x + 5 = 0$

Compare it with

$ax^2 + bx + c = 0$

$a = 1, b = 4, c = 5$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{4-20}}{2}$$

$$= \frac{-4 \pm \sqrt{-16}}{2}$$

Solution is not possible.

31. $x^2 - 2x - 5 = 0$

Solution:

$x^2 - 2x - 5 = 0$

Compare it with

$ax^2 + bx + c = 0$

$a = 1, b = -2, c = -5$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4+20}}{2}$$

$$= \frac{-2 \pm \sqrt{24}}{2}$$

Solution is not possible.

32. $4x^2 - 64 = 0$

Solution:

$4x^2 - 64 = 0$

$4x^2 + 0x - 64 = 0$

Compare it with

$ax^2 + bx + c = 0$

$a = 4, b = 0, c = -64$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-0 \pm \sqrt{(0)^2 - 4(4)(-64)}}{2(4)}$$

$$= \frac{\pm\sqrt{1024}}{8}$$

$$= \pm\frac{32}{8}$$

$x = -\frac{32}{8}$ or $x = +\frac{32}{8}$

$x = -4$ $\quad$ $x = +4$

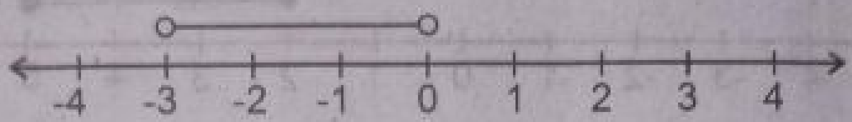
☆☆☆

Solved Section 1.3

Sketch the following intervals.

1. (-3, 0)

Solution

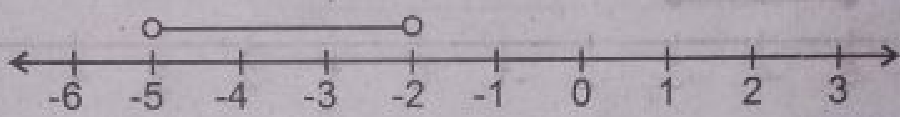


2. (3, 5)

Solution

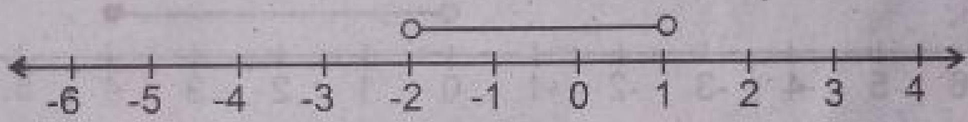
3. (-5, -2)

Solution



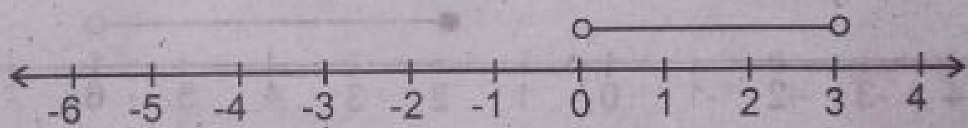
4. (-2, 1)

Solution



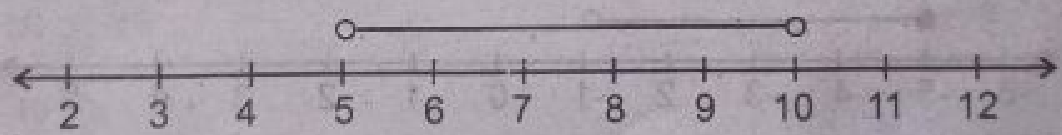
5. (0, 3)

Solution



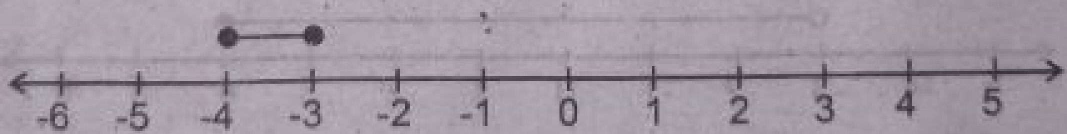
6. (5, 10)

Solution



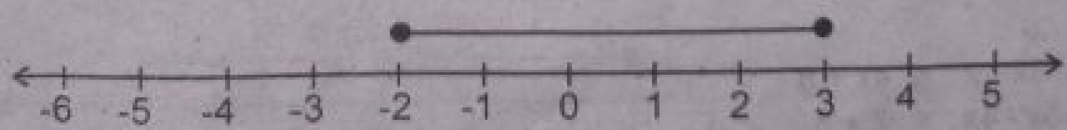
7. [-4, -3]

Solution



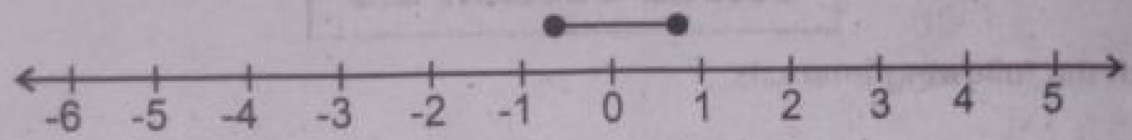
8. [-2, 3]

Solution



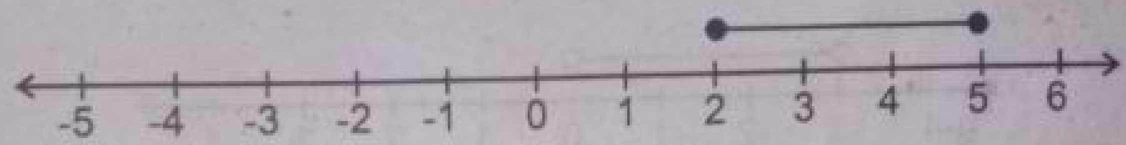
9. [-0.5, 0.5]

Solution



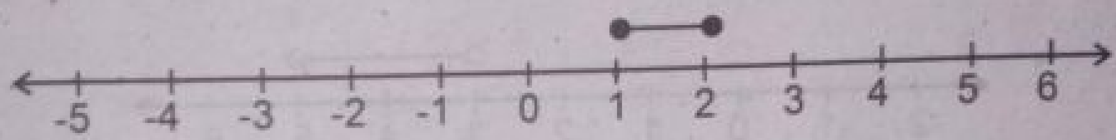
10. [2, 5]

Solution



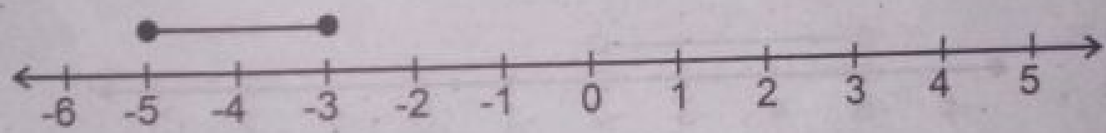
11. [1, 2]

Solution



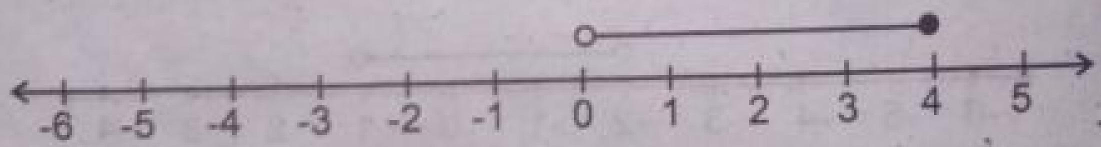
12. [-5, -3]

Solution



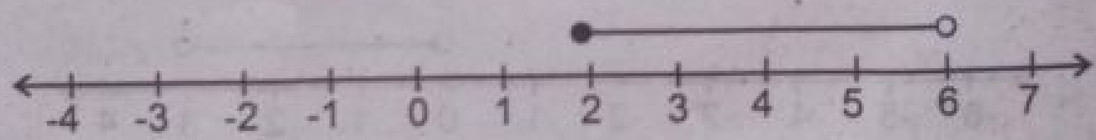
13. (0, 4]

Solution



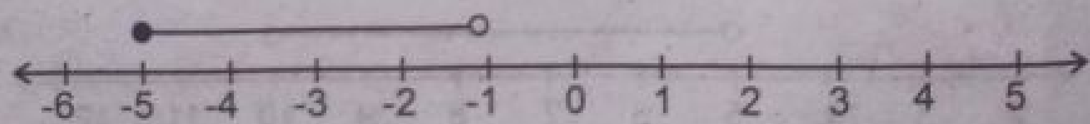
14. [2, 6)

Solution



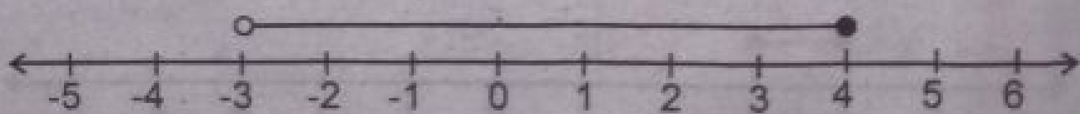
15. [-5, -1)

Solution



16. (-3, 4]

Solution



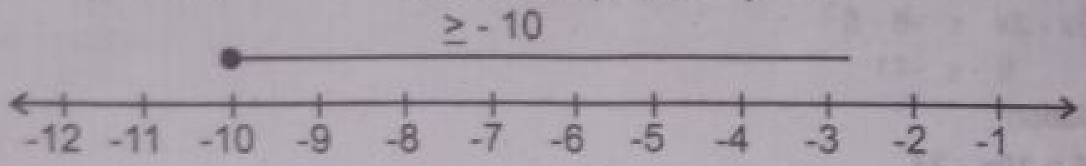
Solve the following inequalities

17. $3x - 2 \leq 4x + 8$

Solution:

$$3x - 2 \leq 4x + 8$$
$$3x - 4x \leq 2 + 8$$

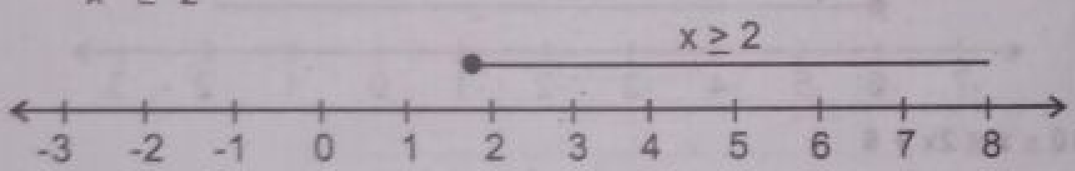
$$-x \leq 10$$
$$(-)(-)x \geq (-)(10)$$ ∵ If we multiply by '___' on btoh sides then replace '≤' by " ≥ "
$$x \geq -10$$



18. $x + 6 \geq 10 - x$

Solution:

$$x + 6 \geq 10 - x$$
$$x + x \geq 10 - 6$$
$$2x \geq 4$$
$$x \geq \frac{4}{2}$$
$$x \geq 2$$



19. $x \geq x + 5$

Solution:

$$x \geq x + 5$$
$$x - x \geq 5$$
$$0 \geq 5$$

No solution.

20. $2x \leq 2x - 10$

Solution:

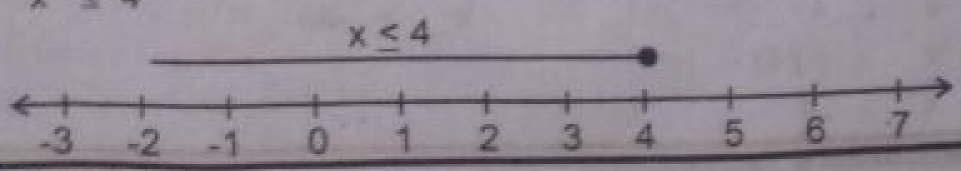
$$2x \leq 2x - 10$$
$$2x - 2x \leq -10$$
$$0 \leq -10$$

No solution.

21. $-4x + 10 \geq -10 + x$

Solution:

$$-4x + 10 \geq -10 + x$$
$$-4x - x \geq -10 - 10$$
$$-5x \geq -20$$
$$-x \geq -4$$
$$(-)(-)x \leq (-)(-)4$$
$$x \leq 4$$



22. $3x + 6 \leq 3x - 5$

Solution:

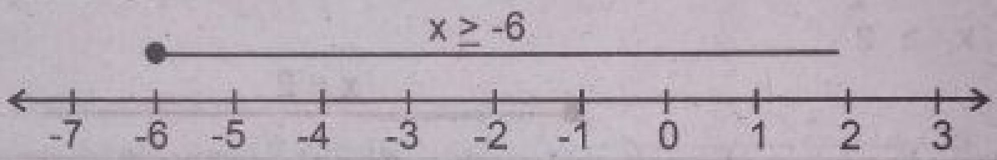
$$3x + 6 \leq 3x - 5$$
$$3x - 3x \leq -6 - 5$$
$$0 \leq -11$$

No solution.

23. $15x + 6 \geq 10x - 24$

Solution:

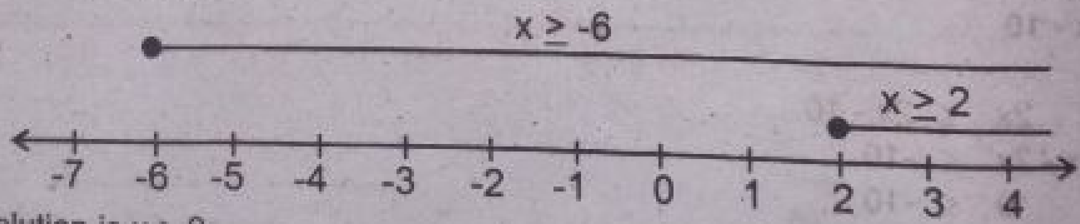
$$15x + 6 \geq 10x - 24$$
$$15x - 10x \geq -24 - 6$$
$$5x \geq -30$$
$$x \geq -\frac{30}{5}$$
$$x \geq -6$$



24. $-4x + 10 \leq x \leq 2x + 6$

Solution:

$$-4x + 10 \leq x \quad \text{and} \quad x \leq 2x + 6$$
$$-4x - x \leq -10 \qquad x - 2x \leq 6$$
$$-5x \leq -10 \qquad -x \leq 6$$
$$-x \leq -2 \qquad (-)(-)x \geq (-)(6)$$
$$(-)(-)x \geq (-)(-)(2) \qquad x \geq -6$$
$$x \geq 2$$



The solution is $x \geq 2$.

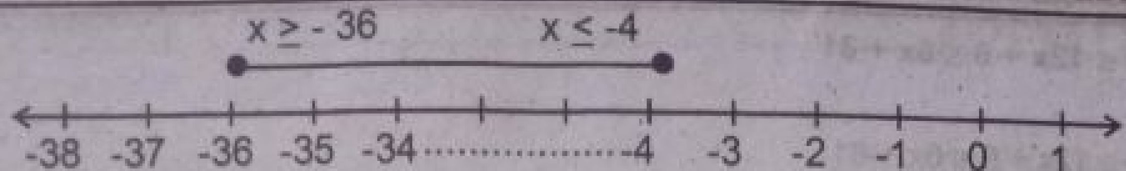
25. $12 \geq x + 16 \geq 2$

Solution:

$12 \geq x + 16 \geq 2$

We can write it as

$$12 \geq x + 16 \quad \text{and} \quad x + 16 \geq -20$$
$$-x \geq 16 - 12 \qquad x \geq -20 - 16$$
$$-x \geq 4 \qquad x \geq -36$$
$$(-)(-)x \geq (-)(4)$$
$$x \geq -4$$



The solution is $-36 \le x \le -4$

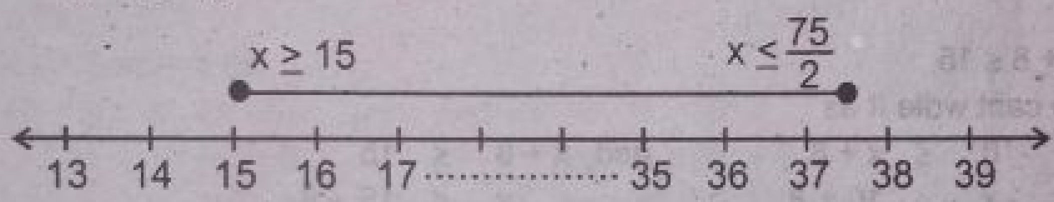
26. $35 \le 2x + 5 \le 80$

Solution:

$35 \le 2x + 5 \le 80$

We can write it as

35	$\le$	$2x + 5$	and	$2x + 5$	$\le$	80
$-2x$	$\le$	$5 - 35$		$2x$	$\le$	$80 - 5$
$-2x$	$\le$	-30		$2x$	$\le$	75
$-x$	$\le$	-15		x	$\le$	$\frac{75}{2}$
$(-)(-)x$	$\ge$	$(-)(-)(15)$				
x	$\ge$	15				



The solution is $15 \le x \le \frac{75}{2}$

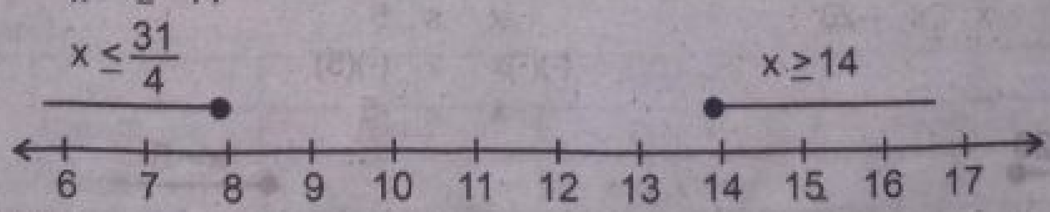
27. $50 \le 4x - 6 \le 25$

Solution:

$50 \le 4x - 6 \le 25$

We can write it as

50	$\le$	$4x - 6$	and	$4x - 6$	$\le$	25
$-4x$	$\le$	$-50 - 6$		$4x$	$\le$	$6 + 25$
$-4x$	$\le$	-56		$4x$	$\le$	31
$-x$	$\le$	-14		x	$\le$	$\frac{31}{4}$
$(-)(-)x$	$\ge$	$(-)(-)(14)$				
x	$\ge$	14				



No solution.

28. 6x - 9 ≤ 12x + 9 ≤ 6x + 81

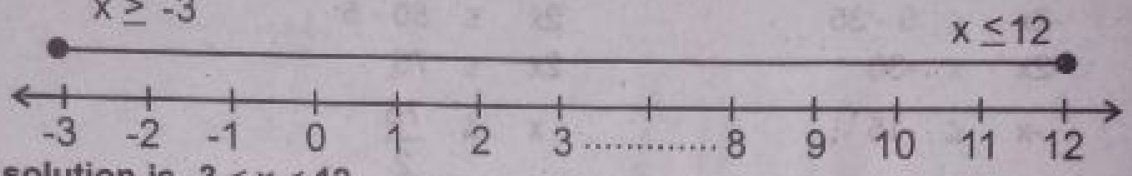
Solution:

6x - 9 ≤ 12x + 9 ≤ 6x + 81

We can write it as

6x - 9	≤	12x + 9	and	12x + 9	≤	6x + 81
6x - 12x	≤	9 + 9		12x-6x	≤	81 - 9
-6x	≤	18		6x	≤	72
-x	≤	3		6x	≤	12
(-)(-)x	≥	(-)(3)				
x	≥	-3				

x ≥ -3



The solution is -3 ≤ x ≤ 12

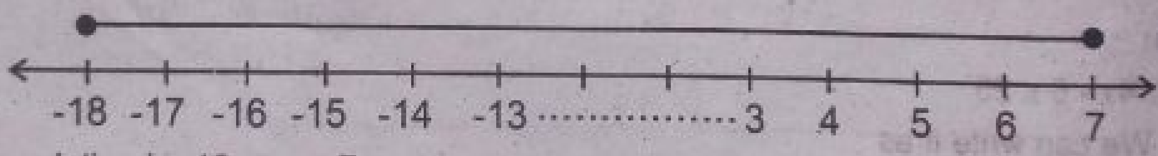
29. -10 ≤ x + 8 ≤ 15

Solution:

-10 ≤ x + 8 ≤ 15

We cant write it as

-10	≤	x + 8	and	x + 8	≤	15
- x	≤	10 + 8		x	≤	15 - 8
-5x	≤	18		x	≤	7
(-)(-)x	≥	(-)(18)				
x	≥	-18				



The solution is -18 ≤ x ≤ 7

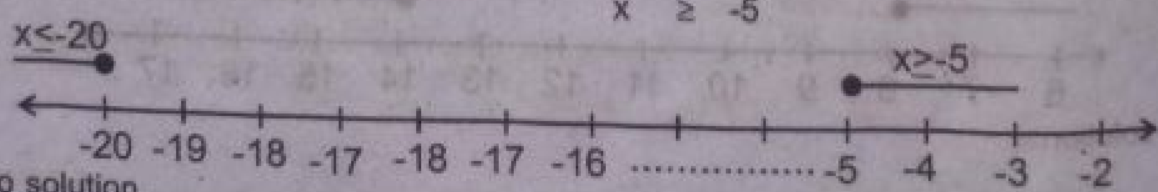
30. 25 ≤ 5 - x ≤ 10

Solution:

25 ≤ 5 - x ≤ 10

We can write it as

25	≤	5 - x	and	5 - x	≤	10
x	≤	5 -25		-x	≤	10 - 5
x	≤	--20		-x	≤	5
				(-)(-)x	≥	(-)(5)
				x	≥	-5



No solution.

31. $0 \geq 20 - x \geq -20$

Solution:

$0 \geq 20 - x \geq -20$

We can write it as

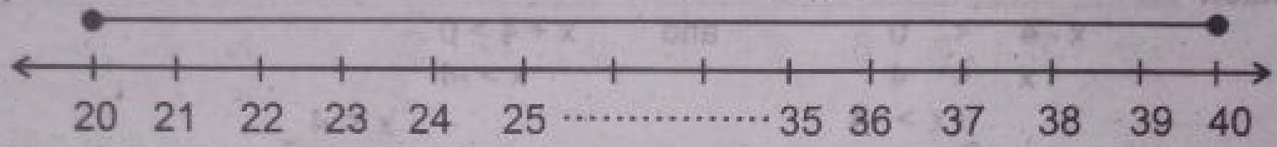
$0 \geq 20 - x$ and $20 - x \geq -20$

$x \geq 20 - 0$ $\quad$ $-x \geq -20 - 20$

$x \geq 20$ $\quad$ $-x \geq -40$

$(-)(-)x \leq (-)(-)(40)$

$x \leq 40$



The solution is $20 \leq x \leq 40$

32. $10 + x \leq 2x - 5 \leq 25$

Solution:

$10 + x \leq 2x - 5 \leq 25$

We cant write it as

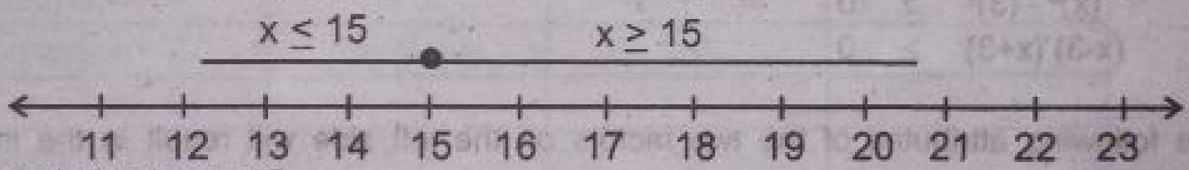
$10 + x \leq 2x - 5$ and $2x - 5 \leq 25$

$x - 2x \leq -10 - 5$ $\quad$ $2x \leq 25 + 5$

$-x \leq -15$ $\quad$ $2x \leq 30$

$(-)(-)x \geq (-)(-)(15)$ $\quad$ $x \leq 15$

$x \geq 15$



The solution is $x = 15$

Solve the following second - degree inequalities.

33. $x^2 - 16 \leq 0$

Solution:

$x^2 - 16 \leq 0$

$(x)^2 - (4)^2 \leq 0$

$(x - 4)(x + 4) \leq 0$

The following attributes of the two factors on the left side will result in the inequality being satisfied.

Condtions	Factors		
	(x - 4)	(x + 4)	Product
Condition - 1	= 0	Any Value	0
Condition - 2	Any Value	= 0	0
Condition - 3	> 0	< 0	< 0
Condition - 4	< 0	> 0	> 0

Condition - 1:

$$x - 4 = 0$$
$$x = 4$$

Condition - 2:

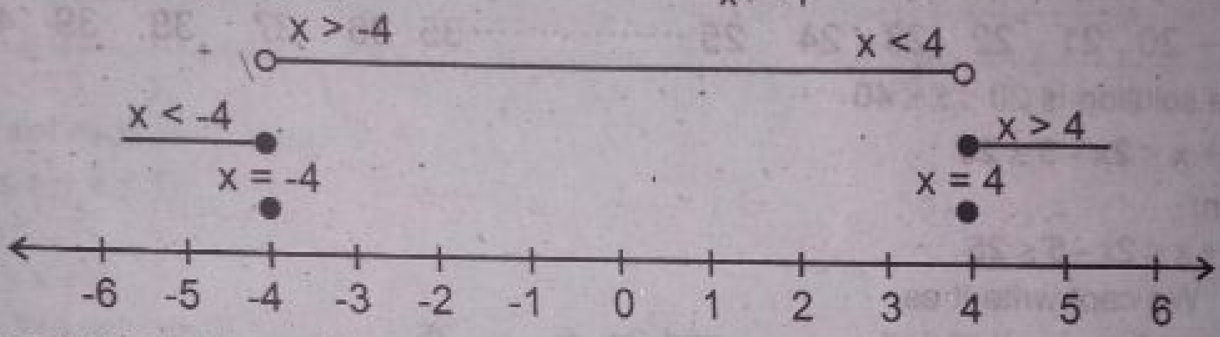
$$x + 4 = 0$$
$$x = -4$$

Condition - 3:

$$x - 4 > 0 \quad \text{and} \quad x + 4 < 0$$
$$x > 4 \quad\quad\quad x < -4$$

Condition - 4:

$$x - 4 < 0 \quad \text{and} \quad x + 4 > 0$$
$$x < 4 \quad\quad\quad x > -4$$



The solution is $-4 \le x \le 4$

34. $x^2 - 9 \ge 0$

Solution:

$$x^2 - 9 \ge 0$$
$$(x)^2 - (3)^2 \ge 0$$
$$(x-3)(x+3) \ge 0$$

The following attributes of the two factors on the left side will result in the inequality being satisfied.

Condtions	Factors (x - 3)	(x + 3)	Product
Condition - 1	= 0	Any Value	0
Condition - 2	Any Value	= 0	0
Condition - 3	< 0	< 0	> 0
Condition - 4	> 0	> 0	> 0

Condition - 1:

$$x - 3 = 0$$
$$x = 3$$

Condition - 2:

$$x + 3 = 0$$
$$x = -3$$

Condition - 3:

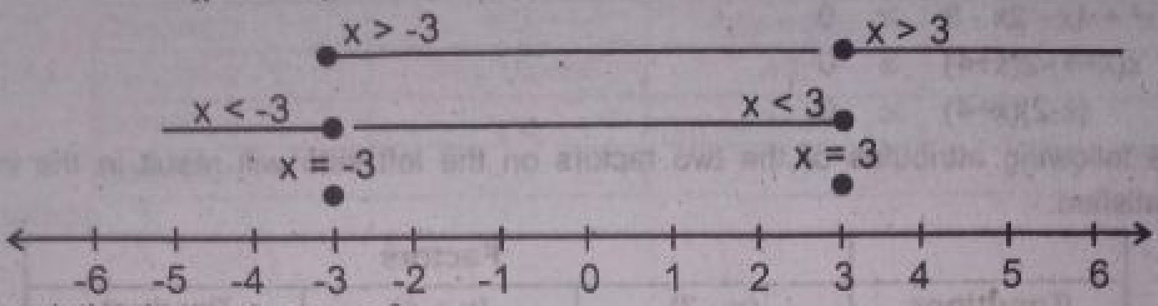
$$x - 3 < 0 \quad \text{and} \quad x + 3 < 0$$

$x < 3$ $\qquad$ $x < -3$

Condition - 4:

$x - 3 > 0$ and $x + 3 > 0$

$x > 3$ $\qquad$ $x > -3$



The solution is $x \le -3$ or $x \le 3$

35. $x^2 + 3x - 18 \le 0$

Solution:

$$x^2 + 3x - 18 \le 0$$
$$x^2 + 6x - 3x - 18 \le 0$$
$$x(x+6)-3(x+6) \le 0$$
$$(x-3)(x+6) \le 0$$

The following attributes of the two factors on the left side will result in the inequality being satisfied.

	Factors		
Condtions	**(x - 3)**	**(x + 6)**	**Product**
Condition - 1	= 0	Any Value	0
Condition - 2	Any Value	= 0	0
Condition - 3	> 0	< 0	< 0
Condition - 4	< 0	> 0	< 0

Condition - 1:

$x - 3 = 0$

$x = 3$

Condition - 2:

$x + 6 = 0$

$x = -6$

Condition - 3:

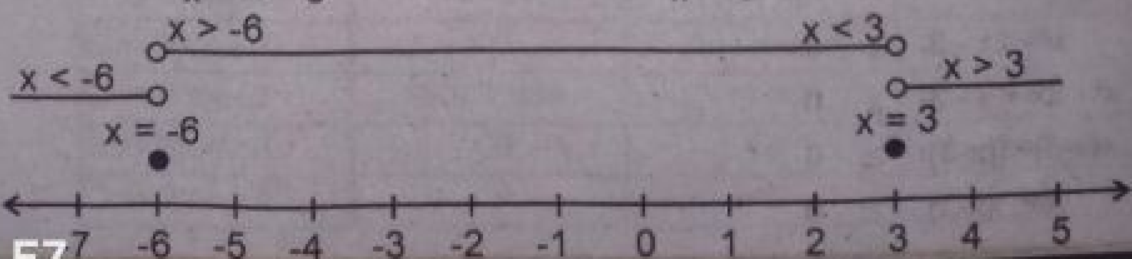
$x - 3 > 0$ and $x + 6 < 0$

$x > 3$ $\qquad$ $x < -6$

Condition - 4:

$x - 3 < 0$ and $x + 6 > 0$

$x < 3$ $\qquad$ $x > -6$



The solution is $-6 \leq x \leq 3$.

36. $x^2 + 2x - 8 \geq 0$

Solution:

$$x^2 + 2x - 8 \geq 0$$
$$x^2 + 4x - 2x - 8 \geq 0$$
$$x(x+4)-2(x+4) \geq 0$$
$$(x-2)(x+4) \geq 0$$

The following attributes of the two factors on the left side will result in the inequality being satisfied.

	Factors		
Condtions	**(x - 2)**	**(x + 4)**	**Product**
Condition - 1	= 0	Any Value	0
Condition - 2	Any Value	= 0	0
Condition - 3	> 0	> 0	< 0
Condition - 4	< 0	< 0	> 0

Condition - 1:

$$x - 2 = 0$$
$$x = 2$$

Condition - 2:

$$x + 4 = 0$$
$$x = -4$$

Condition - 3:

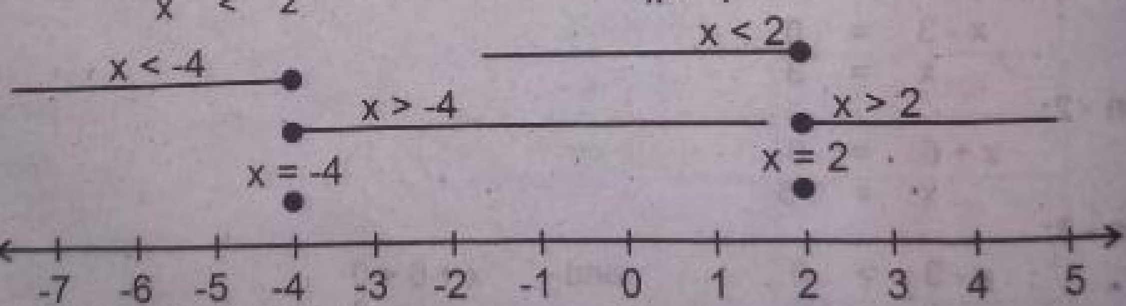
$x - 2 > 0$ and $x + 4 > 0$

$x > 2$ $\quad$ $x > -4$

Condition - 4:

$x - 2 < 0$ and $x + 4 < 0$

$x < 2$ $\quad$ $x < -4$



The solution is $x \leq -4$ or $x \geq 2$.

37. $x^2 - 2x - 3 \geq 0$

Solution:

$$x^2 - 2x - 3 \geq 0$$
$$x^2 - 3x + x - 3 \geq 0$$
$$x(x-3)+1(x-3) \geq 0$$
$$(x+1)(x-3) \geq 0$$

The following attributes of the two factors on the left side will result in the inequality being satisfied.

	Factors		
Condtions	(x + 1)	(x - 3)	Product
Condition - 1	= 0	Any Value	0
Condition - 2	Any Value	= 0	0
Condition - 3	> 0	> 0	> 0
Condition - 4	< 0	< 0	< 0

Condition - 1:

$$x + 1 = 0$$
$$x = -1$$

Condition - 2:

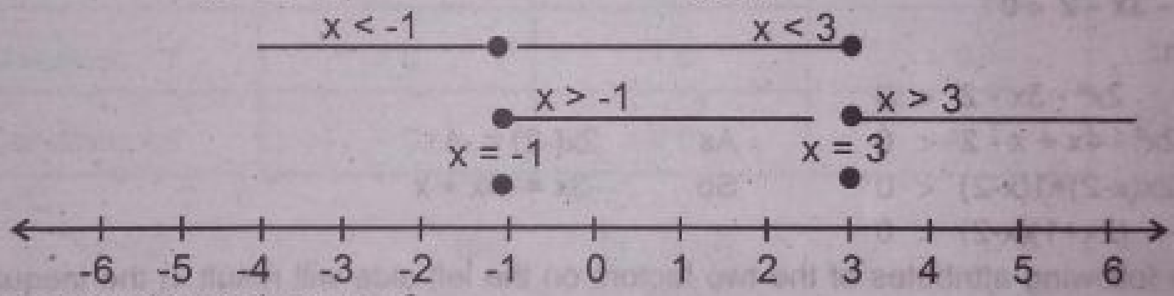
$$x - 3 = 0$$
$$x = 3$$

Condition - 3:

$$x + 1 > 0 \quad \text{and} \quad x - 3 > 0$$
$$x > -1 \qquad\qquad x > 3$$

Condition - 4:

$$x + 1 < 0 \quad \text{and} \quad x - 3 < 0$$
$$x < -1 \qquad\qquad x < 3$$



The solution is $x \leq -1$ or $x \geq 3$.

38. $x^2 + 4x - 12 \leq 0$

Solution:

$$x^2 + 4x - 12 \leq 0$$
$$x^2 + 6x - 2x - 12 \leq 0$$
$$x(x+6)-2(x+6) \leq 0$$
$$(x-2)(x+6) \leq 0$$

The following attributes of the two factors on the left side will result in the inequality being selected.

	Factors		
Condtions	(x - 2)	(x + 6)	Product
Condition - 1	= 0	Any Value	0
Condition - 2	Any Value	= 0	0
Condition - 3	> 0	< 0	< 0
Condition - 4	< 0	> 0	< 0

Condition - 1:

$$x - 2 = 0$$
$$x = 2$$

Condition - 2:

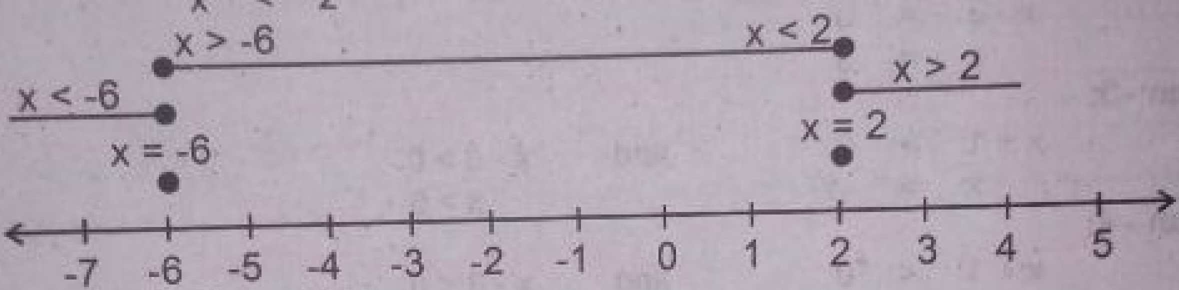
$$x + 6 = 0$$
$$x = -6$$

Condition - 3:

$$x - 2 > 0 \quad \text{and} \quad x + 6 < 0$$
$$x > 2 \qquad\qquad x < -6$$

Condition - 4:

$$x - 2 < 0 \quad \text{and} \quad x + 6 > 0$$
$$x < 2 \qquad\qquad x > -6$$



The solution is $-6 \le x \le 2$.

39. $2x^2 - 3x - 2 < 0$

Solution:

$$2x^2 - 3x - 2 < 0$$
$$2x^2 - 4x + x - 2 < 0 \qquad \text{As} \quad 2x(-2) = -4$$
$$2x(x-2)+1(x-2) < 0 \qquad \text{So} \quad -3x = -4x + x$$
$$(2x+1)(x-2) < 0$$

The following attributes of the two factors on the left side will result in the inequality will be negative if the two factors have the opposite sign.

	Factors		
Condtions	**(2x + 1)**	**(x - 2)**	**Product**
Condition - 1	> 0	< 0	< 0
Condition - 2	< 0	> 0	< 0

Condition - 1:

$$2x + 1 > 0 \quad \text{and} \quad x - 2 < 0$$
$$2x > -1 \qquad\qquad x < 2$$
$$x > -\frac{1}{2}$$

Condition - 2:

$$2x + 1 < 0 \quad \text{and} \quad x - 2 > 0$$
$$2x < -1 \qquad\qquad x > 2$$
$$x < -\frac{1}{2}$$

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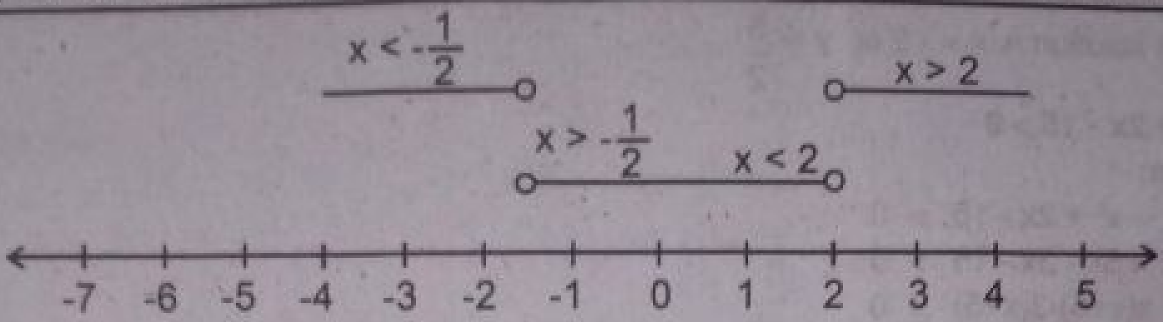


The solution is $-\frac{1}{2} < x < 2.$

40. $2x^2 - x - 10 > 0$

Solution:

$$2x^2 - x - 10 > 0$$
$$2x^2 - 5x + 4x - 10 > 0 \qquad \text{As} \quad 2x(-10) = -20$$
$$x(2x-5)+2(2x-5) > 0 \qquad \text{So} \quad -x = -5x + 4x$$
$$(x+2)(2x-5) > 0$$

The following attributes of the two factors on the left side of the inequality will be positive if the two factors have the same sign.

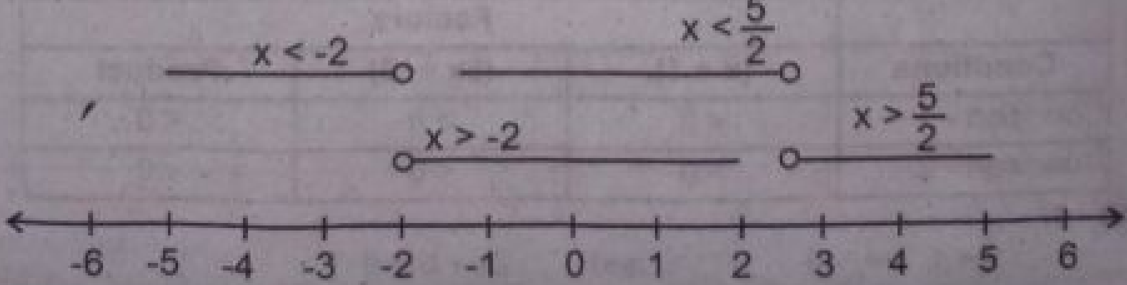
	Factors		
Condtions	**(x + 2)**	**(2x - 5)**	**Product**
Condition - 1	> 0	> 0	> 0
Condition - 2	< 0	< 0	> 0

Condition - 1:

$$x + 2 > 0 \quad \text{and} \quad 2x - 5 > 0$$
$$x > -2 \qquad 2x > 5$$
$$x > \frac{5}{2}$$

Condition - 2:

$$x + 2 < 0 \quad \text{and} \quad 2x - 5 < 0$$
$$x < -2 \qquad 2x < 5$$
$$x < \frac{5}{2}$$



The solution is $x < -2$ or $x > \frac{5}{2}$

41. $x^2 + 2x - 15 > 0$

Solution:

$$x^2 + 2x - 15 > 0$$
$$x^2 + 5x - 3x - 15 > 0$$
$$x(x+5) - 3(x+5) > 0$$
$$(x-3)(x+5) > 0$$

The following attribute of the two factors on the left side of the inequality will be positive if the two factors have the same sign.

	Factors		
Condtions	(x - 3)	(x + 5)	Product
Condition - 1	> 0	> 0	> 0
Condition - 2	< 0	< 0	> 0

Condition - 1:

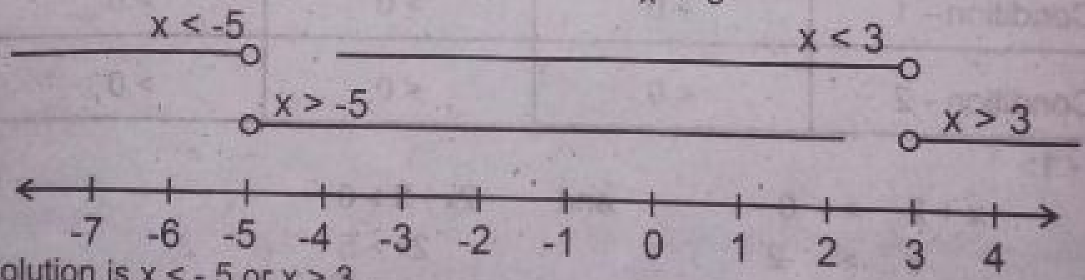
$x - 3 > 0$ and $x + 5 > 0$

$x > 3$ $\quad$ $x > -5$

Condition - 2:

$x - 3 < 0$ and $x + 5 < 0$

$x < 3$ $\quad$ $x < -5$



The solution is $x < -5$ or $x > 3$.

42. $2x^2 + 5x + 3 < 0$

Solution:

$$2x^2 + 5x + 3 < 0$$
$$2x^2 + 3x + 2x + 3 < 0 \qquad \text{As } 2 \times 3 = 6$$
$$x(2x+3) + 1(2x+3) < 0 \qquad \text{So } 5x = 3x + 2x$$
$$(x+1)(2x+3) < 0$$

The following attributes of the two factors on the left side of the inequality will be negative if the two factors have the opposite sign.

	Factors		
Condtions	(x + 1)	(2x + 3)	Product
Condition - 1	< 0	> 0	< 0
Condition - 2	> 0	< 0	< 0

Condition - 1:

$x + 1 < 0$ and $2x + 3 > 0$

$x < -1$ $\qquad$ $2x > -3$

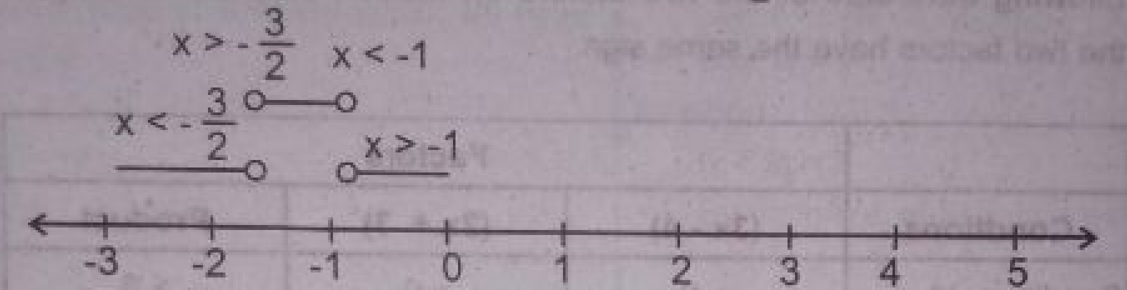
$x > -\frac{3}{2}$

Condition - 2:

$x + 1 > 0$ and $2x + 3 < 0$

$x > -1$ $\qquad$ $2x < -3$

$x < -\frac{3}{2}$



The solution is $-\frac{3}{2} < x < -1$.

43. $4x^2 - 100 < 0$

Solution:

$$4x^2 - 100 < 0$$
$$4(x^2 - 25) < 0$$
$$\Rightarrow \quad x^2 - 25 < 0$$
$$(x)^2 - (5)^2 < 0$$
$$(x-5)(x+5) < 0$$

The following attributes of the two factors on the left side of the inequality will be negative if the two factors have the opposite sign.

	Factors		
Condtions	(x - 5)	(x + 5)	Product
Condition - 1	> 0	< 0	< 0
Condition - 2	< 0	> 0	< 0

Condition - 1:

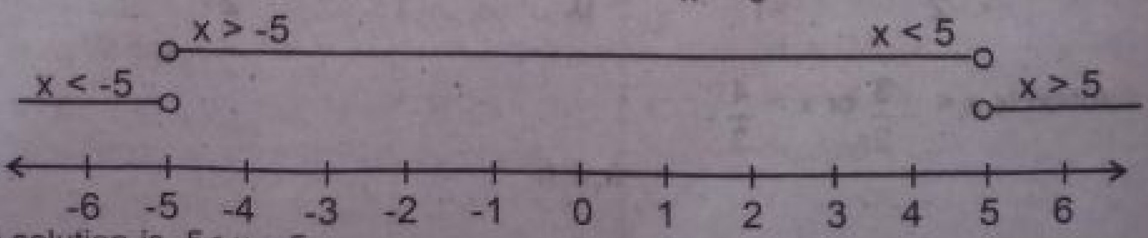
$x - 5 > 0$ and $x + 5 < 0$

$x > -5$ $\qquad$ $x < -5$

Condition - 2:

$x - 5 < 0$ and $x + 5 > 0$

$x < 5$ $\qquad$ $x > -5$



The solution is $-5 < x < 5$.

44. $6x^2 + x - 12 > 0$

Solution:

$$6x^2 + x - 12 > 0 \qquad \text{As } 6 \times (-12) = -72$$
$$6x^2 + 9x - 8x - 12 > 0 \qquad \text{So } +x = +9x - 8x$$
$$3x(2x+3) - 4(2x+3) > 0$$
$$(3x-4)(2x+3) > 0$$

The following attributes of the two factors on the left side of the inequality will be positive if the two factors have the same sign.

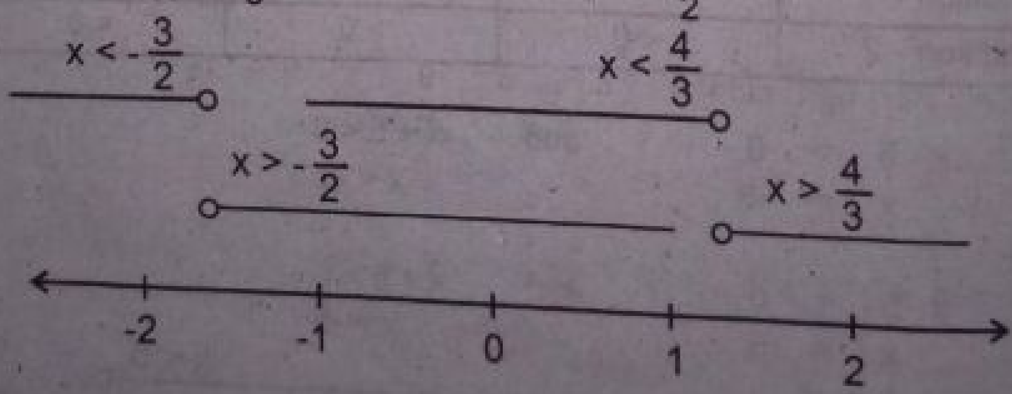
	Factors		
Condtions	**(3x - 4)**	**(2x + 3)**	**Product**
Condition - 1	> 0	> 0	> 0
Condition - 2	< 0	< 0	> 0

Condition - 1:

$$3x - 4 > 0 \qquad \text{and} \qquad 2x + 3 > 0$$
$$3x > 4 \qquad\qquad 2x > -3$$
$$x > \frac{4}{3} \qquad\qquad x > -\frac{3}{2}$$

Condition - 2:

$$3x - 4 < 0 \qquad \text{and} \qquad 2x + 3 < 0$$
$$3x < 4 \qquad\qquad 2x < -3$$
$$x < \frac{4}{3} \qquad\qquad x < -\frac{3}{2}$$



The solution is $x < -\frac{3}{2}$ or $x > \frac{4}{3}$.

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Solved Section 1.4

Solve the following equations.

1. $|x| = 8$

Solution:

$|x| = 8$

By definition of absolute value, we have

$x = \pm 8$

$x = -8, +8$

2. $|x| = 10$

Solution:

$|x| = 10$

By definition of absolute value, we have

$x = \pm 10$

$x = -10, +10$

3. $|x| = -5$

Solution:

$|x| = -5$

By definition of absolute value, if there is a negative sign on right side then solution is not possible.

4. $|x| = -6$

Solution:

$|x| = -6$

By definition of absolute value, if there is a negative sign on right side then solution is not possible.

5. $|x - 6| = 3$

Solution:

$|x - 6| = 3$

By definition of absolute value, we have

$x - 6 = \pm 3$

$x - 6 = -3$ or $x - 6 = +3$

$x = 6 - 3$ $x = 6 + 3$

$x = 3$ $x = 9$

6. $|x - 2| = 4$

Solution:

$|x - 2| = 4$

By definition of absolute value, we have

$x - 2 = \pm 4$

$x - 2 = -4$ or $x - 2 = +4$

$x = 2 - 4$ $x = 2 + 4$

$x = -2$ $x = 6$

7. $|x + 3| = 7$

Solution:

$|x + 3| = 7$

By definition of absolute value, we have

$x + 3 = \pm 7$

$x + 3 = -7$ or $x + 3 = +7$

$x = -3 - 7$ $x = -3 + 7$

$x = -10$ $x = 4$

8. $|2x - 7| = 1$

Solution:

$|2x - 7| = 1$

By definition of absolute value, we have

$2x - 7 = \pm 1$

$2x - 7 = -1$ or $2x - 7 = +1$

$2x = 7-1$ $2x = 7+1$

$2x = 6$ $2x = 8$

$x = \frac{6}{2}$ $x = \frac{8}{2}$

$x = 3$ $x = 4$

9. $|x - 4| = |-3x + 8|$

Solution:

$|x - 4| = |-3x + 8|$

By definition of absolute value, we have

$(x - 4) = \pm(-3x+8)$

$x - 4 = -(-3x+8)$ or $x - 4 = +(-3x+8)$

$x - 4 = 3x - 8$ $x - 4 = -3x + 8$

$x - 3x = 4 - 8$ $x + 3x = 4 + 8$

$-2x = -4$ $4x = 12$

$x = \frac{-4}{-2}$ $x = \frac{12}{4}$

$x = 2$ $\qquad$ $x = 3$

10. |x + 7| = |x - 5|

Solution:

$|x+7| = |x-5|$

By definition of absolute value, we have

$x+7 = \pm(x-5)$

$x+7 = -(x-5)$ or $x+7 = +(x-5)$

$x+7 = -x+5$ $\qquad$ $x+7 = x-5$

$x+7 = -7+5$ $\qquad$ $x-x = -7-5$

$2x = -2$ $\qquad$ $0 = -12$

$x = -\frac{2}{2}$ $\qquad$ No solution

$x = -1$

11. |2x + 5| = |x - 4|

Solution:

$|2x+5| = |x-4|$

By definition of absolute value, we have

$2x+5 = \pm(x-4)$

$2x+5 = -(x-4)$ or $2x+5 = +(x-4)$

$2x+5 = -x+4$ $\qquad$ $2x+5 = x-4$

$2x+x = -5+4$ $\qquad$ $2x-x = -5-4$

$3x = -1$ $\qquad$ $x = -9$

$x = -\frac{1}{3}$

12. |3x - 5| = |2x - 7|

Solution:

$|3x-5| = |2x-7|$

By definition of absolute value, we have

$3x-10 = \pm(2x-7)$

$3x-10 = -(2x-7)$ or $3x-10 = +(2x-7)$

$3x-10 = -2x+7$ $\qquad$ $3x-10 = 2x-7$

$3x+2x = 10+7$ $\qquad$ $3x-2x = 10-7$

$5x = 17$ $\qquad$ $x = 3$

$x = \frac{17}{5}$

13. |5 - 3x| = |-2x + 7|

Solution:

$|5-3x| = |-2x+7|$

By definition of absolute value, we have

$(5-3x) = \pm(-2x+7)$

$5-3x = 2x-7$ or $5-3x = -2x+7$

$-3x-2x = -5-7$ $\qquad$ $-3x+2x = 7-5$

$-5x = -12$ $\qquad$ $-x = 2$

$x = -\frac{12}{-5}$ $\qquad$ $x = -2$

$x = \frac{12}{5}$

14. |x| = |-x + 5|

Solution:

$|x| = |-x+5|$

By definition of absolute value, we have

$x = \pm(-x+5)$

$x = -(-x+5)$ or $x = +(-x+5)$

$x = x-5$ $\qquad$ $x = -x+5$

$x-x = -5$ $\qquad$ $x+x = 5$

$0 = -5$ $\qquad$ $2x = 5$

$x = \frac{5}{2}$

No solution

Solve the following in equilities.

15. |x| < 1

Solution:

$|x| < 1$

By definition of absolute value, we have

$-1 < x < 1$

16. |x| > 8

Solution:

$|x| > 8$

By definition of absolute value, we have

$x < -8$ and $x > 8$

17. |x| > -4

Solution:

$|x| > -4$

By definition of absolute value, we have

$x < -(-4)$ and $x > +(-4)$

$x < 4$ $\qquad$ $x > -4$

−5 < 2x − 3 < +5

or we can write it as x = any real number

18. |x| < -2

Solution

$|x| < -2$

By definition of absolute value, we have

$(-)(-2) < x < (+)(-2)$

$+2 > x > -2$

or $-2 < x < +2$

19. |x| ≤ 2.5

Solution:

$|x| \le 2.5$

By definition of absolute value, we have

$-2.5 \le x \le 2.5$

20. |2x| ≥ 12

Solution:

$|2x| \ge 12$

By definition of absolute value, we have

$2x \le -12$ and $2x \ge 12$

$x \le -\frac{12}{2}$ $x \ge \frac{12}{2}$

$x \le -6$ $x \ge 6$

21. |x-5| < 10

Solution:

$|x-5| < 10$

By definition of absolute value, we have

$-10 < x-5 < 10$

$-10+5 < x < 10+5$

$-5 < x < 15$

22. |4-2x| < 2

Solution:

$|4-2x| < 2$

By definition of absolute value, we have

$-2 < 4-2x < 2$

$-2-4 < -2x < 2-4$

$-6 < -2x < -2$

$-\frac{6}{2} < -x < -\frac{2}{2}$

$-3 < -x < -1$

$(-)(-3) > (-)(-x) > (-)(-1)$

$3 > x > 1$

or $1 < x < 3$

23. |2x - 3| < 5

Solution:

$|2x - 3| < 5$

By definition of absolute value, we have

$2x - 3 < -5$ or $2x-3 > 5$

$2x < -5+3$ $2x > 5+3$

$2x < -2$ $2x > 8$

$x < -\frac{2}{2}$ $x > \frac{8}{2}$

$x < -1$ $x < 4$

24. |3x - 8| > 4

Solution:

$|3x - 8| > 4$

By definition of absolute value, we have

$3x - 8 < -4$ or $3x-8 > 4$

$3x < -4+8$ $3x > 4+8$

$3x < 4$ $2x > 8$

$x < \frac{4}{3}$ $x > \frac{12}{3}$

$x > 4$

25. |y+1| ≤ -9

Solution:

By definition of absolute value, we have

$-(-9) \le y+1 \le +(-9)$

$9 \le y+1 \le -9$

$9-1 \le y \le -9-1$

$8 \le y \le -10$

Which is not correct. So, no solution.

26. |6t - 151| ≤ -6

Solution:

$|6t - 151| \le -6$

By definition of absolute value, we have

$-(-6) \le 6t-15 \le +(-6)$

$6 \le 6t-15 \le -6+15$

$21 \le 6t \le 9$

$$\frac{12}{6} \le t \le \frac{9}{6}$$

$$\frac{7}{2} \le t \le \frac{3}{2}$$

Which is no correct. So, no solution.

27. $\left|\frac{t}{2}\right| \ge 12$

Solution:

$$\left|\frac{t}{2}\right| \ge 12$$

By definition of absolute value, we have

$\frac{t}{2} \le -12$ and $\frac{t}{2} \ge 12$

$t \le -12 \times 2$ $t \ge 2 \times 12$

$t \le -24$ $t \ge 24$

28. $|y - 5| \ge 3$

Solution:

$|y - 5| \ge 3$

By definition of absolute value, we have

$y-5 \le -3$ or $y-5 \ge 3$

$y \le 5-3$ $y \ge 3+5$

$y \le 2$ $y \ge 8$

29. $|x^2-2| \ge 2$

Solution:

$|x^2-2| \ge 2$

By definition of absolute value, we have

$x^2-2 \le -2$ or $x^2-2 \ge 2$

$x^2 \le 2-2$ $x^2 \ge 2+2$

$x^2 \le 0$ $x^2 \ge 4$

$x^2 \le 0$ $x \ge \pm 2$

$x \le -2$ or $x \ge 2$

30. $|x^2-8| \le 8$

Solution:

$|x^2-8| \le 8$

By definition of absolute value, we have

$-8 \le x^2 - 8 \le 8$

$-8+8 \le x^2 \le 8 + 8$

$0 \le x^2 \le 16$

$0 \le x \le 4$

☆☆☆

Solved Section 1.5

Find the midpoint of the line segment connecting the following points.

1. (-1,3) and (4,4)

Solution

Here $(x_1, y_1) = (-1,3)$

and $(x_2, y_2) = (4,4)$

The midpoint of the line segment is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-1+4}{2}, \frac{3+4}{2}\right)$$

$$= \left(\frac{3}{2}, \frac{7}{2}\right)$$

$$= (1.5, 3.5)$$

2. (5,2) and (3,2)

Solution

Here $(x_1, y_1) = (5,2)$

and $(x_2, y_2) = (3,6)$

The midpoint of the line segment is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{5+3}{2}, \frac{2+6}{2}\right)$$

$$= \left(\frac{8}{2}, \frac{8}{2}\right)$$

$$= (4, 4)$$

3. (10,4) and (5,-6)

Solution

Here $(x_1, y_1) = (10,4)$

and $(x_2, y_2) = (5,-6)$

The midpoint of the line segment is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{10+5}{2}, \frac{4+(-6)}{2}\right)$$

$$= \left(\frac{15}{2}, \frac{4-6}{2}\right)$$

$$= \left(\frac{15}{2}, \frac{-2}{2}\right)$$

$$= \left(\frac{15}{2}, -1\right) = (7.5, -1)$$

4. (-1,-3) and (2,15)

Solution

Here $(x_1, y_1) = (-1,-3)$

and $(x_2, y_2) = (2,15)$

The midpoint of the line segment is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-1+2}{2}, \frac{-3+15}{2}\right)$$

$$= \left(\frac{1}{2}, \frac{12}{2}\right)$$

$$= \left(\frac{1}{2}, 6\right)$$

$$= (0.5, 6)$$

5. (20,40) and (-5,-10)

Solution

Here $(x_1, y_1) = (20,40)$

and $(x_2, y_2) = (-5,-10)$

The midpoint of the line segment is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{20+(-5)}{2}, \frac{40+(-10)}{2}\right)$$

$$= \left(\frac{20-5}{2}, \frac{40-10}{2}\right)$$

$$= \left(\frac{15}{2}, \frac{30}{2}\right)$$

$$= (7.5, 15)$$

6. (-5, 24) and (-1, -8)

Solution

Here $(x_1, y_1) = (-5, 24)$

and $(x_2, y_2) = (-1,-8)$

The midpoint of the line segment is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-5+(-1)}{2}, \frac{24+(-8)}{2}\right)$$

$$= \left(\frac{-5-1}{2}, \frac{24-8}{2}\right)$$

$$= \left(-\frac{6}{2}, \frac{16}{2}\right)$$

$$= (-3, 8)$$

7. (0, 6) and (-4, 24)

Solution

Here $(x_1, y_1) = (0, 6)$

and $(x_2, y_2) = (-4, 24)$

The midpoint of the line segment is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{0+(-4)}{2}, \frac{6+24}{2}\right)$$

$$= \left(\frac{-4}{2}, \frac{30}{2}\right)$$

$$= (-2, 15)$$

8. (4, 2) and (-6, -16)

Solution

Here $(x_1, y_1) = (4, 2)$

and $(x_2, y_2) = (-6, -16)$

The midpoint of the line segment is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{4+(-6)}{2}, \frac{2+(-16)}{2}\right)$$

$$= \left(\frac{4-6}{2}, \frac{2-16}{2}\right)$$

$$= \left(\frac{-2}{2}, \frac{-8}{2}\right)$$

$$= (-1, -4)$$

9. (5, 0) and (7, -16)

Solution

Here $(x_1, y_1) = (5, 0)$

and $(x_2, y_2) = (7, -16)$

$d(A,B) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

The midpoint of the line segment is

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{5+7}{2}, \frac{0+(-16)}{2}\right)$

$= \left(\frac{12}{2}, -\frac{16}{2}\right)$

$= (6, -8)$

10. (3, -2) and (-1, 12)

Solution

Here $(x_1, y_1) = (3, -2)$

and $(x_2, y_2) = (-1, 12)$

The midpoint of the line segment is

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{3+(-1)}{2}, \frac{-2+12}{2}\right)$

$= \left(\frac{3-1}{2}, \frac{-2+12}{2}\right)$

$= \left(\frac{2}{2}, \frac{10}{2}\right)$

$= (1, 5)$

11. (6, 3) and (9, -9)

Solution

Here $(x_1, y_1) = (6, 3)$

and $(x_2, y_2) = (9, -9)$

The midpoint of the line segment is

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{6+9}{2}, \frac{3+(-9)}{2}\right)$

$= \left(\frac{6+9}{2}, \frac{3-9}{2}\right)$

$= \left(\frac{15}{2}, -\frac{6}{2}\right)$

$= (7.5, -3)$

12. (0, 4) and (4, 0)

Solution

Here $(x_1, y_1) = (0, 4)$

and $(x_2, y_2) = (4, 0)$

The midpoint of the line segment is

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{0+4}{2}, \frac{4+0}{2}\right)$

$= \left(\frac{4}{2}, \frac{4}{2}\right)$

$= (2, 2)$

13. (-2, -4) and (2, 4)

Solution

Here $(x_1, y_1) = (-2, -4)$

and $(x_2, y_2) = (2, 4)$

The midpoint of the line segment is

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{-2+2}{2}, \frac{-4+4}{2}\right)$

$= \left(\frac{0}{2}, \frac{0}{2}\right)$

$= (0, 0)$

14. (5, 5) and (-2, -2)

Solution

Here $(x_1, y_1) = (5, 5)$

and $(x_2, y_2) = (-2, -2)$

The midpoint of the line segment is

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{5+(-2)}{2}, \frac{5+(-2)}{2}\right)$

$= \left(\frac{5-2}{2}, \frac{5-2}{2}\right)$

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$= \left(\frac{3}{2}, \frac{3}{2}\right)$

$= (1.5, 1.5)$

Find the distance separating the following points.

15. (4, 6) and (0, 0)

Solution

Here $A(x_1, y_1) = A(4, 6)$

$B(x_2, y_2) = B(0, 0)$

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(0-4)^2 + (0-6)^2}$$
$$= \sqrt{(-4)^2 + (-6)^2}$$
$$= \sqrt{16+36}$$
$$= \sqrt{52}$$
$$= 7.21$$

16. (2, 6) and (4, 8)

Solution

Here $A(x_1, y_1) = A(2, 6)$

$B(x_2, y_2) = B(4, 8)$

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4-2)^2 + (8-6)^2}$$
$$= \sqrt{(2)^2 + (2)^2}$$
$$= \sqrt{4+4}$$
$$= \sqrt{8}$$
$$= 2.83$$

17. (0, 0) and (-3, -4)

Solution

Here $A(x_1, y_1) = A(0, 0)$

$B(x_2, y_2) = B(-3, -4)$

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-3-0)^2 + (-4-0)^2}$$
$$= \sqrt{(-3)^2 + (-4)^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25}$$
$$= 5$$

18. (-1, -3) and (4, 3)

Solution

Here $A(x_1, y_1) = A(-1, -3)$

$B(x_2, y_2) = B(4, 3)$

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4-(-1))^2 + (3-(-3))^2}$$
$$= \sqrt{(4+1)^2 + (3+3)^2}$$
$$= \sqrt{(5)^2 + (9)^2}$$
$$= \sqrt{25+81}$$
$$= \sqrt{106}$$
$$= 10.30$$

19. (3, -2) and (-3, 5)

Solution

Here $A(x_1, y_1) = A(3, -2)$

$B(x_2, y_2) = B(-3, 5)$

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-3-3)^2 + (5-(-2))^2}$$
$$= \sqrt{(-6)^2 + (7)^2}$$
$$= \sqrt{36+49}$$
$$= \sqrt{85}$$
$$= 9.22$$

20. (10, 5) and (20, -10)

Solution

Here $A(x_1, y_1) = A(10, 5)$

$B(x_2, y_2) = B(20, -10)$

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(20-10)^2 + (-10-5)^2}$$
$$= \sqrt{(10)^2 + (-15)^2}$$
$$= \sqrt{100+225}$$
$$= \sqrt{325}$$

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= 18.03$

21. (-4, -2) and (6, 10)

Solution

Here $A(x_1, y_1) = A(-4, -2)$

$B(x_2, y_2) = B(6, 10)$

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(6-(-4))^2 + (10-(-2))^2}$$
$$= \sqrt{(6+4)^2 + (10+2)^2}$$
$$= \sqrt{(10)^2 + (12)^2}$$
$$= \sqrt{100+144}$$
$$= \sqrt{244}$$
$$= 15.63$$

22. (3, 12) and (-1, 8)

Solution

Here $A(x_1, y_1) = A(3, 12)$

$B(x_2, y_2) = B(-1, 8)$

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-1-3)^2 + (8-12)^2}$$
$$= \sqrt{(-4)^2 + (-4)^2}$$
$$= \sqrt{16+16}$$
$$= \sqrt{244}$$
$$= 5.66$$

23. (10, 0) and (0, -4)

Solution

Here $A(x_1, y_1) = A(10, 0)$

$B(x_2, y_2) = B(0, -4)$

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(0-10)^2 + (-4-0)^2}$$
$$= \sqrt{(-10)^2 + (-4)^2}$$
$$= \sqrt{100+16}$$
$$= \sqrt{116}$$
$$= 10.77$$

24. (5, 1) and (1, -4)

Solution

Here $A(x_1, y_1) = A(5, 1)$

$B(x_2, y_2) = B(1, -4)$

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(1-5)^2 + (-4-1)^2}$$
$$= \sqrt{(-4)^2 + (-5)^2}$$
$$= \sqrt{16+25}$$
$$= \sqrt{41}$$
$$= 6.40$$

25. (-2, 4) and (1, 0)

Solution

Here $A(x_1, y_1) = A(-2, 4)$

$B(x_2, y_2) = B(1, 0)$

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(1-(-2))^2 + (0-4)^2}$$
$$= \sqrt{(1+2)^2 + (-4)^2}$$
$$= \sqrt{(3)^2 + (-4)^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25}$$
$$= 5$$

26. (2, 2) and (10, 8)

Solution

Here $A(x_1, y_1) = A(2, 2)$

$B(x_2, y_2) = B(10, 8)$

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(10-2)^2 + (8-2)^2}$$
$$= \sqrt{(8)^2 + (6)^2}$$
$$= \sqrt{64+36}$$
$$= \sqrt{100}$$
$$= 10$$

27. (5, 2) and (0, 6)

Solution

Here $A(x_1, y_1) = A(5, 2)$

$B(x_2, y_2) = B(0, 6)$

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(0-5)^2 + (6-2)^2}$$
$$= \sqrt{(-5)^2 + (4)^2}$$
$$= \sqrt{25+16}$$
$$= \sqrt{41}$$
$$= 6.40$$

28. (4, 4) and (-5, -8)

Solution

Here $A(x_1, y_1) = A(4, 4)$

$B(x_2, y_2) = B(-5, -8)$

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-5-4)^2 + (-8-4)^2}$$
$$= \sqrt{(-9)^2 + (-12)^2}$$
$$= \sqrt{81+144}$$
$$= \sqrt{225}$$
$$= 15$$

29. (7, 2) and (-1, 4)

Solution

Here $A(x_1, y_1) = A(7, 2)$

$B(x_2, y_2) = B(-1, 4)$

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-1-7)^2 + (4-2)^2}$$
$$= \sqrt{(-8)^2 + (2)^2}$$
$$= \sqrt{64+4}$$
$$= \sqrt{68}$$
$$= 8.25$$

30. (3, 6) and (-2, 4)

Solution

Here $A(x_1, y_1) = A(3, 6)$

$B(x_2, y_2) = B(-2, 4)$

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-2-3)^2 + (4-6)^2}$$
$$= \sqrt{(-5)^2 + (-2)^2}$$
$$= \sqrt{25+4}$$
$$= \sqrt{29}$$
$$= 5.39$$

Solved Additional Exercise

Solve the following first - degree equations.

(a) $4x - 10 = 8 - 2x$

Solution:

$$4x - 10 = 8 - 2x$$
$$4x + 2x = 8 + 10$$
$$6x = 18$$
$$x = \frac{18}{6}$$
$$x = 3$$

(b) $x - 5 = \frac{(-2x + 10)}{2}$

Solution:

$$x - 5 = \frac{(-2x+10)}{2}$$
$$2(x-5) = (-2x + 10)$$
$$2x - 10 = -2x + 10$$
$$2x + 2x = 10 + 10$$
$$4x = 20$$
$$x = 5$$

(c) $3x + 3 = 3x - 5$

Solution:

$$3x + 3 = 3x - 5$$
$$3x - 3x = -5 - 3$$
$$0 = -8$$

No solution.

Solve the following second - degree equations.

(a) $x^2 + 3x + 2 = 0$

Solution:

$$\begin{aligned} x^2 + 3x + 2 &= 0 \\ x^2 + 2x + x + 2 &= 0 \\ x(x+2)+1(x+2) &= 0 \\ (x+1)(x+2) &= 0 \\ \Rightarrow \quad x+1 &= 0 \quad \text{or} \quad x + 2 = 0 \\ x &= -1 \qquad\quad x = -2 \end{aligned}$$

(b) $3x^2 - 2x + 5 = 0$

Solution:

$$\begin{aligned} 3x^2 - 2x + 5 &= 0 \\ 3x^2 - 5x + 3x + 5 &= 0 \quad \text{As } 3 \times 5 = 15 \\ x(3x-5)+1(3x-5) &= 0 \quad \text{So } -2x = -5x + 3x \\ (x+1)(3x-5) &= 0 \\ \Rightarrow \quad x+1 &= 0 \quad \text{or} \quad 3x - 5 = 0 \\ x &= -1 \qquad\quad 3x = 5 \\ & \qquad\qquad\quad x = \frac{5}{3} \end{aligned}$$

(c) $x^2 + 10x + 25 = 0$

Solution:

$$\begin{aligned} x^2 + 10x + 25 &= 0 \\ x^2 + 5x + 5x + 25 &= 0 \\ x(x+5)+5(x+5) &= 0 \\ (x+5)(x+5) &= 0 \\ \Rightarrow \quad x+5 &= 0 \quad \text{or} \quad x + 5 = 0 \\ x &= -5 \qquad\quad x = -5 \end{aligned}$$

Solve the inequality

$2x - 5 \geq 3x + 2$

Solution:

$$\begin{aligned} 2x - 5 &\geq 3x + 2 \\ 2x - 3x &\geq 5 + 2 \\ -x &\geq 7 \\ (-)(-x) &\leq (-)(7) \\ x &\leq -7 \end{aligned}$$

Solve the inequality

$$10 \leq x + 5 \leq 30$$

Solution:

$$\begin{aligned} 10 \leq x + 5 &\leq 30 \\ 10 - 5 \leq x &\leq 30 - 5 \\ 5 \leq x &\leq 25 \end{aligned}$$

Solve the inequality

$$x^2 + x - 12 \leq 0$$

Solution:

$$\begin{aligned} x^2 + x - 12 &\leq 0 \\ x^2+4x-3x-12 &\leq 0 \\ x(x+4)-3(x+4) &\leq 0 \\ (x-3)(x+4) &\leq 0 \end{aligned}$$

The following attributes of the factors on the left side will result in the inequality being satisfied.

	Factors		
Conditions	**(x-3)**	**(x+4)**	**Product**
Condition-1	= 0	Any value	0
Condition-2	Any value	= 0	0
Condition-3	> 0	< 0	< 0
Condition-4	< 0	> 0	< 0

Condition -1:

$$\begin{aligned} x - 3 &= 0 \\ x &= 3 \end{aligned}$$

Condition -2:

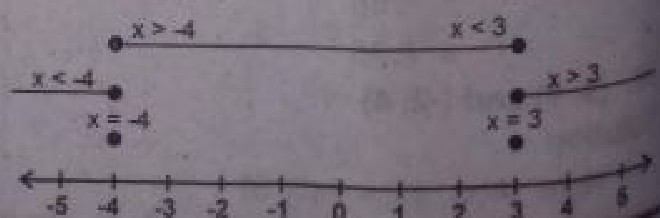
$$\begin{aligned} x + 4 &= 0 \\ x &= -4 \end{aligned}$$

Condition -3:

$$x - 3 > 0 \quad \text{and} \quad x + 4 < 0$$
$$x > 3 \qquad\qquad x < -4$$

Condition -4:

$$x - 3 < 0 \quad \text{and} \quad x + 4 > 0$$
$$x < 3 \qquad\qquad x > -4$$



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The solution is $-4 \le x \le 3$.

Solve the equation.

$|5 - 2x| = 9$

Solution:

$|5 - 2x| = 9$

By the definition of absolute value, we have;

$5 - 2x = \pm 9$

$5 - 2x = -9$ or $5 - 2x = 9$

$-2x = 9 - 5$ $-2x = 9 - 5$

$-2x = -14$ $-2x = 4$

$x = 7$ $x = -2$

Solve the equation.

$|x + 3| = |5 - x|$

Solution:

$|x + 3| = |5 - x|$

By the definition of absolute value, we have;

$x + 3 = \pm(5 - x)$

$x + 3 = -(5 - x)$ or $x + 3 = +(5 - x)$

$x + 3 = -5 + x$ $x + 3 = 5 - x$

$x - x = -5 - 3$ $x + x = 5 - 3$

$0 = -8$ $2x = 2$

$x = 1$

No solution:

Solve the inequality

$|2x + 3| < 5$

Solution:

$|2x + 3| < 5$

By definition of absolute value, we have;

$-5 < 2x + 3 < 5$

$-5 - 3 < 2x < 5 - 3$

$-8 < 2x < 2$

$-\frac{8}{2} < x < \frac{2}{2}$

$-4 < x < 1$

Find the midpoint of the line segment connecting (4,12) and (-2, -18).

Solution:

Here $(x_1, y_1) = (4, 12)$

$(x_2, y_2) = (-2, -18)$

The midpoint of the line segment is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{4 + (-2)}{2}, \frac{12 + (-18)}{2}\right)$$

$$= \left(\frac{4 - 2}{2}, \frac{12 - 18}{2}\right)$$

$$= \left(\frac{2}{2}, -\frac{6}{2}\right)$$

$$= (1, -3)$$

Determine the distance separating (4, 12) and (-3, 6).

Solution:

Here $(x_1, y_1) = (4, -2)$

$(x_2, y_2) = (-3, -6)$

The midpoint of the line segment is

Here $A(x_1, y_1) = A(4, -2)$

$B(x_2, y_2) = B(-3, 6)$

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-3 - 4)^2 + (6 - (-2))^2}$$

$$= \sqrt{(-7)^2 + (8)^2}$$

$$= \sqrt{49 + 64}$$

$$= \sqrt{113}$$

$$= 10.63$$

Section 1.1

Solve the following first-degree equations.

1. 6x - 4 = 5x + 2

Solution:

$6x - 4 = 5x + 2$

$6x - 5x = 4 + 2$

$x = 6$

2. **-10 + 4x = 3x + 8**

Solution:

$$-10 + 4x = 3x + 8$$
$$4x - 3x = 10 + 8$$
$$x = 18$$

3. **4x = 3x + 6**

Solution:

$$4x = 3x + 6$$
$$4x - 3x = 6$$
$$x = 6$$

4. **-2x + 8 = 2x - 4**

Solution:

$$-2x + 8 = 2x - 4$$
$$-2x - 2x = -8-4$$
$$-4x = -12$$
$$x = \frac{-12}{-4}$$
$$x = 3$$

5. **5y = 10y - 30**

Solution:

$$5y = 10y - 30$$
$$5y - 10y = -30$$
$$-5y = -30$$
$$y = \frac{-30}{-5}$$
$$y = 6$$

6. **4(y - 3) = y + 9**

Solution:

$$4y - 12 = y + 9$$
$$4y - y = 12 + 9$$
$$3y = 18$$
$$y = \frac{18}{3}$$
$$y = 6$$

7. **6x + 20 = 40 + 8x**

Solution:

$$6x + 20 = 40 + 8x$$
$$6x - 8x = 40 - 20$$
$$-2x = 20$$
$$x = \frac{20}{-2}$$
$$x = -10$$

8. **15x - 4(2x + 14) = 0**

Solution:

$$15x - 4(2x+14) = 0$$
$$15x - 8x - 56 = 0$$
$$7x - 56 = 0$$
$$7x = 56$$
$$x = \frac{56}{7}$$
$$x = 8$$

9. **-3y - 5(y + 4) = 4**

Solution:

$$-3y - 5(y + 4) = 4$$
$$-3y - 5y - 20 = 4$$
$$-8y = 4 + 20$$
$$-8y = 24$$
$$y = \frac{24}{-8}$$
$$y = -3$$

10. **3(x - 4) +2(2x + 1) = 11**

Solution:

$$3(x-4)+2(2x+1) = 11$$
$$3x-12+4x+2 = 11$$
$$7x-10 = 11$$
$$7x = 11 + 10$$
$$7x = 21$$
$$x = \frac{21}{7}$$
$$x = 3$$

11. **30x + 50(x - 6) = -20**

Solution:

$$30x + 50(x-6) = -20$$
$$30x + 50x - 300 = -20$$
$$80x = -20+300$$
$$80x = 280$$
$$x = \frac{280}{80}$$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{7}{2}$$

12. 4(5-x) + 2x - 10 = -2x + 10

Solution:

$$\begin{aligned} 4(5-x)+2x-10 &= -2x+10 \\ 20-4x+2x-10 &= -2x+10 \\ -2x+10 &= -2x+10 \\ -2x+2x &= -10+10 \\ 0 &= 0 \end{aligned}$$

No solution.

Section 1.2

Solve the following second - degree equations.

13. $x^2 - 36 = 0$

Solution:

$$\begin{aligned} x^2 - 36 &= 0 \\ (x)^2 - (6)^2 &= 0 \\ (x-6)(x+6) &= 0 \\ x-6 &= 0 \quad \text{or} \quad x+6 = 0 \\ x &= 6 \qquad\quad x = -6 \end{aligned}$$

14. $x^2 + 14x + 49 = 0$

Solution:

$$\begin{aligned} x^2 + 14x + 49 &= 0 \\ x^2+7x+7x+49 &= 0 \\ x(x+7)+7(x+7) &= 0 \\ (x+7)(x+7) &= 0 \\ \Rightarrow x + 7 &= 0 \quad \text{or} \quad x + 7 = 0 \\ x &= -7 \qquad\quad x = -7 \end{aligned}$$

15. $x^2 - 5x + 4 = 0$

Solution:

$$\begin{aligned} x^2 - 5x + 4 &= 0 \\ x^2 - 4x - x + 4 &= 0 \\ x(x-4)-1(x-4) &= 0 \\ (x-1)(x-4) &= 0 \\ \Rightarrow x - 1 &= 0 \quad \text{or} \quad x - 4 = 0 \\ x &= 1 \qquad\quad x = 4 \end{aligned}$$

16. $4x^2 + 2x - 30 = 0$

Solution:

$$\begin{aligned} 4x^2+2x-30 &= 0 \\ 4x^2+12x-10x-30 &= 0 \quad \text{As } 4\times(-30)=-120 \\ 4x(x+3)-10(x+3) &= 0 \quad \text{So } +2x=+12x-10x \\ (4x-10)(x+3) &= 0 \\ \Rightarrow 4x - 10 &= 0 \quad \text{or} \quad x + 3 = 0 \\ 4x &= 10 \qquad\quad x = -3 \\ 4x &= \frac{10}{4} \\ x &= \frac{5}{2} \end{aligned}$$

17. $7x^2 - 70 = 21x$

Solution:

$7x^2 - 70 = 21x$

$7x^2 - 21 - 70 = 0$

Compare it with

$ax^2 + bx + c = 0$

$a = 7,\ b = -21,\ c = -70$

We know that

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-21) \pm \sqrt{(-21)^2 - 4(7)(-70)}}{2(7)} \\ &= \frac{21 \pm \sqrt{441 + 1960}}{14} \\ &= \frac{21 \pm \sqrt{2401}}{14} \\ &= \frac{21 \pm 45}{14} \end{aligned}$$

$$x = \frac{21 - 45}{14}, \qquad x = \frac{21 + 45}{14}$$

$$= -\frac{24}{14} \qquad = \frac{66}{14}$$

$$= -\frac{12}{7} \qquad = \frac{33}{7}$$

18. $2x^2 + 3x - 10 = x^2 + 6x + 30$

Solution:

$$2x^2 + 3x - 10 = x^2 + 6x + 30$$
$$2x^2+3x-10-x^2-6x-30 = 0$$
$$2x^2-x^2+3x-6x-10-30 = 0$$
$$x^2-3x-40 = 0$$
$$x^2-8x+5x-40 = 0$$
$$x(x-8)+5(x-8) = 0$$
$$(x+5)(x-8) = 0$$
$$\Rightarrow x+5 = 0 \quad \text{or} \quad x-8 = 0$$
$$x = -5 \quad\quad x = 8$$

19. $-6x^2 + 4x - 10 = 0$

Solution:

$-6x^2 + 4x - 10 = 0$

Compare it with

$ax^2 + bx + c = 0$

$a = -6, b = 4, c = -10$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-4 \pm \sqrt{(4)^2 - 4(-6)(-10)}}{2(-6)}$$
$$= \frac{4 \pm \sqrt{16 - 240}}{-12}$$
$$= \frac{-4 \pm \sqrt{-224}}{-12}$$

Solution is not possible.

20. $-5x^2 + 10x - 20 = 0$

Solution:

$-5x^2 + 10x - 20 = 0$

Compare it with

$ax^2 + bx + c = 0$

$a = -5, b = 10, c = -20$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-10 \pm \sqrt{(10)^2 - 4(-5)(-20)}}{2(-5)}$$
$$= \frac{-10 \pm \sqrt{100 - 400}}{-10}$$
$$= \frac{-10 \pm \sqrt{-300}}{-10}$$

Solution is not possible.

21. $5x^2 - 17.5x - 10 = 0$

Solution:

$5x^2 - 17.5x - 10 = 0$

Compare it with

$ax^2 + bx + c = 0$

$a = -5, b = -17.5, c = -10$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(17.5) \pm \sqrt{(-17.5)^2 - 4(5)(-10)}}{2(5)}$$
$$= \frac{17.5 \pm \sqrt{306.25 + 200}}{10}$$
$$= \frac{17.5 \pm \sqrt{506.25}}{10}$$
$$= \frac{17.5 \pm 22.5}{10}$$
$$x = \frac{17.5 - 22.5}{10}, \quad x = \frac{17.5 + 22.5}{10}$$
$$= \frac{-5}{10} \quad\quad = \frac{40}{10}$$
$$= -0.5 \quad\quad = 4$$

22. $x^2 + 64 = 0$

Solution:

$x^2 + 64 = 0$

$x^2 + 0x + 64 = 0$

Compare it with

$ax^2 + bbx + c = 0$

$a = 1, b = 0, c = 64$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-0 \pm \sqrt{(0)^2 - 4(1)(64)}}{2(1)}$$

$$= \frac{\pm\sqrt{-256}}{2}$$

Solution is not possible.

23. $8x^2 + 2x - 15 = 0$

Solution:

$8x^2 + 2x - 15 = 0$

Compare it with

$ax^2 + bx + c = 0$

$a = 8, b = 2, c = -15$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4(8)(-15)}}{2(8)}$$

$$= \frac{-2 \pm \sqrt{4 + 480}}{16}$$

$$= \frac{-2 \pm \sqrt{484}}{16}$$

$$= \frac{-2 \pm 22}{16}$$

$$= \frac{-2 - 22}{16}, \quad x = \frac{-2 + 22}{16}$$

$$= \frac{-24}{16} \qquad = \frac{20}{16}$$

$$= -\frac{3}{2} \qquad = \frac{5}{4}$$

24. $-x^2 - 2x + 35 = 0$

Solution:

$-x^2 - 2x + 35 = 0$

$-(x^2+2x-35) = 0$

$\Rightarrow x^2 + 2x - 35 = 0$

$x^2+7x-5x-35 = 0$

$x(x+7)-5(x+7) = 0$

$(x-5)(x+7) = 0$

$\Rightarrow x-5 = 0$ or $x + 7 = 0$

$x = 5 \qquad x = -7$

25. $2a^2 + 2a - 12 = 0$

Solution:

$2a^2 + 2a - 12 = 0$

$2a^2 + 2a - 12 = 0$

$2a^2 + 6a - 4a - 12 = 0$ As $2\times(-12)=-24$

$2a(a+3)-4(a+3) = 0$ So $2a = 6a-4a$

$(2a-4)(a+3) = 0$

$\Rightarrow 2a - 4 = 0$ or $a + 3 = 0$

$2a = 4 \qquad a = -3$

$a = \frac{4}{2}$

$a = 2$

26. $5a^2 - 2a + 16 = 0$

Solution:

$5a^2 - 2a + 16 = 0$

$5a^2 - 2a + 16 = 0$

$5a^2 - 10a + 8a + 16 = 0$ As $5\times16 = 80$

$5a(a-2)+8(a+2) = 0$ So $-2a = -10a+8a$

Because factors inside the brackets are not same.

So solution is not possible.

27. $3a^2 - 3a - 18 = 0$

Solution:

$3a^2 - 3a - 18 = 0$

Compare it with

$ax^2 + bx + c = 0$

$a = 3, b = -3, c = -18$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-17)}}{2(3)}$$

$$= \frac{3 \pm \sqrt{9 + 216}}{6}$$

$$= \frac{3 \pm \sqrt{225}}{6}$$

$$= \frac{3 \pm 15}{6}$$

$$x = \frac{3-15}{6}, \quad x = \frac{3+15}{6}$$

$$= \frac{-12}{6} \qquad = \frac{18}{6}$$

$$= -2 \qquad = 3$$

28. $x^2 + 2x - 48 = 0$

Solution:

$x^2 + 2x - 48 = 0$

$$x^2 + 2x - 48 = 0$$
$$x^2 + 8x - 6x - 48 = 0$$
$$x(x+8)-6(x+8) = 0$$
$$(x-6)(x+8) = 0$$
$$\Rightarrow x - 6 = 0 \quad \text{or} \quad x + 8 = 0$$
$$x = 6 \qquad x = -8$$

29. $x^2 - 2x + 10 = 0$

Solution:

$x^2 - 2x + 10 = 0$

Compare it with

$ax^2 + bx + c = 0$

a = 1, b = -2, c = 10

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(10)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4-40}}{2}$$

$$= \frac{2 \pm \sqrt{-36}}{2}$$

Solution is not possible.

30. $5x^2 - 20x + 15 = 0$

Solution:

$5x^2 - 20x + 15 = 0$

Compare it with

$ax^2 + bx + c = 0$

a = 5, b = -20, c = 15

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-20) \pm \sqrt{(-20)^2 - 4(5)(15)}}{2(5)}$$

$$= \frac{20 \pm \sqrt{400-300}}{10}$$

$$= \frac{20 \pm \sqrt{100}}{10}$$

$$= \frac{20 \pm 10}{10}$$

$$x = \frac{20-10}{10}, \quad x = \frac{20+10}{10}$$

$$= \frac{10}{10} \qquad = \frac{30}{10}$$

$$= 1 \qquad = 3$$

Section 1.3

Solve the following inequalities.

31. $x - 8 \le 2x + 4$

Solution:

$x - 8 \le 2x + 4$

$$x - 8 \le 2x + 4$$
$$x - 2x \le 4 + 8$$
$$-x \le 12$$
$$(-)(-x) \ge (-)(12)$$
$$x \ge -12$$

32. $3x + 4 \le 4x - 12$

Solution:

$3x + 4 \le 4x - 12$

$$3x + 4 \le 4x - 2$$
$$3x - 4x \le -4 - 2$$
$$-x \le -6$$
$$(-)(-x) \ge (-)(-6)$$
$$x \ge 6$$

33. $4x + 5 \geq 2x - 3$

Solution:

$4x + 5 \geq 2x - 3$

$4x + 5 \geq -5 - 3$

$2x \geq -8$

$x \geq \frac{-8}{2}$

$x \geq -4$

34. $9x - 5 \geq 6x + 4$

Solution:

$9x - 5 \geq 6x + 4$

$9x - 6x \geq 5 + 4$

$3x \geq 9$

$x \geq \frac{9}{3}$

$x \geq 3$

35. $-2x + 10 \geq x - 8$

Solution:

$-2x + 10 \geq x - 8$

$-2x - x \geq -10 - 8$

$-3x \geq -18$

$-x \geq -6$

$(-)(-x) \leq (-)(-6)$

$x \leq 6$

36. $5x - 4 \geq 3x + 8$

Solution:

$5x - 4 \geq 3x + 8$

$5x - 3x \geq 4 + 8$

$2x \geq 12$

$x \geq 6$

37. $-4 \leq 2x + 2 \leq 10$

Solution:

$-4 \leq 2x + 2 \leq 10$

$-4 - 2 \leq 2x \leq 10 - 2$

$-6 \leq 2x \leq 8$

$-3 \leq x \leq 4$

38. $4 \leq -x + 3 \leq 12$

Solution:

$4 \leq -x + 3 \leq 12$

$4 - 3 \leq -x \leq 12 - 3$

$1 \leq -x \leq 9$

$(-)(1) \geq (-)(-x) \geq (-)(9)$

$-1 \geq x \geq 9$

39. $x + 5 \leq x + 1 \leq 6$

Solution:

$x + 5 \leq x + 1 \leq 6$

We can write it as

$x + 5 \leq x + 1$ or $x + 1 \leq 6$

$x - x \leq -5 + 1$ $x \leq 6 - 1$

$0 \leq -4$ $x \leq 5$

No solution

40. $-x + 3 \leq 2x + 3 \leq 9$

Solution:

$-x + 3 \leq 2x + 3 \leq 9$

We can write it as

$-x + 3 \leq 2x + 3$ or $2x + 3 \leq 9$

$-x - 2x \leq 3 - 3$ $2x \leq 9 - 3$

$-3x \leq 0$ $2x \leq 6$

$(-)(-3x) \geq (-)(0)$ $x \leq 3$

$3x \geq 0$

$x \geq 0$

So $0 \leq x \leq 3$

Solve the following second - degree inequalities.

41. $x^2 - 25 \geq 0$

42. $x^2 - 16 \geq 0$

43. $x^2 - 5x + 4 \leq 0$

44. $x^2 - x - 20 \leq 0$

45. $2x^2 - 5x - 12 \geq 0$

46. $5x^2 - 13x - 6 \geq 0$

47. $12x^2 - 5x - 2 \leq 0$

48. $3x^2 + x - 10 \leq 0$

Solution:

All above questions are same as

(Q-33 to Q-44) (Solved Section 1.3)

Section 1.4

Solve the following questions.

49. $|x - 5| = 4$
50. $|10 - 2x| = 14$
51. $|x + 8| = 2$
52. $|x - 5| = -4$
53. $|x - 4| = |8 - 2x|$
54. $|3x - 6| = |x + 6|$
55. $|x| = |9 - x|$
56. $|2x + 5| = |-x|$

Solution:

All above questions are same as (Q. 1 to 14)

Section 1.4

Solve the following inequalities.

57. $|x| \le 10$
58. $|-x| \ge 12$
59. $|x + 5| \le 8$
60. $|x - 15| \le 10$
61. $|3x - 5| \ge 3$
62. $|2x + 9| \ge 5$
63. $|3x - 6| \le -4$
64. $|5x - 3| \le 9$

Solution:

All above questions are same as (Q.15 to 30)

Section 1.5

Find the midpoint of the line segment connecting the following points.

65. (-3, 10) and (2, -15)
66. (-1, 3) and (1, -9)
67. (4, 4) and (-2, 2)
68. (0, 4) and (2, 0)
69. (4, - 8) and (2, 4)
70. (a, a) and (b, b)
71. (a, b) and (3a, 3a)
72. (a, b) and (-a, -b)

Solution:

All above questions are same as (Q-1 to 14).

Solved Section 1.5

Find the distance separating the following points.

73. (2, 4) and (-4, 6)
74. (-2, 2) and (3, -3)
75. (6, 2) and (3, 6)
76. (-1, -2) and (-4, -6)
77. (10, 5) and (-10, 5)
78. (5, -10) and (20, 10)
79. (a, b) and (a, 3b)
80. (5a, 2b) and (0, 2b)

Solution:

All above questions are same as (Q-15 to 30).

Solved chapter Test

1. Solve the equation $5x = 5x + 10$

Solution:

$$5x = 5x + 10$$
$$5x - 5x = 10$$
$$0 = 10$$

No roots.

2. Solve the equation $x^2 - 2x + 5 = 0$

Solution:

$x^2 - 2x + 5 = 0$

Compare it with

$ax^2 + bx + c = 0$

$a = 1, b = -2, c = 5$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4 - 20}}{2}$$

$$= \frac{2 \pm \sqrt{-16}}{2}$$

No roots.

3. Solve the equation $x^2 - 7x + 12 = 0$

Solution:

$$x^2 - 7x + 12 = 0$$

$$x^2 - 4x - 3x + 12 = 0$$

$$x(x-4) - 3(x-4) = 0$$

$$(x-3)(x-4) = 0$$

$$\Rightarrow \quad x - 3 = 0 \quad \text{or} \quad x - 4 = 0$$

$$x = 3 \qquad x = 4$$

4. Solve the following inequality:

$-2 \le x - 6 \le x + 1$

Solution:

$-2 \le x - 6 \le x + 1$

We cant write it as

$$-2 \le x - 6 \quad \text{or} \quad -6 \le x + 1$$

$$-x \le -6+2 \qquad -x \le +6+1$$

$$-x \le -4 \qquad -x \le +7$$

$$(-)(-x) \ge (-)(-4) \qquad (-)(-x) \ge (-)(+7)$$

$$x \ge 4 \qquad x \ge -7$$

4. Solve the following inequality:

$$x^2 + 3x + 2 \le 0$$

$$x^2 + 3x + 2 \le 0$$

$$x^2 + 2x + x + 2 \le 0$$

$$x(x+2) + 1(x+2) \le 0$$

$$(x+1)(x+2) \le 0$$

The following attributes of two factors on the left side will result in the inequality being satisfied.

	Factors		
Conditions	**(x+1)**	**(x+2)**	**Product**
Condition-1	= 0	Any value	0
Condition-2	Any value	= 0	0
Condition-3	> 0	< 0	< 0
Condition-4	< 0	> 0	< 0

Condition -1:

$$x + 1 = 0$$

$$x = -1$$

Condition -2:

$$x + 2 = 0$$

$$x = -2$$

Condition -3:

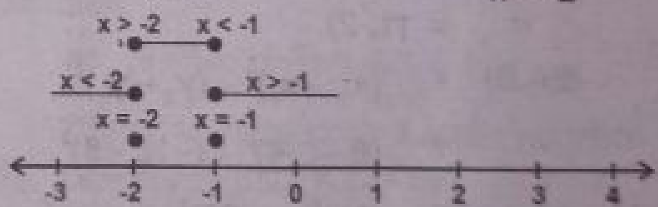
$$x + 1 > 0 \quad \text{and} \quad x + 2 < 0$$

$$x > -1 \qquad x < -2$$

Condition -4:

$$x + 1 < 0 \quad \text{and} \quad x + 2 > 0$$

$$x < -1 \qquad x > -2$$



The solution is $-2 < x < -1$.

6. Solve the equation: $|x - 12| = |4 - x|$

Solution:

$|x - 12| = |4 - x|$

By definition of absolute value, we have

$$x - 12 = \pm(4 - x)$$

$$x - 12 = -(4 - x) \quad \text{or} \quad x - 12 = +(4-x)$$

$$x - 12 = -4 + x \qquad x - 12 = 4 - x$$

$$x - x = -4 + 12 \qquad x + x = 4 + 12$$

0 = 8 $\quad$ 2x = 16

No roots $\quad$ x = 8

7. Solve the following inequality:

$|x + 12| \le 8$

Solution:

$|x + 12| \le 8$

$-8 \le x + 12 \le 8$

$-8 - 12 \le x \le 8 - 12$

$-20 \le x \le -4$

8. Give the points (-4,8) and (6, -12):

(a) Determine the midpoint of the line segment connecting the points.

(b) Determine the distance separating the two points.

Solution:

Here $A(x_1, y_1) = A(-4, 8)$

$B(x_2, y_2) = B(6, -12)$

(a) Mid point $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{-4+6}{2}, \frac{8+(-12)}{2}\right)$

$= \left(\frac{2}{2}, \frac{-4}{2}\right)$

$= (1,-2)$

$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(6-(-4)^2 + (-12-8)^2}$

$= \sqrt{(10)^2 + (-20)^2}$

$= \sqrt{100 + 400}$

$= \sqrt{500}$

$= 22.36$

☆☆☆☆☆☆

Chapter - 2

LINEAR EQUATIONS

Solved Section 2.1

Determine which of the following equations are linear.

1. $-3y = 0$

solution:

$-3y = 0$

The given equation in a linear equation. As we know that.

A linear equation involving two variables x and y has the standard form.

$ax + by = c$

Where a,b and c are constants and a and b cannot both equal to zero.

2. $\sqrt{2x} + 6y = -25$

Solution:

$\sqrt{2x} + 6y = -25$

The given equation is a linear equation.

3. $-5x + 24y = 200$

Solution:

$-5x + 24y = 200$

The given equation is a linear equation.

4. $-x^2 + 3y = 40$

Solution:

$-x^2 + 3y = 40$

The given equation is a non - linear equation.

5. $2x - 3xy + 5y = 10$

Solution:

$2x - 3xy + 5y = 10$

The given equation is a non - linear equation.

6. $\sqrt{4x} - 3y = -45$

Solution:

$\sqrt{4x} - 3y = -45$

The given equation is a non - linear equation.

7. $u - 3v = 20$

Solution:

$u - 3v = 20$

The given equation is a linear equation.

8. $r/2 + S/5 = \frac{2}{7}$

Solution:

$r/2 + S/5 = \frac{2}{7}$

The given equation is a linear equation.

9. $m/2 + (2m - 3n) / 5 = 0$

Solution:

$m/2 + (2m - 3n) / 5 = 0$

The given equation is a linear equation.

10. $(x + 2y)/3 - 3x/4 = 2x - 5y$

Solution:

$(x + 2y)/3 - 3x/4 = 2x - 5y$

The given equation is a linear equation.

11. $40 - 3y = \sqrt{24}$

Solution:

$40 - 3y = \sqrt{24}$

The given equation is a linear equation.

12. $0.0003x - 2.3245y = x+y - 3.2543$

Solution:

$0.0003x - 2.3245y = x+y - 3.2543$

The given equation is a linear equation.

13. $2x_1 - 3x_2 + x_3 = 0$

Solution:

$2x_1 - 3x_2 + x_3 = 0$

The given equation is a linear equation.

14. $(x_1 - 3x_2 + 5x_3 - 2x_4 + x_5)/25 = 300$

Solution:

$(x_1 - 3x_2 + 5x_3 - 2x_4 + x_5)/25 = 300$

The given equation is a linear equation.

15. $(x_1 + x_2 - x_3 x_1) = 5$

Solution:

$(x_1 + x_2 - x_3 x_1) = 5$

The given equation is a non - linear equation.

16. $3x_2 - 4x_1 = 5x_3 + 2x_2 - x_4 + 36$

Solution:

$3x_2 - 4x_1 = 5x_3 + 2x_2 - x_4 + 36$

The given equation is a linear equation.

17. $\sqrt{x^2 + 2xy + y^2} = 25$

Solution:

$\sqrt{x^2 + 2xy + y^2} = 25$

The given equation is a linear equation.

18. $(2x_1 - 3x_2 + x_3)/4 = (x_2 - 2x_4)/5 + 90$

Solution:

$(2x_1 - 3x_2 + x_3)/4 = (x_2 - 2x_4)/5 + 90$

The given equation is a linear equation.

19. **Consider the equation 8x = 120 as two variables equation having the form of linear equation.**

a) **Define a,b and c.**

b) **What pair of values satisfy the equation when y = 10?**

c) **What pair of values satisfy the equation when x = 20?**

d) **Verbalize the somewhat unique nature of the solution set for this equation.**

Solution:

a) $8x = 120$

In the given linear equation.

$a = 8$

$b = 0$

$c = 120$

b) when y = 10

Then $8x = 120$

$x = \frac{120}{8}$

$x = 15$

So the pair of values is (15,10)

c) when x = 20

Then $8y = 120$

$y = \frac{120}{8}$

$y = 15$

So the pair of values is (20,15)

d) when y = 10, then x will be 15

And when x = 20, then y will be 15

So they have a unique nature of solution set.

20. **Rework Example 2 if product A requires 2 hours per unit and product B requries 4 hours per unit.**

Solution:

a) We can define our variables as follow.

x = numbers of units produced of product A.

y = numbers of units produced of product B.

The desired equation has the following structure. Total hours used in producing product A and B = 120 Hence, the equation is $2x + 4y = 120$

b) If 30 units of product B are produced, then y = 30, Therefore

$$2x + 4(30) = 120$$
$$2x + 120 = 120$$
$$2x = 120 - 120$$
$$2x = 0$$
$$x = 0$$

Thus, one pair of values satisfying the equation is (o,30)

c) If manjgement decides to manufacture product A only, no units of product B are produced or if y = 0, then

$$2x + 4(0) = 120$$
$$2x + 0 = 120$$
$$2x = 120$$
$$x = \frac{120}{2}$$
$$x = 60$$

Therefore 60 in the maximum number of units of product A which can be produced using the 120 hours.

If management decides to manufacture product B only, x = 0 and

$$2(0) + 4y = 120$$
$$0 + 4y = 120$$
$$4y = 120$$
$$y = \frac{120}{4}$$
$$y = 30 \text{ units}$$

21. Given the equation $4x_1 - 2x_2 + 6x_3 = 0$

Solution:

a) What values satisfy the equation when $x_1 = 2$ and $x_3 + 1$?

solution:

a) $4x_1 - 2x_2 + 6x_3 = 0$

when $x_1 = 2$, $x_3 = 1$, then By putting the values we get.

$$4(2) - 2x_2 + 6(1) = 0$$
$$8 - 2x_2 + 6 = 0$$
$$-2x_2 + 14 = 0$$
$$-2x_2 = -14$$
$$x_2 = \frac{14}{2}$$
$$x_2 = 7$$

b) Define all elements of the solution set in which he values of two variables equal 0.

Solution:

If two variables are equal to zero, then

When $x_1 = 0$, $x_2 = 0$

$$4x_1 - 2x_2 + 6x_3 = 0$$
$$4(0) - 2(0) + 6x_3 = 0$$
$$0 - 0 + 6x_3 = 0$$
$$6x_3 = 0$$
$$x_3 = 0$$

Hence the all element of solution set will be zero to the given linear equation.

23. **same as above**

Note:

24 to 30 are not important from paper point of view.

Solved Section 2.2

In Exercise 1-20, identify the x and y intercepts for the given linear equations.

1. 3x - 4y = 24

Solution:

3x - 4y = 24 --------- (1)

For x-intercept:

Put y = 0 in eq.(1), we get

$$3x - 4(0) = 24$$

$$3x = 24$$
$$x = \frac{24}{3}$$
$$x = 8 \quad (8,0)$$

For y-intercept:

Put x = 0 in eq.(1), we get

$$3(0) - 4y = 24$$
$$-4y = 24$$
$$y = \frac{24}{-4}$$
$$y = -6 \quad (0,-6)$$

2. -2x + 5y = -20

Solution:

$-2x + 5y = -20$ ---------- (1)

For x-intercept:

Put y = 0 in eq.(1), we get

$$-2x + 5(0) = -20$$
$$-2x = -20$$
$$x = \frac{-20}{-2}$$
$$x = 10 \quad (10,0)$$

For y-intercept:

Put x = 0 in eq.(1), we get

$$2(0) + 5y = -20$$
$$5y = -20$$
$$y = \frac{-20}{5}$$
$$y = -4 \quad (0,-4)$$

3. -x + 3y = 9

Solution:

$-x + 3y = 9$ ---------- (1)

For x-intercept:

Put y = 0 in eq.(1), we get

$$-x + 3(0) = 9$$
$$-x = 9$$
$$x = -9 \quad (-9,0)$$

For y-intercept:

Put x = 0 in eq.(1), we get

$$-(0) + 3y = 9$$
$$3y = 9$$
$$y = \frac{9}{3}$$
$$y = 3 \quad (0,3)$$

4. 4x + 2y = 36

Solution:

$4x + 2y = 36$ ---------- (1)

For x-intercept:

Put y = 0 in eq.(1), we get

$$4x + 2(0) = 36$$
$$4x = 36$$
$$x = \frac{36}{4}$$
$$x = 9 \quad (9,0)$$

For y-intercept:

Put x = 0 in eq.(1), we get

$$4(0) + 2y = 36$$
$$2y = 36$$
$$y = \frac{36}{2}$$
$$y = 18 \quad (0,18)$$

5. -4x = 12

Solution:

$-4x = 12$ ---------- (1)

For x-intercept:

Put y = 0 in eq.(1), we get

$$-4x = 12$$
$$x = \frac{12}{-4}$$
$$x = -3 \quad (-3,0)$$

For y-intercept:

As there is no 'y' variable in eq(1), so there is no y - intercept.

6. -10x + 300 = 0

Solution:

$-10x + 300 = 0$ ---------- (1)

For x-intercept:

Put y = 0 in eq.(1), we get

$$-10x + 300 = 0$$
$$-10x = -300$$
$$x = \frac{-300}{-10}$$
$$x = 30 \quad (30,0)$$

For y-intercept:

As there is no 'y' variable in eq.(1), so there is no y - intercept.

7. x - 2y = 0

Solution:

x - 2y = 0 ---------- (1)

For x-intercept:

Put y = 0 in eq.(1), we get

$$x - 2(0) = 0$$
$$x = 0 \quad (0,0)$$

For y-intercept:

Put x = 0 in eq.(1), we get

$$0 - 2y = 0$$
$$-2y = 0$$
$$y = 0 \quad (0,0)$$

8. 5x - 3y = 0

Solution:

5x - 3y = 0 ---------- (1)

For x-intercept:

Put y = 0 in eq.(1), we get

$$5x - 3(0) = 0$$
$$5x = 0$$
$$x = 0 \quad (0,0)$$

For y-intercept:

Put x = 0 in eq.(1), we get

$$5(0) - 3y = 0$$
$$-3y = 0$$
$$y = 0 \quad (0,0)$$

9. -8x + 5y = -20

Solution:

-8x + 5y = -20 ---------- (1)

For x-intercept:

Put y = 0 in eq.(1), we get

$$-8x + 5(0) = -20$$
$$-8x = -20$$
$$x = \frac{-20}{-8}$$
$$x = \frac{5}{2} \quad \left(\frac{5}{2},0\right)$$

For y-intercept:

Put x = 0 in eq.(1), we get

$$-8(0) + 5y = -20$$
$$5y = -20$$
$$y = -\frac{20}{5}$$
$$y = -4 \quad (0,-4)$$

10. (x + y)/2 = 3r - 2y + 16

Solution:

$$(x+y)/2 = 3x - 2y + 16$$
$$x + y = 2(3x-2y+16)$$
$$x + y = 6x - 4y + 32$$
$$x+y-6x+4y = 32$$
$$-6x+5y = 32 \text{ ---------- } (1)$$

For x-intercept:

Put y = 0 in eq.(1), we get

$$-6x + 5(0) = 32$$
$$-6x = 32$$
$$x = \frac{32}{-6}$$
$$x = -\frac{16}{3} \quad \left(-\frac{16}{3},0\right)$$

For y-intercept:

Put x = 0 in eq.(1), we get

$$-6(0) + 5y = 32$$
$$5y = 32$$
$$y = \frac{32}{5} \quad \left(0,\frac{32}{5}\right)$$

11. 2x - 3y = -18 + x

Solution:

$$2x - 3y = -18 + x$$

$$2x - x - 3y = -18$$
$$x - 3y = -18 \quad \text{----------} (1)$$

For x - intercept:

Put y = 0 in eq.(1), we get

$$x - 3(0) = -18$$
$$x = -18 \quad (-18,0)$$

For y-intercept:

Put x = 0 in eq.(1), we get

$$0 - 3y = -18$$
$$-3y = -18$$
$$y = \frac{-18}{-3}$$
$$y = 6 \quad (0,6)$$

12. -3x + 4y - 10 = 7x - 2y + 50

Solution:

$$-3x + 4y - 10 = 7x - 2y + 50$$
$$-3x-7x+4y-2y = 50 + 10$$
$$-10x + 2y = 60 \quad \text{----------} (1)$$

For x - intercept:

Put y = 0 in eq.(1), we get

$$-10x + 2(0) = 60$$
$$-10x = 60$$
$$x = \frac{60}{-10}$$
$$x = -6 \quad (-6,0)$$

For y-intercept:

Put x = 0 in eq.(1), we get

$$-10(0) + 2y = 60$$
$$2y = 60$$
$$y = \frac{60}{2}$$
$$y = 30 \quad (0,30)$$

13. 15y - 90 = 0

Solution:

$$15y - 90 = 0 \quad \text{------------} (1)$$

For x - intercept:

Put y = 0 in eq.(1), we get

$$15(0) - 90 = 0$$
$$-90 = 0$$

No solution.

For y-intercept:

Put x = 0 in eq.(1), we get

$$15y - 90 = 0$$
$$15y = 90$$
$$y = \frac{90}{15}$$
$$y = 6 \quad (0,6)$$

14. (x-2y)/$_3$ - 12 = (2x +4y)/$_3$

Solution:

$$(x - 2y/3 - 12 = (2x + 4y)/3$$
$$\Rightarrow \quad x - 2y - 36 = 2x + 4y$$
$$x - 2x - 2y - 4y = 36$$
$$-x - 6y = 36 \quad \text{---------} (1)$$

For x - intercept:

Put y = 0 in eq.(1), we get

$$-x - 6(0) = 36$$
$$-x = 36$$
$$x = -36 \quad (-36,0)$$

For y-intercept:

Put x = 0 in eq.(1), we get

$$-(0) - 6y = 36$$
$$-6y = 36$$
$$y = \frac{36}{-6}$$
$$y = -6 \quad (0,-6)$$

15. ax + by = t

Solution:

$$ax + by = t \quad \text{------------} (1)$$

For x - intercept:

Put y = 0 in eq.(1), we get

$$ax + b(0) = t$$
$$ax = t$$
$$x = \frac{t}{a} \quad \left(\frac{t}{a},0\right)$$

For y-intercept:

Put x = 0 in eq.(1), we get

$$a(0) + by = t$$
$$by = t$$
$$y = \frac{t}{b} \qquad \left(0, \frac{t}{b}\right)$$

16. Cx - dy = e

Solution:

Cx - dy = e -------------- (1)

For x - intercept:

Put y = 0 in eq.(1), we get

$$Cx - d(0) = e$$
$$Cx = e$$
$$x = \frac{e}{c} \qquad \left(\frac{e}{c}, 0\right)$$

For y-intercept:

Put x = 0 in eq.(1), we get

$$C(0) - dy = e$$
$$dy = e$$
$$y = -\frac{e}{d} \qquad \left(0, -\frac{e}{d}\right)$$

17. px = q

Solution:

px = q --------- (1)

For x - intercept:

Put y = 0 in eq.(1), we get

$$px = q$$
$$x = \frac{q}{p} \qquad \left(\frac{q}{p}, 0\right)$$

For y-intercept:

Put x = 0 in eq.(1), we get

$$p(0) = q$$
$$0 = q$$

No solution.

18. dx - ey + f = gx - hy

Solution:

dx - ey + f = gx - hy --------- (1)

For x - intercept:

Put y = 0 in eq.(1), we get

$$dx - e(0) + f = gx - h(0)$$
$$dx + f - gx$$
$$dx - gx = -f$$
$$(d-g)x = -f$$
$$-(g-d)x = -f$$
$$x = \frac{f}{(g-d)} \qquad \left(\frac{f}{(g-d)}, 0\right)$$

For y-intercept:

Put x = 0 in eq.(1), we get

$$d(0) - ey + f = g(0) - hy$$
$$-ey + f = -hy$$
$$-ey + hy = -f$$
$$-(ey-hy) = -f$$
$$-(e-h)y = -f$$
$$y = \frac{f}{e-h} \qquad \left(0, \frac{f}{e-h}\right)$$

19. -ry = s

Solution:

-ry = s --------- (1)

For x - intercept:

Put y = 0 in eq.(1), we get

$$-r(0) = s$$
$$0 = s$$

No solution:

For y-intercept:

Put x = 0 in eq.(1), we get

$$-ry = s$$
$$y = -\frac{s}{r} \qquad \left(0, -\frac{s}{r}\right)$$

20. -e + fx - gy = h

Solution:

- e + fx - gy = h

For x - intercept:

Put y = 0 in eq.(1), we get

$$-e + fx - g(0) = h$$

$$-e + fx = h$$
$$fx = e + h$$
$$x = \frac{e+h}{f} \qquad \left(\frac{e+h}{f}, 0\right)$$

For y-intercept:

Put x = 0 in eq.(1), we get

$$-e + f(0) - gy = h$$
$$-e - gy = h$$
$$-gy = e + h$$
$$y = -\frac{e+h}{g} \qquad \left(0, -\frac{e+h}{g}\right)$$

For Exercise 21 - 36, graph the given linear equation.

21. 2x - 3y = -12

Solution:

$2x - 3y = -12$ -------- (1)

For x-intercept:

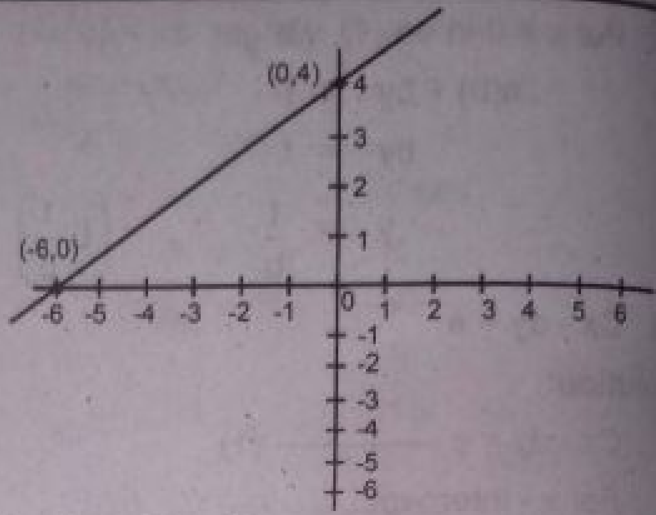
Put y = 0 in eq.(1), we get

$$2x - 3(0) = -12$$
$$2x = -12$$
$$x = \frac{-12}{2}$$
$$x = -6 \qquad (-6,0)$$

For y-intercept:

Put x = 0 in eq.(1), we get

$$2(0) - 3y = -12$$
$$-3y = -12$$
$$y = \frac{-12}{-3}$$
$$y = 4 \qquad (0,4)$$



22. -3x + 6y = -30

Solution:

$-3x + 6y = -30$ -------- (1)

For x-intercept:

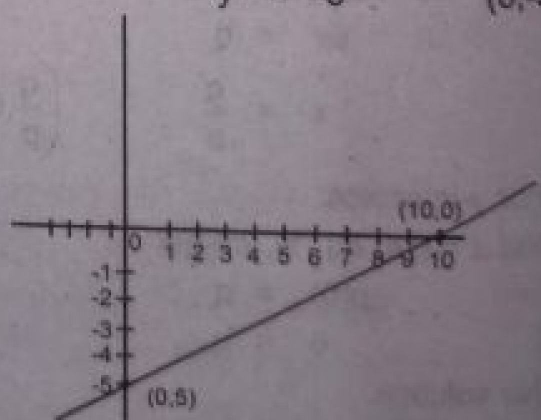
Put y = 0 in eq.(1), we get

$$-3x + 6(0) = -30$$
$$-3x = -30$$
$$x = \frac{-30}{-3}$$
$$x = -0 \qquad (10,0)$$

For y-intercept:

Put x = 0 in eq.(1), we get

$$-3(0) + 6y = -30$$
$$6y = -30$$
$$y = -\frac{30}{6}$$
$$y = -5 \qquad (0,-5)$$



23. x - 2y = -8

24. -8x + 3y = 24

25. -x - 4y = 10

26. 4x + 3y = -36

27. 3x + 8y = 0

28. 10x - 5y = 0

y=mx+k   y-intercept = K = Co.

29. $-5x + 2y = 0$ 30. $8x - 4y = 0$

31. $-4x = 24$ 32. $-2y = -9$

33. $-5y = -17.5$ 34. $8x = 20$

35. $-nx = t$ 36. $my = q$

Solution:

<u>Same as Q - 21 and 22.</u>

37. What is the eqaution of x-axis and the y-axis?

Solution:

The equation of x-axis is y = 0 and the equation of y-axis is x = 0.

In Exercise 38 - 59, compute the slope of the line segment connecting the two points. Interpret the meaning of the slope in each case.

38. (2,8) and (-2, -8)

Solution:

Here $(x_1, y_1) = (2, 8)$

$(x_2, y_2) = (-2, -8)$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-8-8}{-2-2} = \frac{-16}{-4} = 4$$

39. (-3, 10) and (2, -5)

Solution:

Here $(x_1, y_1) = (-3, 10)$

$(x_2, y_2) = (2, -5)$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-10}{2-(-3)} = \frac{-15}{5} = -3$$

40. (3, 5) and (-1, 15)

Solution:

Here $(x_1, y_1) = (3, 5)$

$(x_2, y_2) = (-1, 15)$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-15-5}{-1-3} = \frac{10}{-4} = -\frac{5}{2}$$

41. (10, -3) and (12, 4)

Solution:

Here $(x_1, y_1) = (10, -3)$

$(x_2, y_2) = (12, 4)$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-3)}{12-10} = \frac{7}{2} = 3.5$$

42. (-2, 3) and (1, -9)

Solution:

Here $(x_1, y_1) = (-2, 3)$

$(x_2, y_2) = (1, -9)$

$$m = \frac{\Delta y}{\Delta x}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-9-3}{1-(-2)}$$

$$= -\frac{12}{3}$$

$$= -4$$

43. (5, 8) and (-3, 28)

Solution:

Here $(x_1, y_1) = (5, 8)$

$(x_2, y_2) = (-3, 28)$

$$m = \frac{\Delta y}{\Delta x}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{28-8}{-3-5}$$

$$= \frac{10}{-8}$$

$$= -\frac{5}{4}$$

$$= -1.25$$

44. (4, -3) and (10, -12)

Solution:

Here $(x_1, y_1) = (4, -3)$

$(x_2, y_2) = (10, -12)$

$$m = \frac{\Delta y}{\Delta x}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-12-(-3)}{10-4}$$

$$= \frac{-12+3}{6}$$

$$= -\frac{9}{6}$$

$$= \frac{-3}{2} = -1.5$$

45. (8, -24) and (-5, - 15)

Solution:

Here $(x_1, y_1) = (8, -24)$

$(x_2, y_2) = (-5, -15)$

$$m = \frac{\Delta y}{\Delta x}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-15-(-24)}{-5-8}$$

$$= \frac{-15+24}{-13}$$

$$= \frac{9}{-13} = -0.692$$

46. (-2,8) and (3,8)

Here $(x_1, y_1) = (-2, 8)$

$(x_2, y_2) = (3, 8)$

$$m = \frac{\Delta y}{\Delta x}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{8-8}{-3-(-2)}$$

$$= \frac{0}{5} = 0$$

47. (-5, 4) and (-5, 6)

Here $(x_1, y_1) = (-5, 4)$

$(x_2, y_2) = (-5, 6)$

$$m = \frac{\Delta y}{\Delta x}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6-(-4)}{-5-(-5)}$$

$$= \frac{10}{0} = 0$$

48. (-4, 20) and (-4, 30)

Here $(x_1, y_1) = (-4, 20)$

$(x_2, y_2) = (-4, 30)$

$$m = \frac{\Delta y}{\Delta x}$$
$$= \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{30-20}{-4-(-4)}$$
$$= \frac{10}{-4+44}$$
$$= \frac{10}{0} = 0$$

49. (5, 0) and (-25, 0)
50. (0, 30) and (0, -15)
51. (5, 0) and (0, -10)
52. (a, b) and (-a, b)
53. (0, 0) and (a, b)
54. (d, -c) and (0, 0)
55. (-5, -5) and (5, 5)
56. (3, b) and (-10, b)
57. (-a, -b) and (a, -b)
58. (a + b, c) and (a, c)
59. (c + d, -c-d) and (a+b, -a-b)

Solution:

Same as Q (38 t0 48)

Solved Section 2.3

For Exercise 1 - 24, rewrite each equation in slope - intercept form and determine the slope and y - intercept.

1. **3x - 2y = 15**

Solution:

3x - 2y = 15

-2y = -3x + 15

-2y = -(3x - 15)

2y = 3x - 5

$$\frac{2}{2}y = \frac{3}{2}x - \frac{5}{2}$$
$$y = \frac{3}{2}x + \left(-\frac{5}{2}\right)$$

Compare it with

y = mx + k

$$\Rightarrow \quad m = \frac{3}{2} = 1.50 \text{ and}$$

y - intercept = K = (0,-2.5)

2. **-x + 5y = 27.5**

Solution:

-x + 5y = 27.5

5y = x + 27.5

$$y = \frac{1}{5}x + \frac{27.5}{5}$$
$$y = \frac{1}{5}x + 5.50$$

Compare it with

y = mx + k

$$\Rightarrow \quad m = \frac{1}{5} = 0.20 \text{ and}$$

y - intercept = K = (0,5.50)

3. **4x - 3y = 18**

Solution:

4x - 3y = 18

-3y =-4x + 18

-3y = -(4x - 18)

3y = 4x - 18

$$y = \frac{4}{3}x - \frac{18}{3}$$
$$y = \frac{4}{3}x + (-6)$$

Compare it with

y = mx + k

$$\Rightarrow \quad m = \frac{4}{3} = 1.33 \text{ and}$$

y - intercept = K = (0,-6)

4. **2x - 7y = -21**

Solution:

2x - 7y = -21

$2x - 7y = -21$

$-7y = -2x - 21$

$-7y = -(2x + 21)$

$7y = 2x + 21$

$y = \frac{2}{7}x + \frac{21}{7}$

$y = \frac{2}{7}x + 3$

Compare it with

$y = mx + k$

$\Rightarrow m = \frac{2}{7} = 0.29$ and

y - intercept = K = (0,3)

5. -x + y = 8

Solution:

$-x + y = 8$

$y = x + 8$

Compare it with

$y = mx + k$

$\Rightarrow m = 1$ and

y - intercept = K = (0,8)

6. 2x - y = - 5

Solution:

$2x - y = -5$

$-y = -2x - 5$

$-y = -(2x + 5)$

$y = 2x + 5$

Compare it with

$y = mx + k$

$\Rightarrow m = 2 =$ and

y - intercept = K = (0,5)

7. $(x+2y)/_2 = -6$

Solution:

$(x+2y)/_2 = -6$

$x + 2y = -12$

$2y = -x - 12$

$y = -\frac{1}{2}x - \frac{12}{2}$

$y = -\frac{1}{2}x + (-6)$

Compare it with

$y = mx + k$

$\Rightarrow m = -\frac{1}{2} = -0.5$ and

y - intercept = K = (0,-6)

8. $(-2x + y)/_3 = 2$

Solution:

$(-2x + y)/_3 = 2$

$-2x + y = 2$

$y = 2x + 6$

Compare it with

$y = mx + k$

$\Rightarrow m = 2$ and

y - intercept = K = (0,6)

9. (3x - 5y) = -5

Solution:

$(3x - 5y) = -5$

$-5y = -3x - 5$

$y = \frac{-3x - 5}{-5}$

Compare it with

$y = mx + k$

$\Rightarrow m = \frac{3}{5} = 0.6$ and

y - intercept = K = (0,1)

10. $(-x + 2y)/_4 = 3x - y$

Solution:

$(-x + 2y)/_4 = 3x - y$

$-x + 2y = 4(3x - y)$

$-x + 2y = 12x - 4y$

$4y + 2y = 12x + x$

$6y = 13x$

$y = \frac{13}{6}x$

Compare it with

$$y = mx + k$$

$$\Rightarrow \quad m = \frac{13}{6} = 2.6 \quad \text{and}$$

y - intercept = K = (0,0)

11. 2x = (5x - 2y)/4

Solution:

$$2x = (5x - 2y)/4$$
$$8x = 5x - 2y$$
$$2y = 5x - 8x$$
$$2y = -3x$$
$$y = -\frac{3}{2}x$$

Compare it with

$$y = mx + k$$

$$\Rightarrow \quad m = -\frac{3}{2} \quad \text{and}$$

$$m = -1.5$$ y-intercept=K = (0,0)

12. -(-x + 3y)/2 = 10 - 2x

Solution:

$$-(-x + 3y)/2 = 10 - 2x$$
$$+x - 3y = 2(10 - 2x)$$
$$x - 3y = 20 - 4x$$
$$-3y = 20 - 4x - x$$
$$-3y = 20 - 5x$$
$$y = \frac{20 - 5x}{-3}$$
$$y = -\frac{20}{3} + \frac{5}{3}x$$
$$y = \frac{5}{3}x - 6.67$$
$$y = 1.66x - 6.67$$

Compare it with

$$y = mx + k$$

$$\Rightarrow \quad m = 1.66 \quad \text{and}$$

y - intercept=K = (6,67)

13. 4x - 3y = 0

Solution:

$$4x - 3y = 0$$
$$-3y = -4x$$
$$(-)(-)3y = (-)(-)4x$$
$$3y = 4x$$
$$y = \frac{4}{3}x$$

Compare it with

$$y = mx + k$$

$$\Rightarrow \quad m = \frac{4}{3} = 1.33 \quad \text{and}$$

y - intercept=K = (0,0)

14. 8x + 3y = 24

Solution:

$$8x + 3y = 24$$
$$3y = 24 - 8x$$
$$y = \frac{24}{3} - \frac{8}{3}x$$
$$y = 8 - \frac{8}{3}x$$
$$y = -\frac{8}{3}x + 8$$

Compare with
y = mx + k
y-intercept = k(0,...

Compare it with

$$y = mx + k$$

$$\Rightarrow \quad m = -\frac{8}{3} = -2.67 \quad \text{and}$$

y - intercept=K = (0,8)

15. 3x - 6y + 10 = x

Solution:

$$3x - 6y + 10 = x$$
$$-6y = x - 10 - 3x$$
$$-6y = -2x - 10$$
$$y = \frac{-12x}{-6} - \frac{10}{-6}$$
$$y = \frac{1}{3}x + \frac{5}{3}$$

mx + ny = p
ny = p - mx
y = p/n - mx/n
compare it with
m = -m/n
and
y-intercept = k = 0, p/n

Compare it with

$$y = mx + k$$

$$\Rightarrow \quad m = \frac{1}{3} = 0.33 \quad \text{and}$$

y - intercept=K = (0,1.67)

16. $3y - 5x + 20 = 4x - 2y + 6$
17. $2x + 3y = 4x + 3y$
18. $-5x + y - 12 = 2y - 5x$
19. $8y - 24 = 0$
20. $3x + 6 = 0$
21. $mn + ny = p$
22. $mx - n - 0$
23. $c - dy = 0$
24. $dx = cy - f$

Solution:

Above questions are same as Q(1 to 15) Q(25 to 32) are not important for examination point of view.

Solved Section 2.4

In Exercise 1 - 36, determine the slope - intercept form of the linear equation, given the listed attributes.

1. slope = -2 , y - intercept = (0,10)

Solution:

$m = -2$, $k = 10$

The slope intercept form is

$y = mx + k$

$y = -2x + 10$

2. slope = 4 , y - intercept = (0,-5)

Solution:

$m = 4$, $k = -5$

The slope intercept form is

$y = mx + k$

$= 4x + (-5)$

$= 4x - 5$

3. slope = $\frac{1}{2}$, y - intercept = $(0, \frac{3}{4})$

Solution:

$m = \frac{1}{2}$, $k = \frac{3}{4}$

The slope intercept form is

$y = mx + k$

$= \frac{1}{2}x + \frac{3}{4}$

4. slope = $-\frac{5}{2}$, y - intercept = (0,-20)

Solution:

$m = -\frac{5}{2}$, $k = -20$

The slope intercept form is

$y = mx + k$

$= -\frac{5}{2}x + (-20)$

$= \frac{-5}{2}x - 20$

5. slope = -r , y - intercept = $(0, -\frac{t}{2})$

Solution:

$m = -r$, $k = -\frac{t}{2}$

The slope intercept form is

$y = mx + k$

$= -rx - \frac{t}{2}$

6. slope undefined, infinite number of y-intercepts.

Solution:

Slope $= m = \infty = \frac{1}{0}$ and $k = \infty = \frac{1}{0}$

The slope intercept form is

$y = mx + k$

$= \infty$

7. Slope = -3 , (4,-2) lies on the line.

Solution:

Here $m = -3$, $(x_1, y_1) = (4, -2)$

The point slope formula is

$y - y_1 = m(x - x_1)$

$y-(-2) = -3(x-4)$

$$y+2 = -3x + 12$$
$$y = -3x + 12 - 2$$
$$y = -3x + 10$$

8. Slope = 5 , (-3,12) lies on the line.

Solution:

Here $m = 5$, $(x_1, y_1) = (-3, 12)$

The point slope formula is

$$y - y_1 = m(x - x_1)$$
$$y - 12 = 5[x - (-3)]$$
$$y - 12 = 5(x + 3)$$
$$y - 12 = 5x + 15$$
$$y = 5x + 15 + 12$$
$$y = 5x + 27$$

9. Slope = $\frac{3}{2}$, (-5,-8) lies on the line.

Solution:

Here $m = \frac{3}{2}$, $(x_1, y_1) = (-5, -8)$

The point slope formula is

$$y - y_1 = m(x - x_1)$$
$$y - (-8) = \frac{3}{2}[x-(-5)]$$
$$y + 8 = \frac{3}{2}(x+ 5)$$
$$2(y+8) = 3(x+5)$$
$$2y + 16 = 3x + 15$$
$$2y = 3x + 15 - 16$$
$$2y = 3x - 1$$
$$y = \frac{3}{2}x - \frac{1}{2}$$

10. Slope = $-\frac{1}{2}$, (-4,0) lies on the line.

11. Slope = 2.5, (-2,5) lies on the line.

12. Slope = -3.25,(1.5,-7.5) lies on the line.

13. Slope = 5.6, (2.4,-4.8) lies on the line.

14. Slope = -8.2, (-0.75,16.3) lies on the line.

15. Slope = w, (p, q) lies on line.

16. Slope = -a, (4, -4) lies on line.

Solution:

Above questions are same as Q - 9.

17. Slope undefined, (-3,-5) lies on line.

Solution:

Here $m = \infty \frac{1}{0}$, $(x_1, y_1) = (-3, -5)$

The point slope formula is

$$y - y_1 = m(x - x_1)$$
$$y - (-5) = \frac{1}{0}[x-(3)]$$
$$0(y + 5) = 1 (x + 3)$$
$$0 = x + 3$$
$$\text{or} \quad x + 3 = 0$$
$$x = -3$$

No, point slope form possible.

18. Slope = 0, (20, -10) lies on line.

Solution:

Here $m = 0$, $(x_1, y_1) = (20, -10)$

The point slope formula is

$$y - y_1 = m(x - x_1)$$
$$y - (-10) = 0(x - 20)$$
$$y + 10 = 0$$
$$y = -10$$

19. Slope = 0, (u, v) lies on line.

Solution:

Here $m = 0$, $(x_1, y_1) = (u, v)$

The point slope formula is

$$y - y_1 = m(x - x_1)$$
$$y - v = 0(x - u)$$
$$y - v = 0$$
$$y = v$$

20. Slope undefined, (-t, v) lies on line.

Solution:

<u>Same as Q - 17</u>

21. (-4, 5) and (-2, -3) lies on line.

Solution:

Here $(x_1, y_1) = (-4, 5)$

$(x_2, y_2) = (-2, -3)$

$$m = \frac{\Delta y}{\Delta x}$$
$$= \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-3-5}{-2-(-4)}$$
$$= \frac{-3-5}{-2+4}$$
$$= \frac{-8}{2}$$
$$= -4$$

The slope - intercept formula is

$$y - y_1 = m(x - x_1)$$
$$y - 5 = -4[x - (-4)]$$
$$y - 5 = -4(x + 4)$$
$$y - 5 = -4x - 16$$
$$y = -4x - 16 + 5$$
$$y = -4x - 11$$

22. (3, 2) and (-12, 1) lise on line.

23. (20, 240) and (15, 450) lise on line.

24. (-12, 760) and (8, 1320) lise on line.

25. (0.234,20.75) and (2.642, 18.24) lise on line.

26. (5.76, -2.48) and (3.74, 8.76) lies on line.

27. (a, b) and (c, d) lies on line.

28. (a, -3) and (a, 15) lies on line.

29. (-d, b) and (e, b) lies on line.

30. (p, r) and (-p, r) lies on line.

Solution:

Above question are same as Q - 21.

31. Passes through (2, -4) and is parallel to the line 3x - 4y = 20

Solution:

Here $(x_1, y_1) = (2, -4)$ and $3x - 4y = 20$

$$-4y = -3x + 20$$
$$-4y = -(3x - 20)$$
$$4y = 3x - 20$$
$$y = \frac{3}{4}x - \frac{20}{4}$$
$$y = \frac{3}{4}x - 5$$
$$y = mx + k$$
$$\Rightarrow \quad m = \frac{3}{4}$$

As required line is parallel to the given line, so.

$$m = \frac{3}{4}$$

The slope - intercept formula is

$$y - y_1 = m(x - x_1)$$
$$y - (-4) = \frac{3}{4}(x - 2)$$
$$y + 4 = \frac{3}{4}(x - 2)$$
$$4(y + 4) = 3(x - 2)$$
$$4y + 16 = 3x - 6$$
$$4y = 3x - 6 - 16$$
$$4y = 3x - 22$$
$$y = \frac{3}{4}x - \frac{22}{4}$$
$$y = \frac{3}{4}x - \frac{11}{2}$$

32. Passes through (-2, 10) and is parallel to the line 5x - y = 0

Solution:

Here $(x_1, y_1) = (-2, 10)$

And $5x - y = 0$

$$-y = -5x$$
$$y = 5x + 0$$

Compare it with

$$\Rightarrow \quad y = mx + k$$
$$m = 5$$

As required line is parallel to the given line, so.

$$m = 5$$

The slope - intercept formula is

$$y - y_1 = m(x - x_1)$$
$$y - 10 = 5(x - (-2))$$
$$y - 10 = 5(x + 2)$$
$$y - 10 = 5x + 10$$
$$y = 5x + 10 - 10$$
$$y = 5x + 0$$

33. Passes through (7, 2) and is parallel to the line (a) x = 7 and (b) y = 6

Solution: (a)

Here $(x_1, y_1) = (7, 2)$

And $x = 7$

$$0 = -x + 7$$

It cannot be compared with

$$y = mx + k$$

$\Rightarrow$ $m = \infty$ (Undefined)

The slope - intercept formula is

$$y - y_1 = m(x - x_1)$$
$$y - 2 = \infty(x - 7)$$
$$y - 2 = \frac{1}{0}(x - 7)$$
$$0(y-2) = 1(x - 7)$$
$$0 = x - 7$$

or $x = 7$

y - y1 = m(x - x)

Solution: (b)

Here $(x_1, y_1) = (7, 2)$

And $y = 6$

$$y = 0x + 6$$

Compare it with

$$y = mx + k$$

$\Rightarrow$ $m = 0$

The slope - intercept formula is

$$y - y_1 = m(x - x_1)$$
$$y - 2 = 0(x - 7)$$
$$y - 2 = 0$$
$$y = 2$$

34. Passes through (20, -30) and is parallel to the line 4x + 2y = -18

Solution: (a)

Here $(x_1, y_1) = (20, -30)$

And $4x + 2y = -18$

$$2y = -4x - 18$$
$$y = -\frac{4}{2}x - \frac{18}{2}$$
$$y = -2x - 9$$

Compare it with

$$y = mx + k$$

$\Rightarrow$ $m = -2$

As the required line is perpendicular to the given line, so

$$m = -\frac{1}{m} = -\frac{1}{-2} = \frac{1}{2}$$

The slope - intercept formula is

$$y - y_1 = m(x - x_1)$$
$$y - (-30) = \frac{1}{2}(x - 20)$$
$$y + 30 = \frac{1}{2}(x - 20)$$
$$2(y + 30) = x - 20$$
$$2y + 60 = x - 20$$
$$2y = x - 20 - 60$$
$$2y = x - 80$$
$$y = \frac{1}{2}x - 40$$

35. Passes through (-8, -4) and is perpendicular to the line 8x - 2y = 0

36. Passes through (7, 2) and is perpendicular to the line (a) x = 7 and (b) y = 6

Solution:

Above questions are same as Q - 34.

Q-37 to 42 are not important for examination paper point of veiw.

Section 2.6

The question are not important for examination paper point of view.

Solved Additional Exercise

Verify that $m = \frac{y_2 - y_1}{x_2 - x_1}$ is unaffected by the choice of (x_1, y_1) and (x_2, y_2).

If $(x_1, y_1) = (2,4)$ and $(x_2, y_2) = (5,12)$, then compute slope. Again label (5,12) as (x_1, y_1) and (2,4) as (x_2, y_2) and recompute the slope.

Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-(y_1 - y_2)}{-(x_1 - x_2)}$$

$$= \frac{y_1 - y_2}{x_1 - x_2}$$

Here $(x_1, y_1) = (2, 4)$

$(x_2, y_2) = (5, 12)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{12-4}{5-2}$$

$$= \frac{8}{3}$$

Again, if $(x_1, y_1) = (5, 12)$

$(x_2, y_2) = (2, 4)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4-12}{2-5}$$

$$= \frac{-8}{-3}$$

$$= \frac{8}{3}$$

Choose two points which satisfy the equation $5x + y = 10$ and verify that the slope equals -5.

Solution:

$5x + y = 10$

Let the point (1, 5) and (2, 0) satisfy the above equation.

So $(x_1, y_1) = (1,5)$ and $(x_2, y_2) = (2,0)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0-5}{2-1}$$

$$= \frac{-5}{1}$$

$$= -5$$

Hence proved.

Solved Practice Exercise

Solve Section 2.1:

In Exercise 1 - 12, determine whether the equation is linear

1. $x/3 - y/4 = 2x - y + 12$

Solution:

$$x/3 - y/4 = 2x - y + 12$$

$\Rightarrow$ $4x - 3y = 12(2x - y + 12)$

$4x - 3y = 24x - 12y + 144$

$4x - 3y - 24x + 12y - 144 = 0$

$-20x + 9y - 144 = 0$

$\Rightarrow$ $20x - 9y + 144 = 0$

Hence, the given equation is linear.

2. $(x + 4y)/8 = y$

Solution:

$$(x+4y)/8 = y$$
$$\Rightarrow \quad x + 4y = 8y$$
$$x + 4y - 8y = 0$$
$$x - 4y = 0$$

Hence, the given equation is linear.

3. $2/x - 3/y = 24$

Solution:

$$2/x - 3/y = 24$$
$$\Rightarrow \quad 2y - 3x = 24xy$$

Hence, the given equation is linear.

4. $0.2x - 0.5y = 10 - 4/x$

Solution:

$$0.2x - 0.5y = 10 - 4/x$$
$$\Rightarrow \quad x(0.2x - 0.5y) = 10x - 4$$
$$0.2x^2 - 0.5xy = 10x - 4$$

Hence, the given equation is not linear.

5. $x_1 - x_2/3 + 5x_3 = x_4 - 2x_3$

Solution:

$$x_1 - x_2/3 + 5x_3 = x_4 - 2x_3$$
$$\Rightarrow \quad 3x_1 - x_2 + 15x_3 = 3x_4 - 6x_3$$

Hence, the given equation is not linear.

6. $\frac{2}{x - 3y} = 10 + \frac{x}{3}$

Solution:

$$\frac{2}{x-3y} = 10 + \frac{x}{3}$$
$$\frac{2}{x-3y} = \frac{30+x}{3}$$
$$(30 + x)(x - 3y) = 6$$
$$30x - 90y + x^2 - 3xy = 6$$

Hence, the given equation is not linear.

7. $(x - y + 13)/3 + 5y = -3(x + 12)$

Solution:

$$(x-y+13)/3 + 5y = -3(x + 12)$$
$$\Rightarrow \quad x - y + 13 + 15y = -9(x + 12)$$
$$x - y + 13 + 15y = -9x - 108$$
$$9x + x - y + 15y = -108 - 13$$
$$10x + 14y = -121$$

Hence, the given equation is linear.

8. $x_1 - 4x_2 + 3x_1 x_3 = 5x_3 - 100$

Solution:

$$x_1 - 4x_2 + 3x_1 x_3 = 5x_3 - 100$$
$$x_1 - 4x_2 - 5x_3 + 3x_3 + 3x_1 x_2 + 100 = 0$$

Hence, the given equation is not linear.

9. $\sqrt{10} + 10x - 4y = -4$

Solution:

$$\sqrt{10} + 10x - 4y = -4$$
$$10x - 4y = -4 - \sqrt{10}$$

Hence, the given equation is linear.

10. $\frac{(x_1 - 7x_2 + 5x_3)}{20} = \frac{2}{(x_1 - 3x_2)}$

Solution:

$$\frac{(x_1 - 7x_2 + 5x_3)}{20} = \frac{2}{(x_1 - 3x_2)}$$
$$(x_1 - 3x_2) + (x_1 - 6x_3 + 5x_3) = 40$$
$$x_1^2 - 9x_1x_2 + 5x_1x_3 - 15x_2x_2 + 18x_2^2 = 40$$

Hence, the given equation is not linear.

11. $\sqrt{x^2 - 2x + 1} = \frac{y}{2} = 20 - x + 8y$

Solution:

$$\sqrt{x^2 - 2x + 1} + \frac{y}{2} = 20 - x + 8y$$
$$\Rightarrow \quad \sqrt{x^2 - 2x + 1} + y = 2(20 - x + 8y)$$
$$\sqrt{x^2 - 2x + 1} + y = 40 - 2x + 16y$$
$$2\sqrt{x^2 - 2x + 1} = 40 - 2x |+ 16y - y$$

$$2\sqrt{x^2-2x+1} = 40-2x+15y$$

Hence, the given equation is not linear.

12. $\sqrt{x^2-4x+4} = \sqrt{y^2+6y+9}$

Solution:

$$\sqrt{x^2-4x+4} = \sqrt{y^2+6y+9}$$
$$\sqrt{(x-2)^2} = \sqrt{(y+3)^2}$$
$$x-2 = y+3$$
$$x-y = 2+3$$
$$x-y = 5$$

Hence, the given equation is not linear.

Q-13, 14, 15 are not important for examination paper point of view.

Solved Section 2.2

In Exercise 16 - 28, identify the x and y intercepts if they exists and graph the equation.

16. $-3x = y/2$

Solution:

$$-3x = \frac{y}{2} \quad \text{------------ (1)}$$

For x - intercept:

Put y = 0 in eq.(1), we get

$$-3x = \frac{0}{2}$$
$$-3x = 0$$
$$x = 0 \qquad (0, 0)$$

For y - intercept:

Put x = 0 in eq.(1), we get

$$-3(0) = \frac{y}{2}$$
$$0 = \frac{y}{2}$$
$$\Rightarrow \quad y = 0 \qquad (0, 0)$$

17. $\frac{x}{3} = -4$

Solution:

$$\frac{x}{3} = -4 \quad \text{------------ (1)}$$

For x - intercept:

Put y = 0 in eq.(1), we get

$$\frac{x}{3} = -4$$
$$x = -12 \qquad (-12, 0)$$

For y - intercept:

Put x = 0 in eq.(1), we get

$$\frac{0}{3} = -4$$
$$0 = -12$$

No solution.

18. $(y-4)/_2 = 4x+3$

Solution:

$$(y-4)/_2 = 4x+3$$
$$y-4 = 2(4x+3)$$
$$y-4 = 8x+6$$
$$y = 8x+6+4$$
$$y = 8x+10 \quad \text{-------- (1)}$$

For x - intercept:

Put y = 0 in eq.(1), we get

$$0 = 8x+10$$
$$8x = -10$$
$$x = -\frac{10}{8} = -\frac{5}{4} \qquad \left(-\frac{5}{4}, 0\right)$$

For y - intercept:

Put x = 0 in eq.(1), we get

$$y = 8(0)+10$$
$$y = 10 \qquad (0, 10)$$

19. $3x - 6y = 0$

20. $4x - 2y = -10$

21. $2x - 3y + 20 = -5x + 2y - 8$

22. $5 - 3x + 6y = -x + 5 - 2y$

23. $5y = 2y + 24$

24. $-6x + 24 = -12 + 3x$

25. -2x + 3y = -36
26. (x - 6y)/2 = -3y + 10
27. x + y -20 = 0
28. (2x - 4y)/2 = 10 + (-x + 3y)/3

Solution:

Above questions are same as Q-16,17,18.

In Exercise 29 - 40, compute the slope of the line segment connecting the two points. Interpret the meaning of the slope.

29. (5, 2) and (-10, 5)

Solution:

Here $(x_1, y_1) = (5, 2)$

$(x_2, y_2) = (-10, 5)$

$$m = \frac{\Delta y}{\Delta x}$$
$$= \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{5-2}{-10-5}$$
$$= \frac{3}{-15}$$
$$= -0.2$$

30. (-3, 8) and (1, -14)

Solution:

Here $(x_1, y_1) = (-3, 8)$

$(x_2, y_2) = (1, -14)$

$$m = \frac{\Delta y}{\Delta x}$$
$$= \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-14-8}{1-(-3)}$$
$$= \frac{-22}{4}$$
$$= -\frac{11}{2} = -5.50$$

31. (-b, a) and (-b, 3a)
32. (2a, 3b) and (-3a, 3b)
33. (4, -5) and (-2, 25)
34. (-2, 40) and (3, 75)
35. (4. 38, 2.54) and (-1.24, 6.32)
36. (-15.2, 4.5) and (8.62, -1.6)
37. (m, n) and (-m, -n)
38. (-2a, 4b) and (4b, -2a)
39. (o, t) and (-t, 0)
40. (-4, c) and (-4, b)

Solution:

Above questions are same as Q-29,30.

Solved Section 2.3:

In Exercise 41 - 52, rewrite each question in slope - intercept form and determine the slope and y - intercept.

41. 2x - 5y + 10 = -4y + 2x - 5

Solution:

$$2x - 5y + 10 = -4y + 2x - 5$$
$$-5y + 4y = -2x + 2x - 10 - 5$$
$$-y = 0 - 15$$
$$y = 0 + 15$$

Compare it with

$$y = mx + k$$
$$\Rightarrow m = 0$$

y - intercept = (0, 15)

42. 3x - 8y = 24 + x - 3y

Solution:

$$3x - 8y = 24 + x - 3y$$
$$-8y + 3y = -3x + x + 24$$
$$-5y = -2x + 24$$
$$y = \frac{-2}{-5}x + \frac{24}{-5}$$
$$y = \frac{2}{5}x - \frac{24}{5}$$

Compare it with

$$y = mx + k$$

$\Rightarrow \quad m = \frac{2}{5}$

$$\text{y - intercept} = \left(0, -\frac{24}{5}\right)$$

43. (x - 4y)/3 = (5x - 2y)/2
44. 3x - 6y = 36 + x
45. 8x - 4y = 60 - 3x + y
46. x/2 = 20 - y /3
47. mx - ny = p
48. ax + by = c + dx + ey
49. 30x - 4y + 24 = 8y + 30x - 12
50. -cx + cy = c
51. y/2 + 3x - 10 = (x + y)/2
52. x - 3y = 3y + 5x - 40

Solution:

Above questions are same as Q.41,42.

Q-53, 54, 55, 56, 57 are not important for examination paper point of view.

Solved Exercise 2.4

In Exercise 58 - 73, use the given information to determine the slope - intercept form of the linear equation.

58. Slope undefined and line passes through (-3, 5)

Solution:

$$m = \infty = \frac{1}{0}$$

$$(x_1, y_1) = (-3, 5)$$

The slope - intercept formula is

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{1}{0}(x-(-3)$$

$$0(y - 5) = 1(x + 3)$$

$$0 = x + 3$$

$$x = -3$$

59. Slope undefined and line passes through the origin.

Solution:

$$m = \infty = \frac{1}{0}$$

$$(x_1, y_1) = (0, 0)$$

The slope - intercept formula is

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{1}{0}(x - 0)$$

$$0(y - 0) = 1(x - 0)$$

$$0 = x$$

or $\quad x = 0$

60. Slope equals $\frac{1}{2}$, y - intercept at (0, -20).

Solution:

$$m = \frac{1}{2}, K = -20$$

The slope - intercept formula is

$$y = mx + k$$

$$y = \frac{1}{2}x - 20$$

61. Slope equals zero, y - intercept at (0.5)

62. x intercept at (4, 0) and (-2, 8) lies on line.

63. x intercept at (-3, 0) and (8, -4) lies on line.

64. (-3, 6) and (-1, 2) lie on line.

65. (-2, -18) and (5, 24) lie on line.

66. (-4, 2c) and (10, 2c) lie on line.

67. (3a, -5) and (3a, 10) lie on line.

68. (-2.38, 10.52) and (1.52, 6.54) lie on line.

69. (24.5, -100.6) and (16.2, 36.5) lie on line.

Passes through (-6, 4) and is perpendicular to $3x - 2y = 0$

Passes through (3, 10) and is perpendicular to $4x - 2y = -12$

72. Passes through (-2, 8) and is parallel to $-4x + 8y = 20$

73. Passes through (-4, -1) and is parallel to $8x - 2y = 0$

Solution:

Above questions are same as 58, 59, 60.

Q. 74, 75, 76, 77, 78, 79 are not important for examination paper point of view.

Solved Chapter Test

1. **Given the equation $8x - 2y = -48$**
   (a) Determine the x and y intercept
   (b) Graph the equation

Solution:

$$8x - 2y = -48 \text{ ---------- (1)}$$

For x - intercept

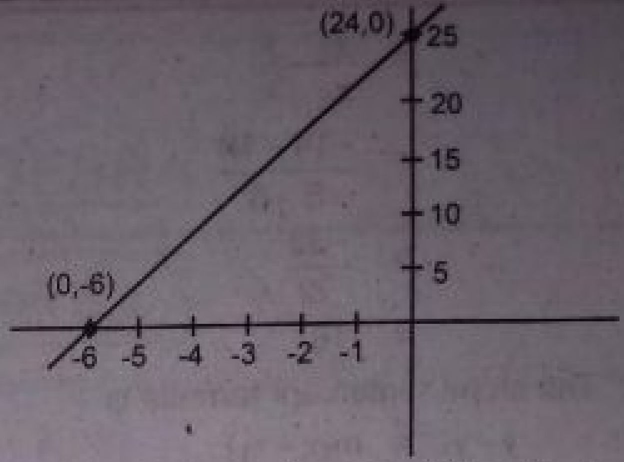
Put $y = 0$ in eq.(1), we get

$$8x - 2(0) = -48$$
$$8x = -48$$
$$x = \frac{-48}{8}$$
$$x = -6 \qquad (-6, 0)$$

For y - intercept

Put $x = 0$ in eq.(1), we get

$$8(0) - 2y = -48$$
$$-2y = -48$$
$$y = 24 \qquad (0,24)$$



2. **Given the equation $(x+y)/3 = 24 - x$.**

(a) Rewrite the equation in slope - intercept form.

(b) Identify the slope and y - intercept.

(c) Interpret the meaning of the slope.

Solution: (a)

$$(x + y)/3 = 24 - x$$
$$x + y = 3(24 - x)$$
$$x + y = 72 - 3x$$
$$y = -3x - x + 72$$
$$y = -4x + 72$$

(b) Compare it with

$$y = mx + k$$
$$\Rightarrow \quad m = -4,$$
$$\text{y-intercept} = k = (0, 72)$$

(c) The slope is negative, indicating that the line segment falls.

3. **Given two points (3, 18) and (5, -14).**

(a) Determine the equation of the straight line which passes through the two points.

(b) Identify the slope, y-intercept and x - intercept.

Solution: (a)

Here $(x_1, y_1) = (3, 18)$

$(x_2, y_2) = (5, -14)$

$$m = \frac{\Delta y}{\Delta x}$$

$= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-14-18}{5-3}$

$= -\frac{32}{2}$

$= -16$

The slope - intercept formula is

$y - y_1 = m(x - x_1)$

$y - 18 = -16(x - 3)$

$y - 18 = -16x + 48$

$y = -16x + 48 + 18$

$y = -16x + 66$ ------------ (1)

(b) Compare above equation with

$y = mx + k$

$\Rightarrow \quad m = -16$

For x - intercept:

Put y = 0 in eq.(1), we get

$0 = -16x + 36$

$16x = 36$

$x = -\frac{36}{16}$

$x = \frac{9}{4} \qquad \left(\frac{9}{4}, 0\right)$

For y - intercept:

Put x = 0 in eq.(1), we get

$y = -16(0) + 36$

$y = 36 \qquad (0, 36)$

Q.4 and Q. 6 are not important for examination paper point of view.

5. Determine the equation of the straight line which is perpendicular to the line 3x - 2y = -28 and which passes through the point (-5, 20).

Solution:

$3x - 2y = -28$

$-2y = -3x - 28$

$-2y = -(3x + 28)$

$\Rightarrow \quad 2y = 3x + 28$

$y = \frac{3}{2}x + \frac{28}{2}$

$y = \frac{3}{2}x + 14$

Compare it with

$y = mx + k$

$\Rightarrow \quad m = \frac{3}{2}$

As the required line is perpendicular to the given line, then

$m = -\frac{1}{m}$

$= -\frac{1}{3/2}$

$= -\frac{2}{3}$

Here $(x_1, y_1) = (-5, 20)$

The slope - intercept formula is

$y - y_1 = m(x - x_1)$

$y - 20 = -\frac{2}{3}(x - (-5))$

$3(y - 20) = -2(x + 5)$

$3y - 60 = -2x - 10$

$2x + 3y = 60 - 10$

$2x + 3y = 50$

☆☆☆☆☆

Chapter - 3

MATRIX ALGEBRA

Solved Section 3.1

Determine the dimension of each of the following matrices and find the transpose.

1. $[8 \quad -8 \quad 5 \quad 3]$

Solutin: $[8 \quad -8 \quad 5 \quad 3]$

Dimension = (1×4)

Transpose = $\begin{bmatrix} 8 \\ -8 \\ 5 \\ 3 \end{bmatrix}$

2. $\begin{bmatrix} 2 & 6 \\ -3 & 8 \end{bmatrix}$

Solution: $\begin{bmatrix} 2 & 6 \\ -3 & 8 \end{bmatrix}$

Dimension = (2×2)

Transpose = $\begin{bmatrix} 2 & -3 \\ 6 & 8 \end{bmatrix}$

3. $\begin{bmatrix} 0 & 1 \\ 5 & 2 \\ -6 & 8 \\ -2 & 4 \end{bmatrix}$

Solution: $\begin{bmatrix} 0 & 1 \\ 5 & 2 \\ -6 & 8 \\ -2 & 4 \end{bmatrix}$

Dimension = (4×2)

Tranpose = $\begin{bmatrix} 0 & 5 & -6 & -2 \\ 1 & 2 & 8 & 4 \end{bmatrix}$

4. $\begin{bmatrix} 2 & 10 & -1 \\ -3 & -5 & 0 \\ 4 & -8 & 2 \end{bmatrix}$

Solution: $\begin{bmatrix} 2 & 10 & -1 \\ -3 & -5 & 0 \\ 4 & -8 & 2 \end{bmatrix}$

Dimension = (3×3)

Transpose = $\begin{bmatrix} 2 & -3 & 4 \\ 10 & -5 & -8 \\ -1 & 0 & 2 \end{bmatrix}$

5. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Dimension = (3×3)

Transpose = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

6. $\begin{bmatrix} -6 & 3 & 2 & 4 \\ 2 & 3 & 3 & 4 \\ 2 & -1 & 5 & 8 \end{bmatrix}$

Solution: $\begin{bmatrix} -6 & 3 & 2 & 4 \\ 2 & 3 & 3 & 4 \\ 2 & -1 & 5 & 8 \end{bmatrix}$

Dimension = (3×4)

Transpose = $\begin{bmatrix} -6 & -2 & 2 \\ 3 & 3 & -1 \\ 2 & 3 & 5 \\ 4 & 4 & 8 \end{bmatrix}$

7. $\begin{bmatrix}1\\2\\3\\4\end{bmatrix}$

Solution:

$$\begin{bmatrix}1\\2\\3\\4\end{bmatrix}$$

Dimension = (4 × 1)

Transpose = $\begin{bmatrix}1 & 2 & 3 & 4\end{bmatrix}$

8. $\begin{bmatrix}1 & 3 & 5 & 7 & 9\\2 & 4 & 6 & 8 & 10\end{bmatrix}$

Solution:

$$\begin{bmatrix}1 & 3 & 5 & 7 & 9\\2 & 4 & 6 & 8 & 10\end{bmatrix}$$

Dimension = (2 × 5)

Tranpose = $\begin{bmatrix}1 & 2\\3 & 4\\5 & 6\\7 & 8\\9 & 10\end{bmatrix}$

9. $\begin{bmatrix}1 & 3 & 5\\6 & 4 & 2\\0 & 1 & 2\\4 & 6 & 3\\5 & 1 & 2\end{bmatrix}$

Solution:

$$\begin{bmatrix}1 & 3 & 5\\6 & 4 & 2\\0 & 1 & 2\\4 & 6 & 3\\5 & 1 & 2\end{bmatrix}$$

Dimension = (5 × 3)

Transpose = $\begin{bmatrix}1 & 6 & 0 & 4 & 5\\3 & 4 & 1 & 6 & 1\\5 & 2 & 2 & 3 & 2\end{bmatrix}$

10. $\begin{bmatrix}6 & 1 & 2 & 3 & 5\\2 & 0 & 4 & 6 & 1\\3 & 1 & -2 & 3 & 5\\4 & 3 & 2 & 1 & 0\end{bmatrix}$

Solution:

$$\begin{bmatrix}6 & 1 & 2 & 3 & 5\\2 & 0 & 4 & 6 & 1\\3 & 1 & -2 & 3 & 5\\4 & 3 & 2 & 1 & 0\end{bmatrix}$$

Dimension = (4 × 5)

Transpose = $\begin{bmatrix}6 & 2 & 3 & 4\\1 & 0 & 1 & 3\\2 & 4 & -2 & 2\\3 & 6 & 3 & 1\\5 & 1 & 5 & 0\end{bmatrix}$

11. Find a (2×4) matrix A for which

aij = $\begin{bmatrix}i+j & \text{if } i=j\\0 & \text{if } i\neq j\end{bmatrix}$

Solution

aij = $\begin{bmatrix}i+j & \text{if } i=j\\0 & \text{if } i\neq j\end{bmatrix}$

Dimension = (2 × 4)

$$\begin{bmatrix}2 & 0 & 0 & 0\\0 & 4 & 0 & 0\end{bmatrix}$$

12. Find a (5×3) matrix B for which

bij = $\begin{bmatrix}i-j & \text{if } i=j\\2i+j & \text{if } i\neq j\end{bmatrix}$

Solution

bij = $\begin{bmatrix}i-j & \text{if } i=j\\2i+j & \text{if } i\neq j\end{bmatrix}$

Dimension = (5 × 3)

$$\begin{bmatrix}0 & 4 & 5\\5 & 0 & 7\\7 & 8 & 0\\9 & 10 & 11\\11 & 12 & 13\end{bmatrix}$$

Solved Section 3.2

Perform the following matrix operations wherever possible.

1. $-\begin{bmatrix}4 & -2\\ 5 & 8\end{bmatrix}-\begin{bmatrix}3 & 12\\ 8 & 4\end{bmatrix}$

Solution:

$$= -\begin{bmatrix}4 & -2\\ 5 & 8\end{bmatrix}-\begin{bmatrix}3 & 12\\ 8 & 4\end{bmatrix}$$

$$= \begin{bmatrix}-4-3 & -(-2)-12\\ -5-8 & -8-4\end{bmatrix}=\begin{bmatrix}-7 & 2-12\\ -13 & -12\end{bmatrix}$$

$$= \begin{bmatrix}-7 & 10\\ -13 & -12\end{bmatrix}$$

2. $\begin{bmatrix}5 & -8\\ 2 & 14\end{bmatrix}+\begin{bmatrix}-6 & -2\\ 10 & -4\end{bmatrix}-\begin{bmatrix}-10 & 5\\ 21 & -8\end{bmatrix}$

Solution:

$$= \begin{bmatrix}5 & -8\\ 2 & 14\end{bmatrix}+\begin{bmatrix}-6 & -2\\ 10 & -4\end{bmatrix}-\begin{bmatrix}-10 & 5\\ 21 & -8\end{bmatrix}$$

$$= \begin{bmatrix}5-6 & -8-2\\ 2+10 & 14-4\end{bmatrix}-\begin{bmatrix}-10 & 5\\ 21 & -8\end{bmatrix}$$

$$= \begin{bmatrix}-1 & -10\\ 12 & 10\end{bmatrix}-\begin{bmatrix}-10 & 5\\ 21 & -8\end{bmatrix}$$

$$= \begin{bmatrix}-1+10 & -10-5\\ 12-21 & 10+8\end{bmatrix}$$

$$= \begin{bmatrix}9 & -17\\ -9 & 18\end{bmatrix}$$

3. $-3\begin{bmatrix}4 & -3\\ -1 & -4\end{bmatrix}+8\begin{bmatrix}12 & 10\\ -2 & -4\end{bmatrix}$

Solution:

$$= -3\begin{bmatrix}4 & -3\\ -1 & -4\end{bmatrix}+8\begin{bmatrix}12 & 10\\ -2 & -4\end{bmatrix}$$

$$= \begin{bmatrix}-12 & 9\\ 3 & 12\end{bmatrix}+\begin{bmatrix}96 & 80\\ -16 & -32\end{bmatrix}$$

$$= \begin{bmatrix}-12+96 & 9+80\\ 3-16 & 12-32\end{bmatrix}$$

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$$= \begin{bmatrix} 84 & 89 \\ -13 & -20 \end{bmatrix}$$

4. $3k\begin{bmatrix} -a & b \\ -b & 2a \end{bmatrix} - 2k\begin{bmatrix} a & b \\ b & 2a \end{bmatrix}$

Solution:

$$= 3k\begin{bmatrix} -a & b \\ -b & 2a \end{bmatrix} - 2k\begin{bmatrix} a & b \\ b & 2a \end{bmatrix}$$

$$= \begin{bmatrix} -3ka & 3kb \\ -3kb & 6ka \end{bmatrix} - \begin{bmatrix} 2ka & 2kb \\ 2kb & 4ka \end{bmatrix}$$

$$= \begin{bmatrix} -3ka - 2ka & 3kb - 2kb \\ -3kb - 2kb & 6ka - 4ka \end{bmatrix}$$

$$= \begin{bmatrix} -5ka & kb \\ -5kb & 2ka \end{bmatrix}$$

5. $5\begin{bmatrix} -2 & 10 \\ 8 & 15 \end{bmatrix} - 3\begin{bmatrix} 20 & -25 \\ -10 & 15 \end{bmatrix}$

Solution:

$$= 5\begin{bmatrix} -2 & 10 \\ 8 & 15 \end{bmatrix} - 3\begin{bmatrix} 20 & -25 \\ -10 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & 50 \\ 40 & 75 \end{bmatrix} - \begin{bmatrix} 60 & -75 \\ -30 & 45 \end{bmatrix}$$

$$= \begin{bmatrix} -10-60 & 50-(-75) \\ 40-(-30) & 75-45 \end{bmatrix}$$

$$= \begin{bmatrix} -70 & 125 \\ 70 & 30 \end{bmatrix}$$

6. $\begin{bmatrix} 7 & 5 \\ 8 & 4 \end{bmatrix} - 8\begin{bmatrix} 4 & 3 \\ 2 & 4 \end{bmatrix} + 6\begin{bmatrix} 1 & -8 \\ 2 & -4 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 7 & 5 \\ 8 & 4 \end{bmatrix} - 8\begin{bmatrix} 4 & 3 \\ 2 & 4 \end{bmatrix} + 6\begin{bmatrix} 1 & -8 \\ 2 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 5 \\ 8 & 4 \end{bmatrix} - \begin{bmatrix} 32 & 24 \\ 16 & 32 \end{bmatrix} + \begin{bmatrix} 6 & -48 \\ 12 & -24 \end{bmatrix}$$

$$= \begin{bmatrix} 7-32 & 5-24 \\ 8-16 & 4-32 \end{bmatrix} + \begin{bmatrix} 6 & -48 \\ 12 & -24 \end{bmatrix}$$

$$= \begin{bmatrix} -25 & -19 \\ -8 & -28 \end{bmatrix} + \begin{bmatrix} 6 & -48 \\ 12 & -24 \end{bmatrix}$$

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$$= \begin{bmatrix} -25+6 & -19-48 \\ -8+12 & -28-24 \end{bmatrix}$$

$$= \begin{bmatrix} -19 & -67 \\ 4 & -52 \end{bmatrix}$$

7. $\begin{bmatrix} 7 & -3 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 7 & -3 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$= [7 \times 4 + (-3)(8)]$$

$$= [28 \ - 24]$$

$$= [4]$$

8. $\begin{bmatrix} 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 8 \\ 5 \\ 4 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 8 \\ 5 \\ 4 \end{bmatrix}$$

$$= [1 \times 8 + (-2)(5) + (-3)(4)]$$

$$= [8 - 10 - 12]$$

$$= [-14]$$

9. $\begin{bmatrix} 3 & -2 \end{bmatrix} \begin{bmatrix} 4 \\ -4 \\ -3 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 3 & -2 \end{bmatrix} \begin{bmatrix} 4 \\ -4 \\ -3 \end{bmatrix}$$

No. of columns of 1st matrix are not equal to no. of rows of 2nd matrix. So product cannot perform.

10. $\begin{bmatrix} 18 & -4 & -6 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 18 & -4 & -6 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

No. of columns of 1st matrix are not equal to no. of rows of 2nd matrix, So product cannot perform.

11. $\begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

Solution:

$$= \begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$= [ax + by]$$

12. $\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1x_1 & a_2x_2 & a_3x_3 \end{bmatrix}$$

13. $\begin{bmatrix} -4 & 2 & -8 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} -4 & 2 & -8 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= [-4\times0 + 2\times1 + (-8)(2) + 4\times3]$$

$$= [0 + 2 - 16 + 12]$$

$$= [-2]$$

14. $\begin{bmatrix} 1 & -8 & 6 & -5 & -2 \end{bmatrix}\begin{bmatrix} 6 \\ -2 \\ 4 \\ 8 \\ 4 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 1 & -8 & 6 & -5 & -2 \end{bmatrix} \begin{bmatrix} 6 \\ -2 \\ 4 \\ 8 \\ 4 \end{bmatrix}$$

$= [1\times6 + (-8)(-2) + 6\times4 + (-5)(8) + (-2)(4)]$

$= [6 + 16 + 24 - 40 - 8]$

$= [46 - 48] = [-2]$

15. $\begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} e \\ f \\ g \\ h \end{bmatrix}$

Solution:

$$= \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} e \\ f \\ g \\ h \end{bmatrix}$$

$= [ae + bf + cg + dh]$

16. $\begin{bmatrix} 1 & 0 & -5 & 0 & 8 \end{bmatrix} \begin{bmatrix} 0 \\ -2 \\ 0 \\ 6 \\ 0 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 1 & 0 & -5 & 0 & 8 \end{bmatrix} \begin{bmatrix} 0 \\ -2 \\ 0 \\ 6 \\ 0 \end{bmatrix}$$

$= [1\times0 + 0\times(-2) + (-5)\times0 + 0\times6 + 8\times0]$

$= [0]$

17. $\begin{bmatrix} 4 & 0 \\ -2 & 9 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ -6 & 8 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 4 & 0 \\ -2 & 9 \end{bmatrix}\begin{bmatrix} 2 & 6 \\ -6 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 4\times2+0\times(-7) & 4\times6+0\times8 \\ (-2)(2)+(9)(-7) & (-2)(6)+9\times8 \end{bmatrix}$$

$$= \begin{bmatrix} 8+0 & 24+0 \\ -4-63 & -12+72 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 24 \\ -67 & 60 \end{bmatrix}$$

18. $\begin{bmatrix} 8 & -3 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} 8 \\ -8 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 8 & -3 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} 8 \\ -8 \end{bmatrix}$$

$$= \begin{bmatrix} 8\times8+(-3)(-8) \\ (-2)(8)+(0)(-8) \end{bmatrix} = \begin{bmatrix} 64+24 \\ -16+0 \end{bmatrix}$$

$$= \begin{bmatrix} 88 \\ -16 \end{bmatrix}$$

19. $\begin{bmatrix} 20 & -8 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 20 & -8 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= [20\times1+(-8)(0) \quad 20\times(0)+(-8)(1)]$$

$$= [20+0 \quad 0-8]$$

$$= [20 \quad -8]$$

20. $\begin{bmatrix} 12 & 10 \\ -1 & -8 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 12 & 10 \\ -1 & -8 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 12\times1+10\times0 & 12\times0+10\times1 \\ (-1)(1)+(-8)(0) & (-1)(0)+(-8)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 12+0 & 0+10 \\ -1-0 & 0-8 \end{bmatrix} = \begin{bmatrix} 12 & 10 \\ -1 & -8 \end{bmatrix}$$

21. $\begin{bmatrix} 10 & -5 \\ 0 & 13 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 10 & -5 \\ 0 & 13 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 10\times1+(-5)(0) & 10\times0+(-5)(1) & 10\times1+(-5)(0) \\ 0\times1+13\times0 & 0\times0+(13)\times(1) & 0\times1+13\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 10-0 & 0-5 & 10-0 \\ 0+0 & 0+13 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -5 & 10 \\ 0 & 13 & 0 \end{bmatrix}$$

22. $\begin{bmatrix} 12 & 0 \\ 4 & 0 \\ -2 & 15 \end{bmatrix}\begin{bmatrix} 7 & 12 \\ 8 & -4 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 12 & 0 \\ 4 & 0 \\ -2 & 15 \end{bmatrix}\begin{bmatrix} 7 & 12 \\ 8 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 12\times7+0\times8 & 12\times12+0\times(-4) \\ 4\times7+0\times8 & 4\times12+0\times(-4) \\ (-2)(7)+15\times8 & (-2)(12)+(15)(-4) \end{bmatrix}$$

$$= \begin{bmatrix} 84+0 & 144+0 \\ 28+0 & 48-0 \\ -14+120 & -24-60 \end{bmatrix}$$

$$= \begin{bmatrix} 84 & 144 \\ 28 & 48 \\ 106 & -84 \end{bmatrix}$$

23. $\begin{bmatrix} 4 & 4 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} 6 & -2 \\ 4 & 20 \\ 8 & 4 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 4 & 4 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} 6 & -2 \\ 4 & 20 \\ 8 & 4 \end{bmatrix}$$

No. of columns in matrix 1 are not equal to the no. of rows in matrix 2, so

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product cannot perform.

24. $\begin{bmatrix}1 & 8 & -2\end{bmatrix}\begin{bmatrix}0 & -2 & 7\\3 & -4 & 10\\1 & 2 & -3\end{bmatrix}$

Solution:

$$= \begin{bmatrix}1 & 8 & -2\end{bmatrix}\begin{bmatrix}0 & -2 & 7\\3 & -4 & 10\\1 & 2 & -3\end{bmatrix}$$

$$= [1\times0 + 8\times3+(-2)(1) \quad 1\times(-2)+8\times(-4)+(-2)(2) \quad 1\times7+8\times10+(-2)(-3)]$$

$$= [0 + 24 -2 \quad -2-32-4 \quad 7+80+6]$$

$$= [22 \quad -38 \quad 93]$$

25. $\begin{bmatrix}2 & -1 & 6\\1 & 0 & -4\\3 & -2 & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 10\\0 & -3 & 0\\1 & 4 & 1\end{bmatrix}$

Solution:

$$= \begin{bmatrix}2 & -1 & 6\\1 & 0 & -4\\3 & -2 & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 10\\0 & -3 & 0\\1 & 4 & 1\end{bmatrix}$$

$$= \begin{bmatrix}2\times1+(-1)(0)+6\times1 & 2\times0+(-1)(-3)+6\times4 & 2\times10+(-1)(0)+6\times1\\1\times1+(0)(0)+(-4)\times1 & 1\times0+(0)(-3)+(-4)(4) & 1\times10+(0)(0)+(-4)(1)\\3\times1+(-2)(0)+(-1)(1) & 3\times0+(-2)(-3)+(-1)(4) & 3\times10+(-2)(0)+(-1)(1)\end{bmatrix}$$

$$= \begin{bmatrix}2-0+6 & 0+3+24 & 20-0+6\\1+0-4 & 0-0-16 & 10+0-4\\3-0-1 & 0+6-4 & 30-0-1\end{bmatrix}$$

$$= \begin{bmatrix}8 & 27 & 26\\-3 & -16 & 6\\2 & 2 & 29\end{bmatrix}$$

26. $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & -2 & 8\\3 & 12 & 4\end{bmatrix}$

Solution:

$$= \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & -2 & 8\\3 & 12 & 4\end{bmatrix}$$

No. of columns of matrix 1 are not equal to the no. of rows of matrix 2, So it cannot perform.

27. $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{21}x_2 \end{bmatrix}$$

28. $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \end{bmatrix}$$

29. $\begin{bmatrix} 2 & -7 & 3 \\ 0 & 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & -1 \\ 2 & 0 \\ 4 & 8 \\ 2 & 6 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 2 & -7 & 3 \\ 0 & 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & -1 \\ 2 & 0 \\ 4 & 8 \\ 2 & 6 \end{bmatrix}$$

No. of columns of matrix 1 are not equal to the no. of rows of matrix 2, So product cannot be performed.

30. $\begin{bmatrix} 5 & -3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 3 & 4 \\ 5 & -6 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 5 & -3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 3 & 4 \\ 5 & -6 \end{bmatrix}$$

No. of columns of matrix 1 are not equal to the no. of rows of matrix, So product cannot perform.

31. $\begin{bmatrix} -2 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -4 & 0 \\ 5 & -2 & 3 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} -2 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 0 \\ 5 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -2\times1+2\times5 & (-2)(-4)+2\times(-2) & (-2)(0)+2\times3 \\ 3\times1+(-1)(5) & (3)(-4)+(-1)(-2) & 3\times(0)+(-1)(3) \end{bmatrix}$$

$$= \begin{bmatrix} -2+10 & 8-4 & 0+6 \\ 3-5 & -12+2 & 0-3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 4 & 6 \\ -2 & -10 & -3 \end{bmatrix}$$

32. $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times1 & 3\times(-2) & 3\times3 \\ 2\times1 & 2\times(-2) & 2\times3 \\ 1\times1 & 1\times(-2) & 1\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -6 & 9 \\ 2 & -4 & 6 \\ 1 & -2 & 3 \end{bmatrix}$$

33. $\begin{bmatrix} 2 & -1 \\ 3 & 2 \\ 0 & -4 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 & 5 & 8 \\ 6 & 2 & 0 & -2 & 0 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 2 & -1 \\ 3 & 2 \\ 0 & -4 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 & 5 & 8 \\ 6 & 2 & 0 & -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times2+(-1)\times6 & 2\times(-1)+(-1)\times2 & 2\times0+(-1)\times0 & 2\times5+(-1)(-2) & 2\times8+(-1)(0) \\ 3\times2+2\times6 & 3\times(-1)+(2)\times2 & 3\times0+2\times0 & 3\times5+2\times(-2) & 3\times8+2\times0 \\ 0\times2+(-4)(6) & 0\times(-1)+(-4)\times2 & 0\times0+(-4)(0) & 0\times5+(-4)(-2) & 0\times8+(-4)(0) \\ 3\times2+(-2)(6) & 3\times(-1)+(-2)\times2 & 3\times0+(-2)(0) & 3\times5+(-2)(-2) & 3\times8+(-2)(0) \end{bmatrix}$$

$$= \begin{bmatrix} 4-6 & -2-2 & 0+0 & 10+2 & 16+0 \\ 6+12 & -3+4 & 0+0 & 15-4 & 24+0 \\ 0-24 & 0-8 & 0+0 & 0+8 & 0+0 \\ 6-12 & -3-4 & 0+0 & 15+4 & 24-0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -4 & 0 & 12 & 16 \\ 18 & 1 & 0 & 11 & 24 \\ -24 & -8 & 0 & 8 & 0 \\ -6 & -7 & 0 & 19 & 24 \end{bmatrix}$$

34. $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \\ g & h \\ i & j \end{bmatrix}$

Solution:

$$= \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \\ g & h \\ i & j \end{bmatrix}$$

Product cannot be performed.

35. $\begin{bmatrix} 2 & 5 & -7 \\ 1 & 0 & -2 \\ 4 & 8 & 2 \end{bmatrix}\begin{bmatrix} -3 & 2 & 5 & 0 \\ 1 & -2 & 3 & -4 \\ -3 & 4 & -2 & 1 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 2 & 5 & -7 \\ 1 & 0 & -2 \\ 4 & 8 & 2 \end{bmatrix}\begin{bmatrix} -3 & 2 & 5 & 0 \\ 1 & -2 & 3 & -4 \\ -3 & 4 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times(-3)+5\times1+(-7)(-3) & 2\times2+5\times(-2)+(-7)(4) & 2\times5+5\times3+(-7)(-2) & 2\times0+5\times(-4)+(-7)(1) \\ 1\times(-3)+0\times1+(-2)(-3) & 1\times2+0\times(-2)+(-2)(4) & 1\times5+0\times3+(-2)(-2) & 1\times0+0\times(-4)+(-2)(1) \\ 4\times(-3)+8\times1+2\times(-3) & 4\times2+8\times(-2)+2\times4 & 4\times5+8\times3+2\times(-2) & 4\times0+8\times(-4)+2\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & -34 & 39 & -27 \\ 3 & -6 & 9 & -2 \\ -10 & 0 & 40 & -30 \end{bmatrix}$$

36. $\begin{bmatrix} 2 & 8 & -1 \\ 0 & 4 & 0 \end{bmatrix}\begin{bmatrix} -1 & 0 & 3 \\ 4 & 0 & -1 \\ -1 & 0 & 1 \end{bmatrix}$

Solution:

$$= \begin{bmatrix} 2 & 8 & -1 \\ 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 3 \\ 4 & 0 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times(-1)+8\times4+(-1)(-1) & 2\times0+8\times0+(-1)(0) & 2\times3+8\times(-1)+(-1)(1) \\ 0\times(-1)+4\times4+0\times(-1) & 0\times0+4\times0+0\times0 & 0\times3+4\times(-1)+0\times(1) \end{bmatrix}$$

$$= \begin{bmatrix} -2+32+1 & 0+0+0 & 6-8-1 \\ 0+16+0 & 0+0+0 & 0-4+0 \end{bmatrix}$$

$$= \begin{bmatrix} 31 & 0 & -3 \\ 16 & 0 & -4 \end{bmatrix}$$

37. Rewrite the following systems of equations in matrix form.

$$x - 3y = 15$$
$$2x + 3y = -10$$

Solution:

$$x - 3y = 15$$
$$2x + 3y = -10$$

In Matrix form:-

$$\begin{bmatrix} 1 & -3 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ -10 \end{bmatrix}$$

38. Rewrite the following systems of equations in matrix form.

$$2x = 4$$
$$3x + 4y = 15$$

Solution:

$$2x = 4$$
$$3x + 4y = 15$$

In Matrix form:-

$$\begin{bmatrix} 2 & 0 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 15 \end{bmatrix}$$

39. Rewrite the following systems of equations in matrix form.

$$5x_1 - 2x_2 + 3x_3 = 12$$
$$3x_1 - 3x_2 - 2x_3 = 15$$

Solution:

$$5x_1 - 2x_2 + 3x_3 = 12$$
$$3x_1 - 3x_2 - 2x_3 = 15$$

In Matrix form:

$$\begin{bmatrix} 5 & -2 & 3 \\ 3 & -1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \end{bmatrix}$$

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40. Rewrite the following systems of equations in matrix form.

$$5x_1 - 8x_2 = 48$$
$$2x_1 - 4x_2 = 25$$

Solution:

$$5x_1 - 8x_2 = 48$$
$$2x_1 - 4x_2 = 25$$

In Matrix form:

$$\begin{bmatrix} 5 & -8 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 48 \\ 25 \end{bmatrix}$$

33. Rewrite the following systems of equations in matrix form.

$$ax_1 + bx_2 = c$$
$$dx_1 + ex_2 = f$$
$$gx_1 + hx_2 = i$$

Solution:

$$ax_1 + bx_2 = c$$
$$dx_1 + ex_2 = f$$
$$gx_1 + hx_2 = i$$

In Matrix form:

$$\begin{bmatrix} a & b \\ d & e \\ g & h \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} c \\ f \\ i \end{bmatrix}$$

42. Rewrite the following systems of equations in matrix form.

$$ax_1 + bx_2 + cx_3 + dx_4 + ex_5 = i$$
$$gx_1 \quad - hx_3 \quad + ix_5 = j$$

Solution:

$$ax_1 + bx_2 + cx_3 + dx_4 + ex_5 = i$$
$$gx_1 \quad - hx_3 \quad + ix_5 = j$$

In Matrix form:-

$$\begin{bmatrix} a & b & c & d & e \\ g & o & -h & o & i \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} i \\ j \end{bmatrix}$$

43. Rewrite the following systems of equations in matrix form.

$$a_1x^2 + a_2x + a_3 = b_1$$
$$a_4x^2 + a_5x + a_6 = b_2$$

Solution:

$$a_1x^2 + a_2x + a_3 = b_1$$
$$a_4x^2 + a_5x + a_6 = b_2$$

In Matrix form:

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix}\begin{bmatrix} x^2 \\ x \\ 1 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

44. Rewrite the following systems of equations in matrix form.

$$a_{11}x^2 + a_{12}x + a_{13} = b_1$$
$$a_{21}x^2 + a_{22}x + a_{23} = b_2$$
$$a_{31}x^2 + a_{32}x + a_{33} = b_2$$

Solution:

$$a_{11}x^2 + a_{12}x + a_{13} = b_1$$
$$a_{21}x^2 + a_{22}x + a_{23} = b_2$$
$$a_{31}x^2 + a_{32}x + a_{33} = b_2$$

In Matrix form:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} x^2 \\ x \\ 1 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

45. Rewrite the following systems of equations in matrix form.

$$5x^3 - 2x^2 + x = 100$$
$$3x^3 = -18$$
$$5x^2 = 125$$

Solution:

$$5x^3 - 2x^2 + x = 100$$
$$3x^3 = -18$$
$$5x^2 = 125$$

In Matrix form:

$$\begin{bmatrix} 5 & -2 & 1 \\ 3 & 0 & 0 \\ 0 & 5 & 0 \end{bmatrix}\begin{bmatrix} x^3 \\ x^2 \\ x \end{bmatrix} = \begin{bmatrix} 100 \\ -18 \\ 125 \end{bmatrix}$$

46. Rewrite the following systems of equations in matrix form.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 = b_2$$

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$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 = b_3$$

$$a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = b_4$$

Solution:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 = b_3$$

$$a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = b_4$$

In Matrix form:-

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

47. If $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$, verify that

(a) A(BC) = (AB) C and

(b) A(B+C) = AB + AC

Solution:

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix},\ B = \begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$$

(a) $A(BC) = (AB)C$

$$\text{L.H.S} = A(BC)$$

$$= \begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4\times1+0\times1 & 4\times1+0\times3 \\ 1\times1+2\times1 & 1\times1+2\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 \\ 3 & 7 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 4 & 4 \\ 3 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times4+1\times3 & 2\times4+1\times7 \\ 3\times4+4\times3 & 3\times4+4\times7 \end{bmatrix}$$

$$= \begin{bmatrix} 8+3 & 8+7 \\ 12+12 & 12+28 \end{bmatrix} = \begin{bmatrix} 11 & 15 \\ 24 & 40 \end{bmatrix}$$

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$$R.H.S = (AB)C$$

$$AB = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times4+1\times1 & 2\times0+1\times2 \\ 3\times4+4\times1 & 3\times0+4\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 8+1 & 0+2 \\ 12+4 & 0+8 \end{bmatrix} = \begin{bmatrix} 9 & 2 \\ 16 & 8 \end{bmatrix}$$

$$A(BC)C = \begin{bmatrix} 9 & 2 \\ 16 & 8 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9\times1+2\times1 & 9\times1+2\times3 \\ 16\times1+8\times1 & 16\times1+8\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 9+2 & 9+6 \\ 16+8 & 16+24 \end{bmatrix} = \begin{bmatrix} 11 & 15 \\ 24 & 40 \end{bmatrix}$$

Hence Proved.

$$A(BC) = (AB)\,C$$

(b) $A(B+C) = AB + AC$

$$L.H.S = A(B+C)$$

$$B + C = \begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 1 \\ 2 & 5 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times5+1\times2 & 2\times1+1\times5 \\ 3\times5+4\times2 & 3\times1+4\times5 \end{bmatrix}$$

$$= \begin{bmatrix} 10+2 & 2+5 \\ 15+8 & 3+20 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 7 \\ 23 & 23 \end{bmatrix}$$

$$R.H.S = AB + AC$$

$$AB = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 2 \\ 16 & 8 \end{bmatrix}$$

$$AC = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 5 \\ 7 & 15 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 9 & 2 \\ 16 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 5 \\ 7 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 7 \\ 23 & 23 \end{bmatrix}$$

Hence Proved.

$$A(B + C) = AB + AC$$

Solved Section 3.3

Find the determinant of each of the following matrices.

1. A = [-5]

Solution:

$$A = [-5]$$

$$\Delta = |A| = -5$$

2. A = [b]

Solution:

$$A = [b]$$

$$\Delta = |A| = b$$

3. $T = \begin{bmatrix} 8 & 3 \\ -2 & -4 \end{bmatrix}$

Solution:

$$T = \begin{bmatrix} 8 & 3 \\ -2 & -4 \end{bmatrix}$$

$$\Delta = |T| = \begin{bmatrix} 8 & 3 \\ -2 & -4 \end{bmatrix}$$

$$= 8\times(-4)-(-2)(3)$$

$$= -32 + 6$$

$$= -26$$

4. $S = \begin{bmatrix} -7 & 12 \\ -4 & 8 \end{bmatrix}$

Solution:

$$S = \begin{bmatrix} -7 & 12 \\ -4 & 8 \end{bmatrix}$$

$$\Delta = |S| = \begin{bmatrix} -7 & 12 \\ -4 & 8 \end{bmatrix}$$

$$= (-7)(8)-(-4)(12)$$

$$= -56 + 48$$

$$= -8$$

5. N = [28]

Solution:

$$N = [28]$$

$$\Delta = |N| = 28$$

6. T = [-a]

Solution:

$$T = [-a]$$

$$\Delta = |T| = -a$$

7. $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Solution:

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Delta = |B| = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 1 \times 1 - 0 \times 0$$

$$= 1 - 0$$

$$= 1$$

8. $A = \begin{bmatrix} a & a \\ a & a \end{bmatrix}$

Solution:

$$A = \begin{bmatrix} a & a \\ a & a \end{bmatrix}$$

$$\Delta = |A| = \begin{bmatrix} a & a \\ a & a \end{bmatrix}$$

$$= a \times a - a \times a$$

$$= a^2 - a^2$$

$$= 0$$

9. $C = \begin{bmatrix} 2 & -6 & 10 \\ 4 & 0 & -2 \\ 3 & -2 & 8 \end{bmatrix}$

Solution:

$$C = \begin{bmatrix} 2 & -6 & 10 \\ 4 & 0 & -2 \\ 3 & -2 & 8 \end{bmatrix}$$

$$\Delta = |C| = \begin{bmatrix} 2 & -6 & 10 \\ 4 & 0 & -2 \\ 3 & -2 & 8 \end{bmatrix}$$

$$= 2\begin{vmatrix} 0 & -2 \\ -2 & 8 \end{vmatrix} - (-6)\begin{vmatrix} 4 & -2 \\ 3 & 8 \end{vmatrix} + 10\begin{vmatrix} 4 & 0 \\ 3 & -2 \end{vmatrix}$$

$$= 2(0 \times 8-(-2)(-2)) + 6\ (4 \times 8-(-2) \times 3) + 10(4 \times (-2) -0 \times 3)$$

$$= 2(0 - 4) +6\ (32 + 6) + 10(- 8 - 0)$$

$$= 2(-4) + 6(38) + 10(-8)$$

$$= -8 + 228 - 80$$

$$= 140$$

10. $B = \begin{bmatrix} 4 & -3 & 10 \\ -2 & 0 & -1 \\ 7 & 12 & 8 \end{bmatrix}$

Solution:

$$B = \begin{bmatrix} 4 & -3 & 10 \\ -2 & 0 & -1 \\ 7 & 12 & 8 \end{bmatrix}$$

$$\Delta = |B| = \begin{vmatrix} 4 & -3 & 10 \\ -2 & 0 & -1 \\ 7 & 12 & 8 \end{vmatrix}$$

$$= 4\begin{vmatrix} 0 & -1 \\ 12 & 8 \end{vmatrix} - (-3)\begin{vmatrix} -2 & -1 \\ 7 & 8 \end{vmatrix} + 10\begin{vmatrix} -2 & 0 \\ 7 & 12 \end{vmatrix}$$

$= 4(0 \times 8-(-1) \times 12) + 3(-2 \times 8-(-1) \times 7) + 10(-2 \times 12-0 \times 7)$

$= 4(0 + 12) + 3(-16+7) + 10(-24 -0)$

$= 4(12) +3(-9) + 10(-24)$

$= 48 - 27 - 240$

$= -219$

11. $D = \begin{bmatrix} 1 & -2 & 8 \\ -2 & 10 & -5 \\ 4 & -8 & 12 \end{bmatrix}$

Solution:

$$D = \begin{bmatrix} 1 & -2 & 8 \\ -2 & 10 & -5 \\ 4 & -8 & 12 \end{bmatrix}$$

$$\Delta = |D| = \begin{vmatrix} 1 & -2 & 8 \\ -2 & 10 & -5 \\ 4 & -8 & 12 \end{vmatrix}$$

$$= 1\begin{vmatrix} 10 & -5 \\ -8 & 12 \end{vmatrix} - (-2)\begin{vmatrix} -2 & -5 \\ 4 & 12 \end{vmatrix} + 8\begin{vmatrix} -2 & 10 \\ 4 & -8 \end{vmatrix}$$

$= 1(10 \times 12-(-5)(-8)) + 2(-2 \times 12-(-5) \times 4) + 8(-2 \times(-8)-10\times 4)$

$= 1(120 - 40) + 2(-24 + 20) + 8(16 - 40)$

$= 1(80) +2(-4) + 8(-24)$

$= 80 - 8 - 192$

$= -120$

12. $A = \begin{bmatrix} 3 & 10 & 3 \\ 2 & -6 & -5 \\ 1 & -3 & 8 \end{bmatrix}$

Solution:

$$A = \begin{bmatrix} 3 & 10 & 3 \\ 2 & -6 & -5 \\ 1 & -3 & 8 \end{bmatrix}$$

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$$\Delta = |A| = \begin{vmatrix} 3 & 10 & 3 \\ 2 & -6 & -5 \\ 1 & -3 & 8 \end{vmatrix}$$

$$= 3\begin{vmatrix} -6 & -6 \\ -3 & 8 \end{vmatrix} - 10\begin{vmatrix} 2 & -6 \\ 1 & 8 \end{vmatrix} + 8\begin{vmatrix} 2 & -6 \\ 1 & -3 \end{vmatrix}$$

$= 3(-6\times8-(-6)(-3)) +10(2\times8 -(-6)\times1) + 8(2\times(-3)-(-6)\times1)$

$= 3(-48-18) - 10(16+6) + 8(-6 + 6)$

$= 3(-66) -10(22) + 8(0)$

$= -198 - 220 + 0$

$= -418$

13. $A = \begin{bmatrix} 2 & 0 & 8 \\ 4 & -1 & -2 \\ 0 & 5 & -4 \end{bmatrix}$

Solution:

$$A = \begin{bmatrix} 2 & 0 & 8 \\ 4 & -1 & -2 \\ 0 & 5 & -4 \end{bmatrix}$$

$$\Delta = |A| = \begin{vmatrix} 2 & 0 & 8 \\ 4 & -1 & -2 \\ 0 & 5 & -4 \end{vmatrix}$$

$$= 2\begin{vmatrix} -1 & -2 \\ 5 & -4 \end{vmatrix} - 0\begin{vmatrix} 4 & -2 \\ 0 & -4 \end{vmatrix} + 8\begin{vmatrix} 4 & -1 \\ 0 & 5 \end{vmatrix}$$

$= 2(-1\times -4-(-2)\times5) -0(4 \times -4 -(-2) \times 0) +8(4 \times 5 -(-1) \times 0)$

$= 2(4 + 10) -0(-8 + 0) + 8(20 + 0)$

$= 2(14) -0(-8) + 9(20)$

$= 28 - 0 + 160$

$= 188$

14. $B = \begin{bmatrix} 2 & 4 & 7 \\ -1 & 3 & 2 \\ 4 & -2 & 0 \end{bmatrix}$

Solution:

$$B = \begin{bmatrix} 2 & 4 & 7 \\ -1 & 3 & 2 \\ 4 & -2 & 0 \end{bmatrix}$$

$$\Delta = |B| = \begin{vmatrix} 2 & 4 & 7 \\ -1 & 3 & 2 \\ 4 & -2 & 0 \end{vmatrix}$$

$$= 2\begin{vmatrix} 3 & 2 \\ -2 & 0 \end{vmatrix} - 4\begin{vmatrix} -1 & 2 \\ 4 & 0 \end{vmatrix} + 7\begin{vmatrix} -1 & 3 \\ 4 & -2 \end{vmatrix}$$

$= 2(3 \times 0-(-2) \times 2) -4(-1\times 0 -2 \times 4) +7(-1 \times(-2) -3 \times 4)$

$= 2(0 + 4) -4(0 + 8) + 7(2 - 12)$

$= 2(4) -4(-8) + 7(-10)$

$= 8 + 32 - 70$

$= -30$

15. $C = \begin{bmatrix} -1 & -2 & -3 \\ 3 & -4 & 6 \\ 0 & 0 & 8 \end{bmatrix}$

Solution:

$$B = \begin{bmatrix} -1 & -2 & -3 \\ 3 & -4 & 6 \\ 0 & 0 & 8 \end{bmatrix}$$

$$\Delta = |B| = \begin{vmatrix} -1 & -2 & -3 \\ 3 & -4 & 6 \\ 0 & 0 & 8 \end{vmatrix}$$

$$= -1\begin{vmatrix} -4 & 6 \\ 0 & 8 \end{vmatrix} - (-2)\begin{vmatrix} 3 & 6 \\ 0 & 8 \end{vmatrix} + (-3)\begin{vmatrix} 3 & -4 \\ 0 & 0 \end{vmatrix}$$

$= -1(-4\times8 - 0\times6) +2(3\times2 - 6\times0) -33(3\times0 -(-4) \times0)$

$= -1(-32 - 0) +2(24 - 0) -3(0 + 0)$

$= -1(-32) +2(24) -3(0)$

$= 32 + 48 - 0$

$= 80$

16. $D = \begin{bmatrix} 2 & 6 & -5 \\ 5 & 0 & 10 \\ 3 & 2 & -3 \end{bmatrix}$

Solution:

$$D = \begin{bmatrix} 2 & 6 & -5 \\ 5 & 0 & 10 \\ 3 & 2 & -3 \end{bmatrix}$$

$$\Delta = |D| = \begin{vmatrix} 2 & 6 & -5 \\ 5 & 0 & 10 \\ 3 & 2 & -3 \end{vmatrix}$$

$$= 2\begin{vmatrix} 0 & 10 \\ 2 & -3 \end{vmatrix} - 6\begin{vmatrix} 5 & 10 \\ 3 & -3 \end{vmatrix} + (-5)\begin{vmatrix} 5 & 0 \\ 3 & 2 \end{vmatrix}$$

$= 2(0\times-3 - 10\times2) -6(5\times(-3) - 10\times3) -5(5\times2 - 0\times3)$

$= 2(0 - 20) -6(-15 - 30) -5(10 - 0)$

$= 2(-20) -6(-45) -5(10)$

$= -40 + 270 - 50$

$= 180$

Find the matrix of cofactors for each of the following matrices.

17. $\begin{bmatrix} 8 & -2 \\ 10 & -4 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 8 & -2 \\ 10 & -4 \end{bmatrix}$

The Cofactors are

$a'_{11} = (-1)^{1+1}(-4) = +(-4) = -4$

$a'_{12} = (-1)^{1+2}(-10) = -(10) = -10$

$a'_{21} = (-1)^{2+1}(-2) = -(-2) = +2$

$a'_{22} = (-1)^{2+2}(8) = +(8) = 8$

The cofactor matrix is

$$A_c = \begin{bmatrix} -4 & -10 \\ 2 & 8 \end{bmatrix}$$

18. $\begin{bmatrix} -1 & -2 \\ -4 & 10 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} -1 & -2 \\ -4 & 10 \end{bmatrix}$

The Cofactors are

$a'_{11} = (-1)^{1+1}(10) = +(10) = 10$

$a'_{12} = (-1)^{1+2}(-4) = -(-4) = 4$

$a'_{13} = (-1)^{1+3}(-2) = -(-2) = 2$

$a'_{22} = (-1)^{2+2}(-1) = +(-1) = -1$

The cofactor matrix is

$$A_C = \begin{bmatrix} 10 & 4 \\ 2 & -1 \end{bmatrix}$$

Q - 19, 20 are same Q - 17, 18.

21. $\begin{bmatrix} 2 & -4 & -2 \\ -2 & 0 & 4 \\ 4 & 3 & -3 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 2 & -4 & -2 \\ -2 & 0 & 4 \\ 4 & 3 & -3 \end{bmatrix}$

The Cofactors are

$a'_{11} = (-1)^{1+1}\begin{vmatrix} 0 & 4 \\ 3 & -3 \end{vmatrix} = +(0-12) = -12$

$a'_{12} = (-1)^{1+2}\begin{vmatrix} -2 & 4 \\ 4 & -3 \end{vmatrix} = -(6-16) = -(-10) = 10$

$a'_{13} = (-1)^{1+3}\begin{vmatrix} -2 & 0 \\ 4 & -3 \end{vmatrix} = +(-6-0) = -6$

$a'_{21} = (-1)^{2+1}\begin{vmatrix} -4 & -2 \\ 3 & -3 \end{vmatrix} = -(12+6) = -18$

$a'_{22} = (-1)^{2+2}\begin{vmatrix} 2 & -2 \\ 4 & -3 \end{vmatrix} = +(-6+8) = 2$

$a'_{23} = (-1)^{3+2}\begin{vmatrix} 2 & -4 \\ 4 & -3 \end{vmatrix} = -(6+16) = -22$

$a'_{31} = (-1)^{3+2}\begin{vmatrix} -4 & -2 \\ 0 & 4 \end{vmatrix} = +(-16-0) = -16$

$a'_{32} = (-1)^{3+2}\begin{vmatrix} 2 & -2 \\ -2 & 4 \end{vmatrix} = -(8-4) = -4$

$a'_{33} = (-1)^{3+3}\begin{vmatrix} 2 & -4 \\ -2 & 0 \end{vmatrix} = +(0-8) = -8$

The cofactor matrix is

$$A_c = \begin{bmatrix} -12 & 10 & -6 \\ -18 & 2 & -22 \\ -16 & -4 & -8 \end{bmatrix}$$

22. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Solution:

$$\text{Let } A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The Cofactors are

$$a'_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = +(1-0) = 1$$

$$a'_{12} = (-1)^{1+2} \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = -(0-0) = 0$$

$$a'_{13} = (-1)^{1+3} \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = +(0-0) = 0$$

$$a'_{21} = (-1)^{2+1} \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = -(0-0) = 0$$

$$a'_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = +(1-0) = 1$$

$$a'_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = -(0-0) = 0$$

$$a'_{31} = (-1)^{3+1} \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = +(0-0) = 0$$

$$a'_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = -(0-0) = 0$$

$$a'_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = +(1-0) = 1$$

The cofactor matrix is

$$A_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q - 23 to Q - 34 are same as Q - 17 to Q - 22.

35 to 52. Using the matrix of cofactors found, respectively, in Exercise 17 - 34, find the determinant of the original matrix.

35. The cofactor matrix in Q - 17 is

$$A_c = \begin{bmatrix} -4 & -10 \\ 2 & 8 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} -4 & -10 \\ 2 & 8 \end{bmatrix}$$

$$= (-4)(8) - (-10)(2)$$

$$= -32 + 20$$

$$= -12$$

The original matrix is

$$A = \begin{bmatrix} 8 & -2 \\ 10 & -4 \end{bmatrix}$$

$$\Delta = (8)(-4) - (-2)(10)$$

$$= -32 + 20$$

$$= -12$$

36. The cofactor matrix in Q - 18 is

$$A_c = \begin{bmatrix} 10 & 4 \\ 2 & -1 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 10 & 4 \\ 2 & -1 \end{bmatrix}$$

$$= (10)(-1) - (2)(4)$$

$$= -10 - 8$$

$$= -18$$

The original matrix is

$$A = \begin{bmatrix} -1 & -2 \\ -4 & 10 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} -1 & -2 \\ -4 & 10 \end{bmatrix}$$

$= (-1)(10) - (-2)(-4)$
$= -10 - 8$
$= -18$

Q - 37 to Q - 52 are same as Q - 35, 36.

53. Find the determinent of

$$A = \begin{bmatrix} 2 & 7 & -2 & 0 \\ 1 & -2 & -3 & 0 \\ 3 & 3 & 6 & 9 \\ 6 & -3 & -2 & 0 \end{bmatrix}$$

Solution:

$$|A| = \begin{bmatrix} 2 & 7 & -2 & 0 \\ 1 & -2 & -3 & 0 \\ 3 & 3 & 6 & 9 \\ 6 & -3 & -2 & 0 \end{bmatrix}$$

Expand from Column 4.

$$= 0 - 0 + 9 \begin{vmatrix} 2 & 7 & -2 \\ 1 & -2 & -3 \\ 6 & -3 & -2 \end{vmatrix} - 0$$

$$= 9\left[2\begin{vmatrix} -2 & -3 \\ -3 & -2 \end{vmatrix} - 7\begin{vmatrix} 1 & -3 \\ 6 & -2 \end{vmatrix} + (-2)\begin{vmatrix} 1 & -2 \\ 6 & -3 \end{vmatrix}\right]$$

$= 9\,[2(4-9) - 7(-2+18) - 2(-3+12)]$
$= 9\,[2(-5) - 7(16) - 2(9)]$
$= 9\,[-10 - 112 - 18]$
$= 9\,[-140]$
$= -1260$

Q - 54 to 56 are same as Q - 53.

In the following exercises, solve the system of equations by using Cramer's rule.

57. $3x_1 - 2x_2 = -13$
$4x_1 + 6x_2 = 0$

Solution:

$3x_1 - 2x_2 = -13$
$4x_1 + 6x_2 = 0$

In Matrix form:

$$\begin{bmatrix} 3 & -2 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} -13 \\ 0 \end{bmatrix}$$

According to Cramer's rule:-

$$x_i = \frac{\Delta_i}{\Delta}$$

So

$$x_1 = \frac{\begin{vmatrix} -13 & -2 \\ 0 & 6 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ 4 & 6 \end{vmatrix}}$$

$$= \frac{(-13)\times 6-(-2)\times 0}{3\times 6-(-2)\times 4}$$

$$= \frac{-78+0}{18+8} = -\frac{78}{26} = -3$$

And

$$x_2 = \frac{\begin{vmatrix} 3 & -13 \\ 4 & 0 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ 4 & 6 \end{vmatrix}}$$

$$= \frac{3\times 0-(-13)\times 4}{3\times 6-(-2)\times 4}$$

$$= \frac{0+52}{18+8} = \frac{52}{26} = 2$$

58. $5x_1 - 4x_2 = -8$

$3x_1 + 5x_2 = 47$

Solution:

$5x_1 - 4x_2 = -8$

$3x_1 + 5x_2 = 47$

In Matrix form:

$$\begin{bmatrix} 5 & -4 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} -8 \\ 47 \end{bmatrix}$$

According to Cramer's rule:-

$$x_1 = \frac{\begin{vmatrix} -8 & -4 \\ 47 & 5 \end{vmatrix}}{\begin{vmatrix} 5 & -4 \\ 3 & 5 \end{vmatrix}}$$

$$= \frac{-8\times 5-(-4)\times 47}{5\times 5-(-4)\times 3}$$

$x_1 = \frac{\Delta i}{\Delta}$

$$= \frac{-40+188}{25+12} = \frac{148}{37} = 4$$

$$x_2 = \frac{\begin{vmatrix} 5 & -8 \\ 3 & 47 \end{vmatrix}}{\begin{vmatrix} 5 & -4 \\ 3 & 5 \end{vmatrix}}$$

$$= \frac{5\times47-(-8)\times3}{5\times5-(-4)\times3}$$

$$= \frac{235+24}{25+12} = \frac{259}{37} = 7$$

59. $x_1 - 5x_2 = -85$
$2x_1 + 4x_2 = 40$

Solution:

$$x_1 - 5x_2 = -85$$
$$2x_1 + 4x_2 = 40$$

In Matrix form:

$$\begin{bmatrix} 1 & -5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -85 \\ 40 \end{bmatrix}$$

According to Cramer's rule:-

$$x_1 = \frac{\begin{vmatrix} -85 & -5 \\ 40 & 4 \end{vmatrix}}{\begin{vmatrix} 1 & -5 \\ 2 & 4 \end{vmatrix}}$$

$$= \frac{-85\times4-(-5)\times40}{1\times4-(-5)\times2}$$

$$= \frac{-340+200}{4+10} = -\frac{140}{14} = -10$$

$$x_2 = \frac{\begin{vmatrix} 1 & -85 \\ 2 & 40 \end{vmatrix}}{\begin{vmatrix} 1 & -5 \\ 2 & 4 \end{vmatrix}}$$

$$= \frac{1\times40-(-85)\times2}{1\times4-(-5)\times2}$$

$$= \frac{40+170}{4+10} = \frac{210}{14} = 15$$

Q.No. 60 to 62 are same as Q. No. 57 to 59.

63. $x_1 + 3x_2 - 2x_3 = 17$

$2x_1 - 4x_2 + x_3 = -16$

$5x_1 + 2x_2 - 4x_3 = 21$

Solution:

$$x_1 + 3x_2 - 2x_3 = 17$$
$$2x_1 - 4x_2 + x_3 = -16$$
$$5x_1 + 2x_2 - 4x_3 = 21$$

In Matrix form:

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & -4 & 1 \\ 5 & 2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 17 \\ -16 \\ 21 \end{bmatrix}$$

According to Cramer's Rule:-

$$x_1 = \frac{\begin{vmatrix} 17 & 3 & -2 \\ -16 & -4 & 1 \\ 21 & 2 & -4 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & -2 \\ 2 & -4 & 1 \\ 5 & 2 & -4 \end{vmatrix}}$$

$$= \frac{17\begin{vmatrix} -4 & 1 \\ 2 & -4 \end{vmatrix} - 3\begin{vmatrix} -16 & 1 \\ 21 & -4 \end{vmatrix} + (-2)\begin{vmatrix} -16 & -4 \\ 21 & 2 \end{vmatrix}}{1\begin{vmatrix} -4 & 1 \\ 2 & -4 \end{vmatrix} - 3\begin{vmatrix} 2 & 1 \\ 5 & -4 \end{vmatrix} + (-2)\begin{vmatrix} 2 & -4 \\ 5 & 2 \end{vmatrix}}$$

$$= \frac{17(16-2) - 3(64-21) - 2(-32+84)}{1(16-2) - 3(-8-5) - 2(4+20)}$$

$$= \frac{17(14) - 3(43) - 2(52)}{1(14) - 3(-13) - 2(24)}$$

$$= \frac{238 - 129 - 104}{14 + 39 - 48} = \frac{238 - 233}{53 - 48}$$

$$= \frac{5}{5} = 1$$

$$x_2 = \frac{\begin{vmatrix} 1 & 17 & -2 \\ 2 & -16 & 1 \\ 5 & 21 & -4 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & -2 \\ 2 & -4 & 1 \\ 5 & 2 & -4 \end{vmatrix}}$$

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$$= \frac{1\begin{vmatrix}-16 & 1\\ 21 & -4\end{vmatrix} - 17\begin{vmatrix}2 & 1\\ 5 & -4\end{vmatrix} + (-2)\begin{vmatrix}2 & -16\\ 5 & 21\end{vmatrix}}{1\begin{vmatrix}-4 & 1\\ 2 & -4\end{vmatrix} - 3\begin{vmatrix}2 & 1\\ 5 & -4\end{vmatrix} + (-2)\begin{vmatrix}2 & -4\\ 5 & 2\end{vmatrix}}$$

$$= \frac{1(64-21)-17(-8-5)-2(42+80)}{1(16-2)-3(-8-5)-2(4+20)}$$

$$= \frac{1(43)-17(-13)-2(122)}{1(14)-3(-13)-2(24)}$$

$$= \frac{43+221-224}{14+39-48} = \frac{264-244}{53-48}$$

$$= \frac{20}{5} = 4$$

$$x_3 = \frac{\begin{vmatrix}1 & 3 & 17\\ 2 & -4 & -16\\ 5 & 2 & 21\end{vmatrix}}{\begin{vmatrix}1 & 3 & -2\\ 2 & -4 & 1\\ 5 & 2 & -4\end{vmatrix}}$$

$$= \frac{1\begin{vmatrix}-4 & -16\\ 2 & 21\end{vmatrix} - 3\begin{vmatrix}2 & -16\\ 5 & 21\end{vmatrix} + 17\begin{vmatrix}2 & -4\\ 5 & 2\end{vmatrix}}{1\begin{vmatrix}-4 & 1\\ 2 & -4\end{vmatrix} - 3\begin{vmatrix}2 & 1\\ 5 & -4\end{vmatrix} - 2\begin{vmatrix}2 & -4\\ 5 & 2\end{vmatrix}}$$

$$= \frac{1(-84+32)-3(42+80)+17(4+20)}{1(16-2)-3(-8-5)-2(4+20)}$$

$$= \frac{1(-52)-3(122)+17(24)}{1(14)-3(-13)-2(24)}$$

$$= \frac{-52-366+408}{14+39-48} = \frac{-418+408}{53-48}$$

$$= -\frac{10}{5} = -2$$

$x_1 = 1$, $x_2 = 4$, $x_3 = -2$

<u>Q - 64 to Q - 68 are same as Q - 63.</u>

Solved Section 3.5

Determine the inverse, if it exists, for the following matrices, using the gaussion procedure.

1. $\begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}$

Solution:

By Gaussian procedure, we have

$$\left[\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 2 & -3 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & -1 & -2 & 1 \end{array}\right] \quad R_2 - 2R_1$$

$$\left[\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & -1 & 2 & -1 \end{array}\right] \quad -R_2$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 3 & -1 \\ 0 & 1 & 2 & -1 \end{array}\right] \quad R_2 + R_1$$

The inverse is

$$\begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}$$

2. $\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$

Solution:

By Gaussian procedure, we have

$$\left[\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 4 & 7 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 4 & 7 & 0 & 1 \end{array}\right] \quad \frac{1}{2}R_1$$

$$\left[\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -2 & 1 \end{array}\right] \quad R_2 - 4R_1$$

$$\left[\begin{array}{cc|cc} 1 & 0 & \frac{7}{2} & -\frac{3}{2} \\ 0 & 1 & -2 & 1 \end{array}\right] \quad R_1 - \frac{3}{2}R_2$$

The inverse is

$$\begin{bmatrix} \frac{7}{2} & -\frac{3}{2} \\ -2 & 1 \end{bmatrix}$$

Q - 3 to Q - 6 are same as Q - 1, 2

7. $\begin{bmatrix} 0 & 3 & 1 \\ 1 & 1 & 0 \\ 2 & 3 & 3 \end{bmatrix}$

Solution:

$$\left[\begin{array}{ccc|ccc} 0 & 3 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 3 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 3 & 1 & 1 & 0 & 0 \\ 2 & 3 & 3 & 0 & 0 & 1 \end{array}\right] \quad R_{12}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & -2 & 1 \end{array}\right] \quad R_3 - 2R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & -2 & 1 \\ 0 & 3 & 1 & 1 & 0 & 0 \end{array}\right] \quad R_{23}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & -2 & 1 \\ 0 & 0 & 1 & 1 & -3 & 0 \end{array}\right] \quad R_3 - 3R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 0 & 3 & -1 \\ 0 & 1 & 3 & 0 & -2 & 1 \\ 0 & 0 & 1 & 1 & -3 & 0 \end{array}\right] \quad R_1 - R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 0 & 3 & -1 \\ 0 & 1 & 0 & -3 & 7 & 1 \\ 0 & 0 & 1 & 1 & -3 & 0 \end{array}\right] \quad R_2 - 3R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -6 & -1 \\ 0 & 1 & 0 & -3 & 7 & 1 \\ 0 & 0 & 1 & 1 & -3 & 0 \end{array}\right] \quad R_1 + 3R_3$$

The inverse is

$$\begin{bmatrix} 3 & -6 & -1 \\ -1 & 7 & 1 \\ 1 & -3 & 0 \end{bmatrix}$$

8. $\begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & -1 \\ -1 & 0 & 2 \end{bmatrix}$

Solution:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ -1 & 1 & -1 & 0 & 1 & 0 \\ -1 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 1 & 0 \\ -1 & 0 & 2 & 0 & 0 & 1 \end{array}\right] \quad R_2 + R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right] \quad R_3 + R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 & 1 & 2 \end{array}\right] \quad R_2 + 2R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & 1 & 2 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 & 1 & 2 \end{array}\right] \quad R_1 + R_3$$

The inverse is

$$\begin{bmatrix} 4 & 1 & 2 \\ 1 & 1 & 0 \\ 3 & 1 & 2 \end{bmatrix}$$

<u>Q - 9 to Q - 12 are same as Q - 7, 8.</u>

Determine the inverse of the following matrices by using the matrix of cofactors approach.

13. $\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

$$A^{-1} = \frac{1}{|A|} \text{ Adj } A$$

$$|A| = \begin{vmatrix} 3 & 7 \\ 2 & 5 \end{vmatrix}$$

$$= (3)(5) - (7)(2)$$

$$= 15 - 14$$

$$= 1$$

The cofactor matrix A_c is $A_c = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

The corresponding adjoint matrix is

$$\text{Adj } A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

So $$A^{-1} = \frac{1}{1}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

<u>Q - 14, 17, 18, 19, 20 are same as Q - 13</u>

15. $\begin{bmatrix} 3 & 5 & 2 \\ 4 & 1 & 0 \\ -9 & -15 & -6 \end{bmatrix}$

Solution:

Let $$A = \begin{bmatrix} 3 & 5 & 2 \\ 4 & 1 & 0 \\ -9 & -15 & -6 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{ Adj } A$$

$$|A| = \begin{vmatrix} 3 & 5 & 2 \\ 4 & 1 & 0 \\ -9 & -15 & -6 \end{vmatrix}$$

$$= 3\begin{vmatrix} 1 & 0 \\ -15 & -6 \end{vmatrix} - 5\begin{vmatrix} 4 & 0 \\ -9 & -6 \end{vmatrix} + 2\begin{vmatrix} 4 & 1 \\ -9 & -15 \end{vmatrix}$$

$$= 3(-6-0) - 5(-24-0) + 2(-60+9)$$

$$= 3(-6) - 5(-24) + 2(-51)$$

$$= -18 + 120 - 102$$

$$= 120 - 120$$

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$= 0$

As $|A| = 0$, no inverse exists.

21. $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 0 & -4 \\ 1 & 2 & 5 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 0 & -4 \\ 1 & 2 & 5 \end{bmatrix}$

$A^{-1} = \dfrac{1}{|A|} \text{ Adj } A$

$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 0 & -4 \\ 1 & 2 & 5 \end{vmatrix}$

$= 1\begin{vmatrix} 0 & -4 \\ 2 & 5 \end{vmatrix} - 1\begin{vmatrix} 3 & -4 \\ 1 & 5 \end{vmatrix} + 1\begin{vmatrix} 3 & 0 \\ 1 & 2 \end{vmatrix}$

$= 1(0+8) - 1(15+4) + 1(6-0)$

$= 1(8) - 1(19) + 1(6)$

$= 8 - 19 + 6$

$= 14 - 15$

$= -5$

The Cofactors are

$a'_{11} = (-1)^{1+1} \begin{vmatrix} 0 & -4 \\ 2 & 5 \end{vmatrix} = +(0 + 8) = 8$

$a'_{12} = (-1)^{1+2} \begin{vmatrix} 3 & -4 \\ 1 & 5 \end{vmatrix} = -(15 + 4) = -19$

$a'_{13} = (-1)^{1+3} \begin{vmatrix} 3 & 0 \\ 1 & 2 \end{vmatrix} = +(6 - 0) = 6$

$a'_{21} = (-1)^{2+1} \begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix} = -(5 - 2) = -3$

$a'_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 1 & 5 \end{vmatrix} = +(5 - 1) = 4$

$$a'_{23} = (-1)^{2+3}\begin{vmatrix}1 & 1\\1 & 2\end{vmatrix} = -(2-1) = -1$$

$$a'_{31} = (-1)^{3+1}\begin{vmatrix}1 & 1\\0 & -4\end{vmatrix} = +(-4-0) = -4$$

$$a'_{32} = (-1)^{3+2}\begin{vmatrix}1 & 1\\3 & -4\end{vmatrix} = -(4-3) = 7$$

$$a'_{33} = (-1)^{3+3}\begin{vmatrix}1 & 1\\3 & 0\end{vmatrix} = +(0-3) = -3$$

The cofactor matrix is

$$A_c = \begin{bmatrix}8 & -19 & 6\\-3 & 4 & -1\\-4 & 7 & -3\end{bmatrix}$$

The adjoint matrix is the transpose of A_c, or

$$\text{Adj } A = \begin{bmatrix}8 & -3 & -4\\-19 & 4 & 7\\6 & -1 & -3\end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{ Adj } A$$

$$= \frac{1}{-5}\begin{bmatrix}8 & -3 & -4\\-19 & 4 & 7\\6 & -1 & -3\end{bmatrix}$$

$$= \begin{bmatrix}-\frac{8}{5} & \frac{-3}{-5} & \frac{-4}{-5}\\-\frac{19}{-5} & \frac{4}{-5} & \frac{7}{-5}\\\frac{6}{-5} & \frac{-1}{-5} & \frac{-3}{-5}\end{bmatrix}$$

$$= \begin{bmatrix}-1.6 & 0.6 & 0.8\\3.8 & -0.8 & -1.4\\-1.2 & 0.2 & 0.6\end{bmatrix}$$

22.

Solution:

Same as Q - 21

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Using the results of Exercise 1 to 22, determine the solution to the systems of equations in Exercise 23 -44, respectively (if one exists).

23. $x_1 - x_2 = -1$

$2x_1 - 3x_2 = -5$

Solution:

$$x_1 - x_2 = -1$$
$$2x_1 - 3x_2 = -5$$

Let $A = \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad B = \begin{bmatrix} -1 \\ -5 \end{bmatrix}$

So, $AX = B$

$$X = \frac{1}{A} B$$
$$X = A^{-1} B$$

And $A^{-1} = \frac{1}{|A|} \text{Adj } A$

$$|A| = \begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix}$$
$$= (1)(-3) - (-1)(2)$$
$$= -3 + 2$$
$$= -1$$

$$\text{Adj } A = \begin{bmatrix} -3 & 1 \\ -2 & 1 \end{bmatrix}$$

So, $A^{-1} = \frac{1}{-1}\begin{bmatrix} -3 & 1 \\ -2 & 1 \end{bmatrix}$

$$= -\begin{bmatrix} -3 & 1 \\ -2 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}$$

$$X = A^{-1} B$$
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ -5 \end{bmatrix}$$
$$= \begin{bmatrix} -3 & +5 \\ -2 & +5 \end{bmatrix}$$
$$= \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$\Rightarrow \quad x_1 = 2, \; x_2 = 3$

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Q - 24, 25, 26, 27, 28 are same as Q - 23.

29.

$$3x_2 + x_3 = 1$$
$$x_1 + x_2 = 2$$
$$2x_1 + 3x_2 + 3x_3 = 7$$

Solution:

Let $A = \begin{bmatrix} 0 & 3 & 1 \\ 1 & 1 & 0 \\ 2 & 3 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}$

So, $AX = B$

$$X = \frac{1}{A} B$$

$$X = A^{-1} B$$

And $A^{-1} = \frac{1}{|A|} B$

$$|A| = \begin{vmatrix} 0 & 3 & 1 \\ 1 & 1 & 0 \\ 2 & 3 & 3 \end{vmatrix}$$

$$= 0\begin{vmatrix} 1 & 0 \\ 3 & 3 \end{vmatrix} - 3\begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$$

$$= 0(3 - 0) - 3(3 - 0) + 1(3 - 2)$$
$$= 0(3) - 3(3) + 1(1)$$
$$= 0 - 9 + 1$$
$$= -8$$

The Cofactors are

$$a'_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 0 \\ 3 & 3 \end{vmatrix} = +(3 - 0) = 3$$

$$a'_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} = -(3 - 0) = -3$$

$$a'_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = +(3 - 2) = 1$$

$$a'_{21} = (-1)^{2+1} \begin{vmatrix} 3 & 1 \\ 3 & 3 \end{vmatrix} = -(9 - 3) = -6$$

$$a'_{22} = (-1)^{2+2} \begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} = +(0 - 2) = -2$$

$$a'_{23} = (-1)^{2+3} \begin{vmatrix} 0 & 3 \\ 2 & 3 \end{vmatrix} = -(0 - 6) = 6$$

$$a'_{31} = (-1)^{3+1} \begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix} = +(0 - 1) = -1$$

$$a'_{32} = (-1)^{3+2} \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -(0 - 1) = 1$$

$$a'_{33} = (-1)^{3+3} \begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix} = +(0 - 3) = -3$$

The cofactor matrix is

$$A_C = \begin{bmatrix} 3 & -3 & 1 \\ -6 & -2 & 6 \\ -1 & 1 & -3 \end{bmatrix}$$

The adjoint matrix is the transpose of A_c, or

$$\text{Adj } A = \begin{bmatrix} 3 & -6 & -1 \\ -3 & -2 & -1 \\ 1 & 6 & -3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-8} \begin{bmatrix} 3 & -6 & -1 \\ -3 & -2 & 1 \\ 1 & 6 & -3 \end{bmatrix}$$

$$X = A^{-1} B$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{-8} \begin{bmatrix} 3 & -6 & -1 \\ -3 & -2 & 1 \\ 1 & 6 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}$$

$$= \frac{1}{-8} \begin{bmatrix} 3 & -12 & -7 \\ -3 & -4 & +7 \\ 1 & +12 & -21 \end{bmatrix}$$

$$= \frac{1}{-8} \begin{bmatrix} 3 & - & 19 \\ 7 & - & 7 \\ 13 & - & 21 \end{bmatrix}$$

$$= \frac{1}{-8} \begin{bmatrix} -16 \\ 0 \\ 8 \end{bmatrix}$$

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$$a'_{23} = (-1)^{2+3}\begin{vmatrix} 0 & 3 \\ 2 & 3 \end{vmatrix} = -(0-6) = 6$$

$$a'_{31} = (-1)^{3+1}\begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix} = +(0-1) = -1$$

$$a'_{32} = (-1)^{3+2}\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -(0-1) = 1$$

$$a'_{33} = (-1)^{3+3}\begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix} = +(0-3) = -3$$

The cofactor matrix is

$$A_c = \begin{bmatrix} 3 & -3 & 1 \\ -6 & -2 & 6 \\ -1 & 1 & -3 \end{bmatrix}$$

The adjoint matrix is the transpose of A_c, or

$$\text{Adj } A = \begin{bmatrix} 3 & -6 & -1 \\ -3 & -2 & -1 \\ 1 & 6 & -3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-8}\begin{bmatrix} 3 & -6 & -1 \\ -3 & -2 & 1 \\ 1 & 6 & -3 \end{bmatrix}$$

$$X = A^{-1} B$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{-8}\begin{bmatrix} 3 & -6 & -1 \\ -3 & -2 & 1 \\ 1 & 6 & -3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}$$

$$= \frac{1}{-8}\begin{bmatrix} 3 & -12 & -7 \\ -3 & -4 & +7 \\ 1 & +12 & -21 \end{bmatrix}$$

$$= \frac{1}{-8}\begin{bmatrix} 3 & - & 19 \\ 7 & - & 7 \\ 13 & - & 21 \end{bmatrix}$$

$$= \frac{1}{-8}\begin{bmatrix} -16 \\ 0 \\ 8 \end{bmatrix}$$

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$$= \begin{bmatrix} \frac{-16}{-8} \\ \frac{0}{-8} \\ \frac{8}{-8} \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

$\Rightarrow \quad x_1 = 2, \; x_2 = 0, \; x_3 = -1$

Q - 30 to Q - 44 are same as Q - 29.
Q - 45 to Q - 48 are not necessary for paper point of view.

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Chapter - 4

Introduction to Probability Theory

Solved Section 4.1

In Exercise 1 - 5 redefine each set using the descriptive property method:

1. **A = {1, 3, 5, 7, 9, 11, 13, 17, 19}**

Solution:

A = {a/a is a positive odd integer less than 20}

2. **S = {- 3, 3, - 2, 2, - 1, 1, 0}**

Solution:

S = { 3, 2, 1, 0, 1, 2, 3}

S = { S/S is a set of integers between - 3 to 3}

3. **V = {a, e, i, o, u}**

Solution:

V = {v/v is a set of vowel}

4. **S = {0, -1, -4, -9, -16, - 25, - 36}**

Solution:

S = {0, 1, 4, 9, 16, 25, 36}

S = {S/S is the square of integer with negative sign less than and equal to 0}.

5. **C = {-1, -8, -27, -64}**

Solution:

C = { 1, 8, 27, 64}

= {(1)3 = 1, (2)3 = 8, (3)3 = 27, (4)3 = 64}

={C/C is the cube of negative integers less than - 5}.

In exercise 6 -10 redefine each by enumeration.

6. **A = {a/a is a negative odd integer greater than - 10}.**

Solution:

A = { 9, -7, -5, ...}

7. **B = {b/b is a positive integer less than 8}.**

Solution:

B = {1, 2, 3, 4, 5, 6, 7}

8. **C = {c/c is the name of a day of the week}**

Solution:

C = {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

9. **B = {b/ when a = 2, a + 3b = - 7}**

Solution:

Given a = 2 and a + 3b = 7

2 + 3b = 7

3b = 7 - 2

3b = 9

b =

b = {- 3}

10. M = {m/m is the fourth power of a negative integer greater than - 6}

Solution:

M = {(-5)^4 = 625, (4)^4 = 256, (3)4 = 81,........}

11. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and B = {b/b is a positive even integer less than 10}, find B .

Solution:

U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

B = {0, 2, 4, 6, 8,}

B = U - B

= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} {0, 2, 4, 6, 8,}

= {1, 3, 5, 7, 9, 10}

12. If U equals the set of students in mathematics class P is the students who fail the course, define P' .

Solution:

U = The set of students in mathematics class

P = The set of students who fail the course

P' = U - P

= The set of students who pass the course.

13. If U = {x/x is an integer greater than 6 but less than 14} and S = {7, 9, 10, 12, 13}, find S.

Solution:

U = {7, 8, 9, 10, 11, 12, 13}

S' = {7, 9, 10, 12, 13}

S = U S

= {7, 8, 9, 10, 11, 12, 13} {7, 9, 10, 12, 13}

= {8, 11}

14. If U is the set consisting of all positive integers and T' equals the set consisting of all positive even integers, find T.

Solution:

U = {1, 2, 3, 4, 5, 6,........}

T = {2, 4, 6, 8,10,.........}

T = U - T

= {1, 2, 3, 4, 5, 6,........} {2, 4, 6, 8, 10,.........}

= {1, 3, 5, 7, 9,............}

15. If U = {x/x is a positive integer less than 20}, A = {1, 5, 9, 19}
B = (b/b is positive odd integers less than 11}, and
C = {c/c is positive odd integers less than 20} find all subset relationships which exist among U, A, B and C.

Solution:

U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19}

A = {1, 5, 9, 19},

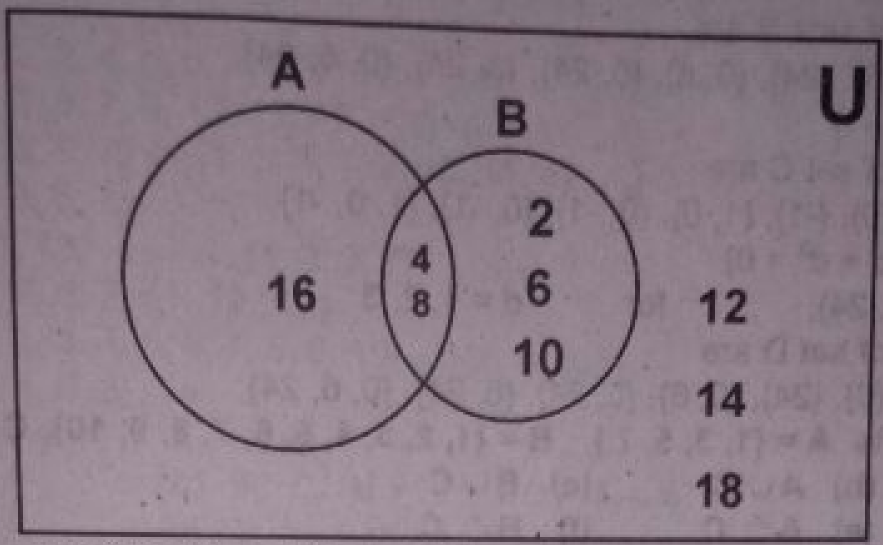
B = {1, 3, 5, 7, 9,},

C = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}

$A \subset U$, $B \subset U$, $C \subset U$, $A \subset C$, $B \subset C$.

16. Given U = {2, 4, 6, 8, 10, 12, 14, 16, 18}, A = {4, 8, 16}, B = {2, 4, 6, 8, 10}
Draw a Venn diagram representing the sets.

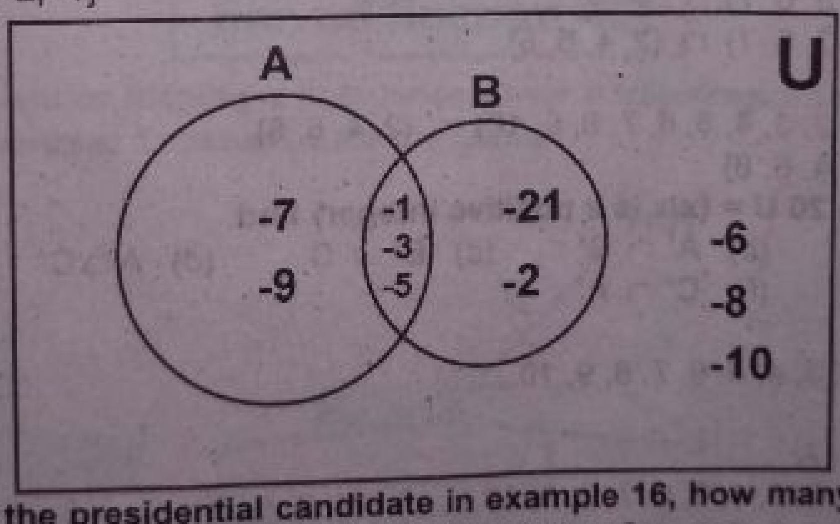
Solution:



17. If U = {x/x is a negative integer than - 11},
A = {a/a is a negative odd integer greater than - 10} and
B = {b/b is a negative integer greater than- 6}, draw a Venn diagram representing the sets.

Solution:

U = { 10, 9, 8, 7, 6, 5, 4, 3, 2, 1},
A = { 9, 7, 5, 3, 1}
B = { 5, 4, 3, 2, 1}



18. If we refer to the presidential candidate in example 16, how many combinations of three cities might he and his staff consider visiting?

Solution:

Same as Q - 19

19. Given the following sets, (a) state which, if any, are equal and (b) define all subset relationships among A, B, C and D.
A = {0, 1, -1}, B = {b/b^3 - b = 0}, C = {-1, 0, -1}, D = {d/-d + d^3 = 0}

Solution:

(a) A and C, B and D are equal sets.
(b) A = {0, 1, -1}

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Subsets of set A are

Φ , {0}, {1}, {-1}, {0, 1}, {0, -1}, {1, -1}, {0, 1, -1}

B = {b/b^3- b = 0} = {0, 6, 24} for b = 1, 2, 3

Subsets of sets B are

Φ , {0}, {6}, {24}, {0, 6}, {0, 24}, {6, 24}, {0, 6, 24}

C = {1, 0, -1}

Subsets of set C are

Φ , {1}, {0}, {-1}, {1, 0}, {0, -1}, {0, -1}, {1, 0, -1}

D = {d/ d + d^3 = 0}

D = {0, 6, 24}, for d = 1, 2, 3

Subsets of set D are

Φ , {0}, {6}, {24}, {0, 6}, {0, 24}, {6, 24}, {0, 6, 24}

20. Given the sets A = {1, 3, 5, 7,}, B = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, C = {2, 4, 6, 8}. Find

(a) $A \cup B$ (b) $A \cup C$ (c) $B \cup C$

(d) $A \cap B$ (e) $A \cap C$ (f) $B \cap C$

Solution:

(a) $A \cup B$ = {1, 3, 5, 7} $\cup$ {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

(b) $A \cup C$ = {1, 3, 5, 7} $\cup$ {2, 4, 6, 8}
= {1, 2, 3, 4, 5, 6, 7, 8}

(c) $B \cup C$ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} $\cup$ {2, 4, 6, 8}
= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

(d) $A \cap B$ = {1, 3, 5, 7} $\cap$ {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
= {1, 3, 5, 7}

(e) $A \cap C$ = {1, 3, 5, 7} $\cap$ {2, 4, 6, 8}
= { }

(f) $B \cap C$ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} $\cap$ {2, 4, 6, 8}
= {2, 4, 6, 8}

21. If in Exercise 20 U = {x/x is a positive integer} find

(a) $A \cap A'$ (b) $A' \cap B'$ (c) $B' \cup C$ (d) $A \cap C'$

(e) $B' \cup A$ (f) $C' \cap A'$

Solution:

U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10.......}

(a) $A \cap A'$ =

A' = U - A
= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10.......} - {1, 3, 5, 7}
= {2, 4, 6, 8, 9, 10.......}

$A \cap A'$ = {1, 3, 5, 7} {2, 4, 6, 8, 10, ...}
= { }

(b) $A' \cap B'$ =

B' = U - B
= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ...} - {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
= {11, 12, ...}

$A' \cap B'$ = {2, 4, 6, 8, 9, 10, 11, 12, ...} - {11, 12, ...}
= {11, 12, ...}

(c) B' ∪ C' = {11, 12,.....} ∪ {2, 4, 6, 8}
= {2, 4, 6, 8, 11, 12, ...}

(d) A ∩ C' =
C' = U - C
= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, ...} - {2, 4, 6, 8}
= {1, 3, 5, 7, 9, 10, 11, 12, ...}
A ∩ C' = {1, 3, 5, 7} ∩ {1, 3, 5, 7, 9, 10, 11, 12,....}
= {1, 3, 5, 7}

(e) B' ∪ A = {11, 12, ...} ∪ {1, 3, 5, 7}
= {1, 3, 5, 7, 11, 12......}

(f) B ∩ C = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} ∩ {2, 4, 6, 8}
= {2, 4, 6, 8}

22. **Given any set A:**

(a) A ∪ U = **(b) A' ∩ U =** **(c) A ∪ϕ =**
(d) A' ∪ϕ = **(e) U' =**

Solution:

(a) A ∪ U = U
(b) A' ∩ U = A'
(c) A ∪ϕ = A
(d) A' ∪ϕ = A'
(e) U' = ϕ

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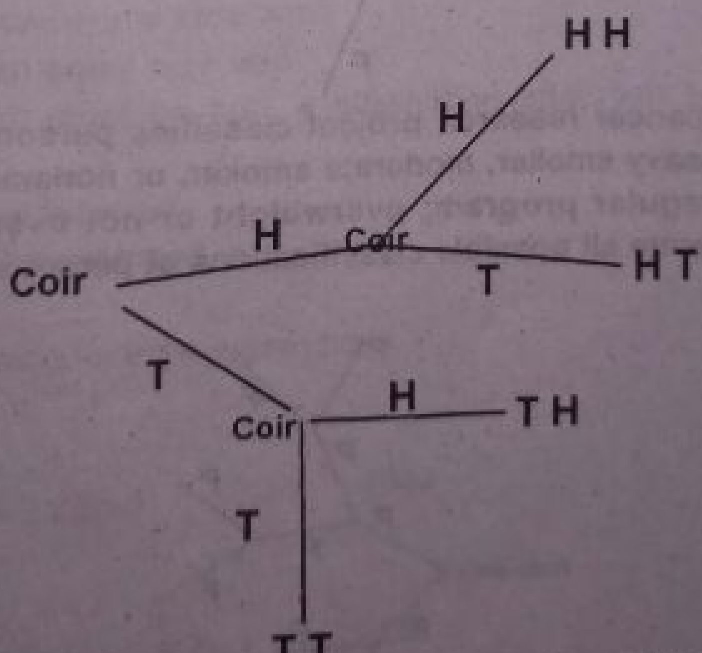
Solved Section 4.2

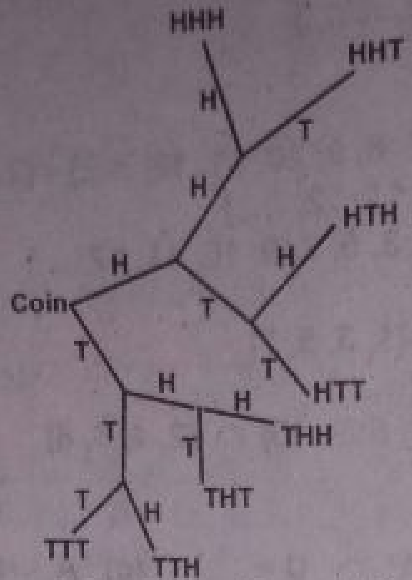
1. **A game consists of flipping a coin twice. Draw a tree diagram which enumerates all possible combined outcomes for the game.**

Solution:



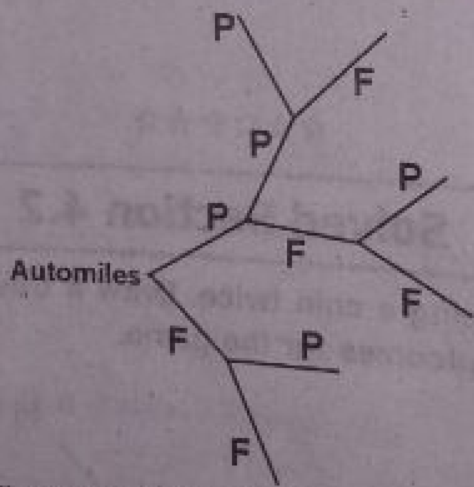
2. **A game consists of flipping a coin three times in a row. Draw a tree diagram which enumerates all possible combined outcomes for the game.**

Solution:



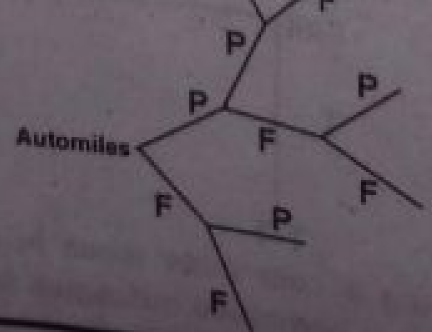
3. **Emissions control an automobile inspection station inspects vehicles for level of air pollution emissions. Vehicles either pass (P) or fail (F) the inspection. Draw decision tree which diagram which enumerates all possible combined outcomes for the game.**

Solution:



4. **Health Profile A cancer research project classifies persons in four categories: male or female; heavy smoker, moderate smoker, or nonsmoker; regular exercise program or no regular program; overweight or not overweight. Draw a tree diagram to enumerate all possible classifications of persons.**

Solution:



Use the fundamental counting principle to solve exercises 5 - 8.

5. **A license plate consists of two letters followed by three single- digit numbers. Determine the number of different license plate codes which are possible.**

Solution:

Here n = 3, r = 2

$$^{n}C_{p} = {^{3}P_{2}} = \frac{3!}{(3-2)!} = \frac{3!}{1!} = \frac{3\times2\times1}{1} = 6$$

6. **College Admissions the admissions office at a local university classifies applicants as male or female; in - state or out- of -state; preferred college within the university (Engineering, Business, Liberal Arts, Education, and Pharmacy); above- average, average, or below- average SAT cores; and request for financial aid or no request for financial aid. Determine the number of possible applicant classification.**

Solution:

Here n = 5, r = 2

$$^{n}P_{r} = {^{5}P_{2}} = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5\times4\times3}{3!} = 20$$

7. **A student is planning his schedule for the fall. For the five courses he is considering there are three possible English instructors, six sociology instructors, four mathematics instructors, eight history instructors, and five political science instructors. Determine the number of different sets of instructors possible for his fall schedule.**

Solution:

Same as above

8. **Determine the number of possible seven- digit telephone number if none of the first three digits can equal zero and:**

(a) **Any digits can equal zero and:**

(b) **The first digit must be odd, a alternating after that between even and odd digits.**

(c) **All digits must be even.**

(d) **No digit can be repeated.**

Solution:

Evaluate the following factorial expressions.

9. 7!

Solution:

$7! = 7\times6\times5\times4\times3\times2\times1 = 5040$

10. 9!

Solution:

$9! = 9\times8\times7\times6\times5\times4\times3\times2\times1 = 362880$

11. 15!

Solution:

$$15! = 15\times14\times13\times12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1 = 1.3077\times10^{12}$$

$$= 1,307,700,000,000$$

12. (15 - 8)!

Solution:

$$(15-8)! = 7! = 7\times6\times5\times4\times3\times2\times1 = 5040$$

13. $\frac{7!}{4!}$

Solution:

$$\frac{7!}{4!} = \frac{7\times6\times5\times4!}{4!} = 7\times6\times5 = 210$$

14. $\frac{15!}{6!}$

Solution:

$$\frac{15!}{6!} = \frac{15\times14\times13\times12\times11\times10\times9\times8\times7\times6}{6!} = 1,816,214,400$$

15. $\frac{8!.5!}{6!}$

Solution:

$$\frac{8!.5!}{6!} = \frac{8\times7\times6!\times5\times4\times3\times2!}{6!} = 6720$$

16. $\frac{15!.8!}{10!}$

Solution:

$$\frac{15!.8!}{10!} = \frac{15\times14\times13\times12\times11\times10!\times8\times7\times6!\times5\times4\times3\times2\times1}{8!} = 1.4529\times10^{10}$$

17. $\frac{10!}{3!.6!}$

Solution:

$$\frac{10!}{3!.6!} = \frac{10\times9\times8\times7\times6!}{3\times2\times1\times6!} = 840$$

18. $\frac{10!}{8!.2!}$

Solution:

$$\frac{10!}{8!.2!} = \frac{10\times9\times8!}{8\times2\times1} = 45$$

19. $\frac{8!}{0!.5!}$

Solution:

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$$\frac{8!}{0!.5!}=\frac{8\times7\times6\times5!}{1\times5!}=336$$

20. $\frac{9!}{3!.5!}$

Solution:

$$\frac{9!}{3!.5!}=\frac{9\times8\times7\times6\times5!}{3\times2\times1\times5!}=504$$

In Exercise 21 - 28, evaluate each symbol.

21. ${}_6P_6$

Solution:

$${}_6P_6=\frac{6!}{(6-6)!}=\frac{6\times5\times4\times3\times2\times1}{1}=720$$

22. ${}_7P_3$

Solution:

$${}_7P_3=\frac{7!}{(7-3)!}=\frac{7!}{4!}=\frac{7\times6\times5\times4!}{4!}=210$$

23. ${}_8P_6$

Solution:

$${}_8P_6=\frac{8!}{(8-6)!}=\frac{8!}{2!}=\frac{8\times7\times6\times5\times4\times3\times2!}{2!}=20160$$

24. ${}_9P_4$

Solution:

$${}_9P_4=\frac{9!}{(9-4)!}=\frac{9!}{5!}=\frac{9\times8\times7\times6\times5!}{5!}=3024$$

25. $\binom{5}{5}$

Solution:

$$\binom{5}{5}=\frac{5!}{5!(5-5)!}=\frac{5!}{5!\times1!}=1$$

26. $\binom{6}{4}$

Solution:

$$\binom{6}{4}=\frac{6!}{4!(6-4)!}=\frac{6!}{4!\times2!}=\frac{6\times5\times4!}{4!\times2!}=15$$

27. $\binom{8}{4}$

Solution:

$$\binom{8}{4}=\frac{8!}{4!(8-4)!}=\frac{8!}{4!\times 4!}=\frac{8\times7\times6\times5\times4!}{4\times3\times2\times1\times4!}=70$$

28. $\binom{7}{3}$

Solution:

$$\binom{7}{3}=\frac{7!}{3!(7-3)!}=\frac{7!}{3!\times 4!}=\frac{7\times6\times5\times4!}{3\times2\times1\times4!}=35$$

29. Ten horses are to be placed within a starting gate for a major sweepstakes race. How many different starting arrangements are possible?

Solution:

$$10! = 10\times9\times8\times7\times6\times5\times4\times3\times2\times1$$
$$= 3{,}628{,}8000$$

30. A political candidate wishes to visit eight different states. In how many different orders can she visit these states?

Solution:

$$8! = 8\times7\times6\times5\times4\times3\times2\times1$$
$$= 40,\ 320$$

31. The same political candidate in exercises 30 has time and funds to visit only four states. How many different combinations of four states can she visit?

Solution:

$$^{8}C_{4}=\frac{8!}{4!(8-4)!}=\frac{8!}{4!\times 4!}=\frac{8\times7\times6\times5\times4!}{4\times3\times2\times1\times4!}=70$$

32. A credit card company issues credit cards which have a three- letter prefix as part of the card number. A sample card number is ABC 1234.

(a) If each letter of the prefix is to be different, how many prefixes are possible?

(b) If each of the four numerals following the prefix is to be different, how many different four- digit sequences are possible?

Solution:

Same as 29 to 31.

33. Eight astronauts are being considered for the next flight team. If a flight team consists of three members, how many different combinations of astronauts could be considered?

Solution:

Same as 29 to 31.

34. A portfolio management expert is considering 30 stocks for investment. Only 15 stocks will be selected for inclusion in a portfolio. How many different

combinations of stocks can be considered?

Solution:

Same as 29 to 31.

35. Four persons are to be selected for the board of directors of a local hospital. If twelve candidates have been selected, how many different groups of four could be selected for the board?

Solution:

Same as 29 to 31.

36. Given a committee of ten persons, in how many ways can we select a chairperson, vice chairperson, and recording secretary.

Solution:

Same as 29 to 31.

37. Six airline companies have submitted applications for for operating over a new international route. Only two of the companies will be awarded permits to operate over the route. How many different sets of airlines could be selected?

Solution:

$$C_2^6 = \frac{6!}{2!(6-2)!} = \frac{6!}{2! \times 4!} = \frac{6\times5\times4\times3\times2\times1}{2\times1\times4\times3\times2\times1} = 15$$

38. Medical research A major research foundation I considering funding a set of medical research projects. Fifteen applications have been submitted, but only six will receive funding. How many different set of projects could be funded?

Solution:

$$C_6^{15} = \frac{15!}{6!(15-6)!} = \frac{15!}{6! \times 9!} = \frac{15\times14\times13\times12\times11\times10\times9!}{6\times5\times4\times3\times2\times1\times9!} = 5005$$

39. A bridge hand consists of 13 cards. How many different bridge hands can be dealt from a deck consisting of 52 cards?

Solution:

$$C_{13}^{52} = \frac{52!}{13!(52-13)!} = \frac{52!}{13 \times 39!} = 6.3501356\times10^{11}$$

40. Design Team The president of a major corporation has decided to undertake the development of a major new product which will give the corporation a significant competitive edge. The president wants to appoint a special product design team which will consist of three engineers, one marketing research analyst, one financial analyst, and two production supervisors. There are eight engineers four marketing research analysts, ix financial analysts, and five production supervisors being considered for the tem. How many different design teams could be created?

Solution:

Same as above questions

41. Education The chairperson of a high school mathematics department wants to

select eight seniors, six juniors, five sophomores, and four freshmen for the high school mathematics team. Ten seniors, eight juniors, eight sophomores, and six freshmen have applied for the team and have qualified on the basis of their mathematics grades. How many different teams could the chairperson select from this group?

Solution:

Same as above questions

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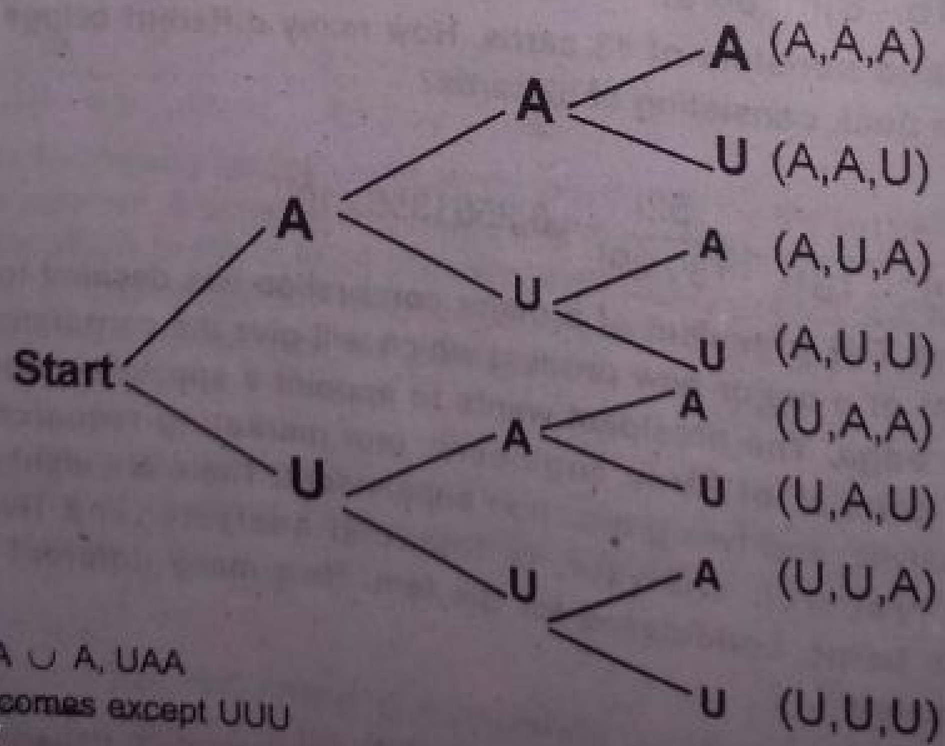
Solved Section 4.3

1. **Pollution monitoring a water quality inspector I conducting an experiment where he samples the water from various wells to see if it has acceptable (A) or unacceptable (U) levels of contaminants. Suppose the inspector is going to inspect three wells, one after the other, and record the quality of the water for each.**
   (a) **Determine the sample space S for this experiment.**
   (b) **Construct a tree diagram enumerating the possible outcomes.**
   (c) **What simple outcomes are included in the event "exactly two acceptable wells"?**
   (d) **What simple outcomes are included in the event "at least one acceptable well"?**

Solution:

(a) S = {AAA, AAU, A ∪ A, A ∪ U, UAA, UAU, UUA, UUU}

(b)



(c) AAU, A ∪ A, UAA

(d) All outcomes except UUU

2. Recidivism A criminal justice researcher is studying the rate of recidivism (repeat offenders) for child molestation. He is conducting an experiment where he examines the criminal record for persons convicted of child molestation. If a person has been convicted more than once, he is classified as a "recidivist" (R). If a person has not been convicted more than once for child molestation, he is classified as a "nonrecidivist" (N) in the experiment. If the researcher examines the records of three offenders, (a) deter mine the sample space S for the experiment, (b) construct a tree diagram enumerating the possible outcomes for the experiment, (c) determine the set of simple outcomes included in the event 'two or fewer recidivists," and (d) determine the set of simple outcomes included in the event "two or fewer recidivists," and (d) determine the set of simple outcomes included in the event "at least one recidivist."

Solution:

Same as Q - 1

3. Table 13.5 indicates some characteristics of a pool of 1,000 applicants for an administrative position. Applicants are classified by sex and by highest educational degree received. Suppose that one applicant is to be selected at random in an experiment. The samples pace S for this experiment consists of the simple outcomes S = {MC, MH, MN, FC, FH, FN}.

(a) Determine the set of simple outcomes which are used to define the compound event "male applicant" (M)

(b) Determine the set of simple outcomes which are used to define the compound event "highest degree of applicant is college degree" (C).

Solution:

(a) M = {MC, MH, MN}

(b) C = {MC, FC}

4. In exercise 3, determine, for each of the following et of events, whether they are mutually exclusive and / or collectively exhaustive.

(a) {M, F, H}

(b) {C, H, N, M, F}

(c) {MC, MH, MN, F}

(d) {MC, FC, C, H, N}

(e) {M, FC, FH}

Solution:

(a) Mutually exclusive

(b) Collectively exhaustive

(c) Collectively exhaustive

(d) Collectively exhaustive

(e) Mutually exclusive

5. In Exercise 3, suppose that one applicant is selected at random (each having an equal chance of being selected). What is the probability that the applicant selected will (a) be a female, (b) have a high school diploma as the highest

degree, (c) be a male with no degrees, and (d) be a female with a college degree?

Solution:

(a) Let 'A' be the event that the applicant selected will be a female.

$$P(A) = \frac{510}{1000}$$

(b) Let 'B' be the event that the applicant selected will have a high school diploma as the highest degree.

$$P(B) = \frac{310}{1000}$$

(c) Let 'C' be the event that the applicant selected will be a male with no degree.

$$P(C) = \frac{40}{1000}$$

(d) Let 'D' be the event that the applicant selected will be female with a college degree.

$$P(D) = \frac{275}{1000}$$

6. Table 13.6 indicates some characteristics of 10,000 borrower from a major financial institution. Borrowers are classified according to the type of loan (personal or business) and level of risk. Suppose that an experiment is to be conducted where one borrower's account is selected at random. The sample space S for this experiment consists of the simple outcomes S = {PL, PA, PH, BL, BA, BH}.

(a) Determine the set of simple outcomes which are used to define the compound event "personal loan" (P).

(b) Determine the set of simple outcomes which are used to define the compound event "average - risk loan" (A).

Solution:

(a) P = {PL, PA, PH}

(b) A = {PA, BA}

7. In Exercise 6, determine, for each of the following sets of event, whether they are mutually exclusive and / or collectively exhaustive.

(a) {P, L, A, H} **(b) {PL, BL, PA, BA, H}**

(c) {P, B, H} **(d) {B, PL, PH, BL, BH}**

(e) {PL, BL, PA, BH}

Solution:

(a) Mutually exclusive (b) Collectively exclusive

(c) Mutually exclusive (d) Collectively exclusive

(e) Collectively exclusive

8. In Exercise 6, suppose that one account is selected at random (each having an equal chance of selection). What is the probability that the account selected will (a) be m the average-risk category, (b) be a personal loan, (c) be a business loan

in the high-risk category, and (d) be a personal loan with a low risk?

Solution:

(a) Let 'A' be the event that the account selected will be in the average risk category.

$$P(A)=\frac{4500}{10000}$$

(b) Let 'B' be the event that the account selected will be a personal loan.

$$P(B)=\frac{7600}{10000}$$

(c) Let 'C' be the event that the account selected will be a business loan in the high risk. $P(C)=\frac{800}{10000}$

(d) Let 'A' be the event that the account selected will be a personal loan with a low risk. $P(D)=\frac{1600}{10000}$

9. **Child Care Alternatives In 1987, the U.S. Census Bureau estimated that 9.1 million children under age five required primary child care because of employed mothers. Table 13.7 indicates the child care alternatives and the Census Bureau's estimates of the number of children using each type. If a child from this group is selected at random, what is the probability that (a) the child is cared for in his or her home, (6) the child cares for himself or herself, and (c) the child is cared for at work by the mother?**

Solution:

Same as Q - 6, 7, 8

10. **Aging U.S. Population A U.S. Census Bureau study reveals that the average age the population la increasing. The bureau estimates that by the year 2000, there will 105.6 million households. Table 13.8 indicates projections regarding the age of the head of household. If m the year 2000 a household is selected at random, what is the probability that the head of household will be (a) of age 65 or older, (b) of age 26 - 34, (c) of are 35 or older, and (d) of age 45 or younger?**

Solution:

Same as Q - 6, 7, 8

11. **Cardiac Care in order So support its request for a cardiac intensive care unit, the emergency room at a major urban hospital has gathered data on the number of heart attack victims seen. Table 13.9 indicates the probabilities of different numbers of heart attack victims being treated in the emergency room on a typical day. For a given day, what is the probability that (a) five or fewer victims will be seen, (b) five or more victims will be seen, and (c) no more than seven victims will be seen?**

Solution:

Same as Q - 6, 7, 8

12. Fire Protection The number of fire alarms pulled each hour fluctuates in a particular city. Analysts have estimated the probability of different numbers of alarms per hour as shown in Table 13.10. In any given hour, what is the probability that (a) more than 8 alarms will be pulled, (6) between 8 and 10 alarms (inclusive) will be pulled, and (c) no more than 9 alarms will be pulled?

Solution:

Same as Q - 6, 7, 8

13. A card is to be drawn at random from a well-shuffled deck. What is the probability that the card will be (c) a lung or jack, (ft) a face card (Jack, queen, or king), (c) a 7 or a spade, and (d) a face card or a card from a red suit?

Solution:

Same as Q - 6, 7, 8s

14. A survey of 2,000 consumers was conducted to determine their purchasing behavior regarding two leading soft drinks- It was found that during the past month 800 persons had purchased brand A, 300 had purchased brand J3, and 100 had purchased both brand A and brand-B. If a person is selected at random from this group (assuming equal chance of selection for each person), what is the probability that the person (a) would have purchased brand A during the past month, (6) would have purchased brand B but not brand A, (c) would have purchased brand. A, brand B, or both, and (d) would not have purchased either brand?

Solution:

$n(S) = 2000,\ n(A) = 800,\ n(B) = 300,\ n(A \cap B) = 100$

$n(A') = n(S) - n(A) = 2000 - 800 = 1200$

$n(B') = n(S) - n(B) = 2000 - 300 = 1700$

$$P(A) = \frac{n(A)}{n(S)} = \frac{800}{2000},\ P(B) = \frac{n(B)}{n(S)} = \frac{300}{2000}$$

$$P(A) = \frac{n(A')}{n(S)} = \frac{1200}{2000},\ P(B') = \frac{n(B')}{n(S)} = \frac{1700}{2000}$$

(a) $P(A) = \frac{n(A)}{n(S)} = \frac{800}{2000}$

(b) $P(A \cap B') = P(A).P(B') = \frac{300}{2000} \times \frac{1700}{2000} = \frac{510000}{4000000} = \frac{51}{400}$

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{800}{2000} + \frac{300}{2000} - \frac{100}{1000} = \frac{1000}{2000} = \frac{1}{2}$

(d) $P(A' \cap B') = P(A')\,P(B') = \frac{1200}{2000} \times \frac{1700}{2000} - \frac{2040000}{4000000} = \frac{204}{400} = \frac{51}{100}$

15. Vitamin C Research In recent years there has been much controversy about the

possible benefits of using supplemental doses of vitamin C. Claims have been made by proponents of vitamin C that supplemental doses will reduce the incidence of the common cold and influenza (flu). A test group of 3,000 persons received supplemental doses of vitamin C for a period of 1 year. During this period it was found that 800 such people had one or more colds, 250 people suffered from influenza, and 150 people suffered from both colds and influenza. If a person is selected at random from this test group (assuming equal likelihood of selection), what is the probability that the person (a) would have had one or more colds but not influenza, (b) would have had both colds and influenza, (c) would have had one or more colds but no influenza, or influenza but no colds, and (d) would have Buffered neither colds nor influenza?

Solution:

Same as Q - 14

16. The sample space for an experiment consists of five simple events E_1, E_2, E_3, E_4, and E_5, These events are mutually exclusive. The probabilities of occurrence of these events are P (E_1) = 20 P (E_2) = 15, P (E_3) .25, P (E_4) = 30 and P (E_6) .10. Several compound events can be denned for this experiment. They are $F = \{E_1, E_2, E_3\}$, $G = \{E_1, E_3, E_5\}$, $H = \{E_4, E_5\}$.

Determine

(a) P (F), (b) P (G). (c) P (H), (d) P (G′), (e) P (F G),(f) P $(G \cup H)$,(g) P $(F \cap H)$, and (h) P $(F \cap G)$.

Solution:

(a) P (F) $= P\{E_1, E_2, E_3\} = (E_1)\,P(E_2)\,P(E_3) = (.20)\,(.15)\,(.25) = 0.0075$

(b) P (G) $= P\{E_1, E_3, E_5\} = (E_1)\,P(E_3)\,P(E_5) = (.20)\,(.25)\,(.10) = 0.0050$

(c) P (H) $= P\{E_4, E_5\} = (E_4)\,P(E_5) = (0.30)\,(0.10) = 0.030$

(d) P (G′) $= 1 - P(G) = 1 - 0.0050 = 0.0050$

(e) $P(F \cup G) = P(F) + P(G) = 0.0075 + 0.0050 = 0.0125$

(f) $P(G \cup H) = P(G) + P(H) = 0.0050 + 0.0030 = 0.0080$

(g) $P(F \cap H) = P(F)\,P(H) = (0.0075)\,(0.030) = 0.000225$

(h) $P(F \cap G) = P(F)\,P(G) = 0.0075)\,(0.0050) = 0.0000375$

17. The sample space for an experiment consists of four simple events E_1, E_2, E_3 and E_4 which are mutually exclusive.The probabilities of occurrence of these events are P (E_1) = . 2, P (E_2) = .1, P (E_3) = .4, and P (E_4) = .3. Several compound events can be defined for this experiment. They include.

$A = \{E_1, E_2, E_3\}$ $B = \{E_2, E_4\}$, $C = \{E_1, E_3, E_4\}$

Determine(a) P (A), (b) P (A′), (c) P (B), (d) P (C), (e) P (C′), (f) $(A \cup B)$, (g) P $(B \cup C)$, and (h) P $(A \cap C)$.

Solution:

Same as Q - 16

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Solved Section 4.4

1. The probability that a machine will produce a defective part equals.15 If the process is changed by statistical independence, what is the probability that (a) two items in succession will not be defective, (b) the first three items are not defective and the fourth is detective, and (c) five consecutive items will not be defective?

Solution:

$P(D) = 0.15, P(D') = 1 - 0.15 = 0.85$

(a) $(D_1' \cap D_2') = P(D_1')P(D_2') =$ (0.85) (0.85) = 0.7225

(b) $P(D_1' \cap D_2' \cap D_3' \cap D_4') =$ (0.85)2(0.85)(0.85)(0.15)= 0.0921

(c) $P(D_1' \cap D_2' \cap D_3' \cap D_4' \cap D_5') = P(D_1')P(D_2')P(D_3')P(D_4')P(D_5')$

$= (0.885)(0.85)(0.85)(0.85) = 0.4437$

2. IRS Audit An income tax return be audited by the federal government and/ or by the state. The probability that an individual tax return will be audited by the federal government is 03. The probability that it will be audited by the state is .04. Assume that audit decisions are made independent of one another at the federal and state levels.

(a) What is the probability of being audited by both agencies?

(b) What is the probability of a state audit but not a federal audit?

(c) What is the probability of not being audited?

Solution:

Let 'A' be the event that an individual tax return will be audited by federal government and 'B' bet the event that tax return will be audited by the state.

$P(A) = 0.03, P(A') = 1 - P(A) = 1 - 0.03 = 0.97$

$P(B) = 0.04, P(B') = 1 - P(B) = 1 - 0.04 = 0.96$

(a) $P(AB) = P(A)P(B) = (0.03)(0.04) = 0.0012$

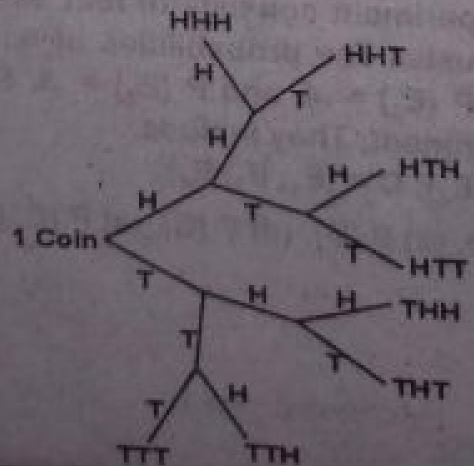
(b) $P(A \cap B') = P(A)P(B') = (0.03)(0.96) = 0.0291$

(c) $P(A' \cap B') = P(A')P(B') = (0.97)(0.96) = 0.9312$

3. A coin is weighted such that P (H) = .45 and P (T) = .55. Construct a probability tree denoting all possible outcomes if the coin is tossed three times. What is the probability of two tails in three tosses? Two heads?

Solution:

$P(H) = 0.45, P(T) = 0.55$



P (2 tails in three tosses) = {T TH, T H T, HTT} = $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$

P (2 heads) = {HHT, HTH, THH} = $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$

Q - 3, 4, 5, are same as Q - 1, 2

4. **Five cards are selected at random from a deck of 52. If the drawn cards are not replaced in the deck, what is the probability of selecting an ace, king, ace, jack, and ace, in that order?**

Solution:

Q - 3, 4, 5, are same as Q - 1, 2

5. **Table 13.12 summarizes the results of a recent survey of attitudes regarding nuclear war. The question asked was, "How likely do you believe it is that a nuclear war will occur during the next 10 years?" If a respondent is selected at random from the sample of 8,000, what are the following probabilities?**
   - **(a) The respondent is 30 years or older.**
   - **(b) The respondent believes nuclear war is "likely."**
   - **(c) The respondent is between the ages of 30 and 39 and believes that nuclear war is "very likely."**
   - **(d) The respondent is between the ages of 20 and 39 and believes that nuclear war is "unlikely."**
   - **(e) The respondent believes that nuclear war is "unlikely," given that he or she is between the ages of 20 and 29.**
   - **(f) The respondent is 40 years of age of older, given that he or she believes nuclear war is "unlikely."**

Solution:

Q - 3, 4, 5, are same as Q - 1, 2

6. **A television game show contestant has earned the opportunity to win some prizes. The contestant is shown 10 boxes, 4 of which contain prizes. If the contestant is allowed to select any 4 of the boxes, what is the probability that (a) four prizes will be selected, (b) no prizes will be selected, and (c) the first 3 boxes selected contain no prizes but the 4th box does?**

Solution:

Prizes	No Prize	Total
4	6	10

N = 10, n = 4

(a) $P(X = 4 \text{ Prizes}) = \frac{C_4^4 \times C_0^6}{10_{C_4}} = \frac{1\times 1}{210} = \frac{1}{210}$

(b) $P(X = 0 \text{ Prizes}) = \frac{C_0^4 \times C_{C_4}^6}{C_4^{10}} = \frac{1\times 15}{210} = \frac{15}{210}$

(c) P(First three contains no prize and 4th box contain prize)

$$= \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8} \times \frac{6}{7}$$

$$= \frac{144}{5040}$$

7. **For the previous exercise, draw a probability tree which summarizes the different out comes possible when selecting 4 boxes at random and their associated probabilities. What is the probability that at least one prize will be won? Exactly one prize?**

Solution:

Same as Q - 6

8. **Refer to Example 34. Construct a probability tree which summarizes all outcomes and their probabilities for the selection of two balls.**

Solution:

Same as Q - 6

9. **Given that cards are selected at random, without replacement, from a standard 52-card deck, determine the probability that (a) the first 2 cards are hearts, (b) the first is a spade, the second a club, the third a heart, and the fourth a diamond, (c) 3 aces are selected in a row, and (d) no aces are included in the first 4 cards.**

Solution:

Same as Q - 6

10. **A graduating class consists of 52 percent women and 48 percent men. Of the men, 20 percent are engineering majors. If a graduate is selected at random from the class, what is the probability the student is a male engineering major? A male majoring in some thing other than engineering?**

Solution:

% Women	% Men	Total
0.52	0.48	1

P (Men Engineering majors) = 0.48 20% = 0.48 0.20 = 0.096

P (Male other than Engineering majors) = 1 - 0.096 = 0.0904

11. **Table 13.13 summarizes the results of a recent health survey. Persona who were suffering from heart disease, cancer, or diabetes were asked whether there had been any known, history of the disease in their family. If ft person is selected at random from this sample of 4,000, what is the probability that:**
   - (a) **The person has cancer?**
   - (b) **The person had a family history of their particular disease?**
   - (c) **The person has cancer and had no family history of the disease?**
   - (d) **The person has diabetes, given that the person selected has a family history of their disease.**
   - (e) **The person has no family history of the disease, given that the person selected has heart disease.**

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Solution:

(a) Let 'A' be the event that a person has cancer.

$$P(A) = \frac{1200}{4000} = 0.30$$

(b) Let 'B' be the event that a person had a family history of their particular disease.

$$P(B) = \frac{1700}{4000} = 0.425$$

(c) Let 'C' be the event that a person has cancer and no family history the disease.

$$P(C) = \frac{760}{4000} = 0.19$$

(d) Let 'D' be the event that a person has diabetes, and let 'E' given that the person selected has a family history of their disease.

$$P(D) = \frac{P(D \cap E)}{P(E)} = \frac{389/4000}{1700/4000} = \frac{380}{1700} = 0.22$$

(e) Let 'F' be the event that a person has no family history of the disease, and let 'G' given that the person selected has heart disease.

$$P\left(F/G\right) = \frac{P(F \cap G)}{P(G)} = \frac{920/4000}{2300/4000} = \frac{920}{2300} = 0.40$$

12. **A sample of 800 parts has been selected from three product lines and inspected by the quality control department. Table 13.14 summarizes the results of the inspection. If a part is selected at random from this sample, what is the probability that.**

(a) **The part is of the product 1 type?**
(b) **The part is unacceptable?**
(c) **The part is an acceptable unit of product 3?**
(d) **The part is an unacceptable unit of product I?**
(e) **The part is acceptable, given that the selected part is a unit of product 2?**
(f) **The part is product 1, given that the selected part is acceptable?**
(g) **The part is product 3, given that the selected part is unacceptable?**

Solution:

Same as Q - 11

13. **A pool of applicants for a welding job consists of 30 percent women and 70 percent men. f the women, 60 percent have college degrees. Of the men, 40 percent have college degrees. What is the probability that a randomly selected applicant will be (a) a woman, with a college degree and (b) a man without a college degree?**

Solution:

Same as Q - 11

14. Suppose that B and F are events where P (E) - .4, P (F) = .3, and P (E $\cup$ F) = .6. Determine (a) P (E $\cap$ F), (b) P (E\F), and (c) P (F\E).

Solution:

P (E) = 0.4, P (F) = 0.3, P (E $\cup$ F) = 0.6

(a) P (E $\cap$ F) = P (E) P (F) = (0.4) (0.3) = 0.12

(b) $P\left(E/F\right)=\frac{P(E\cap F)}{P(F)}=\frac{0.12}{0.3}=0.4$

(c) $P\left(F/E\right)=\frac{P(E\cap F)}{P(E)}=\frac{0.12}{0.4}=0.3$

15. Suppose that G and H are events where P(G) = 0.25, P(H) .45, and P(G $\cup$ H) = .55. Determine (a) P (G $\cap$ H), (b) P (G\H), and (c) P (H\G).

Solution:

Same as Q - 14

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Chapter - 5

Probability Distributions

Solved Section 5.1

1. Given the following random variables for a series of experiments, which are discrete and which are continuous?
   (a) The weights of students at a high school
   (b) The number of cigarettes smoked on a daily basis
   (c) The body temperature of a person at any given time
   (d) The length of a newborn baby
   (e) The number of applications for welfare received each day by a social agency
   (f) The amount of water used by a community each day
   (g) The number of grains of a and on a given beach each day
   (h) The length of life of a size AA battery

Solution:

(a) Continuous Variable (b) Discrete Variable
(c) Continuous Variable (d) Continuous Variable
(e) Discrete Variable (f) Continuous Variable
(g) Discrete Variable (h) Continuous Variable

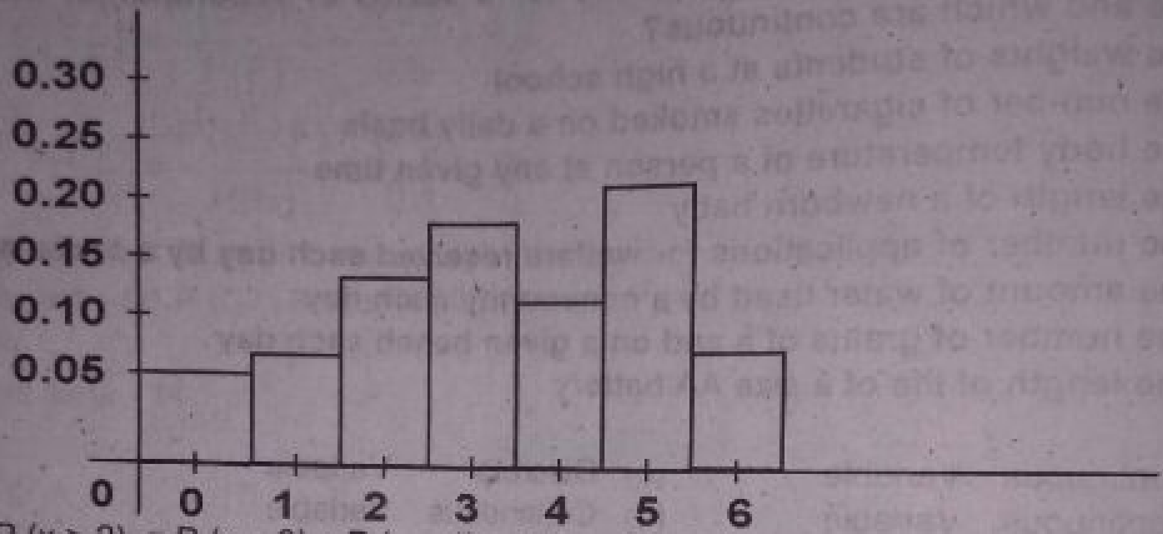
2. Public Works The director of public works for a New England city has checked the city records to determine the number of major snowstorms which have occurred in each of the last 50 years. Table 14.10 presents a frequency distribution summarizing the findings.
(a) Construct the probability distribution for this study.
(b) Draw a histogram for this distribution.
(c) What is the probability that there will be more than two major storms in a given year? Three or fewer?

Solution: (a)

X	f	P(x)
0	3	$\frac{3}{60} = 0.050$
1	5	$\frac{5}{60} = 0.083$
2	10	$\frac{10}{60} = 0.167$
3	13	$\frac{13}{60} = 0.217$
4	8	$\frac{8}{60} = 0.133$

5	16	$\frac{16}{60} = 0.267$
6	5	$\frac{5}{60} = 0.083$
-	$\Sigma f = 60$	$\Sigma p(x) = 1$

(b)



(c) $P(x > 2) = P(x = 3) + P(x = 4) + P(x = 5) + P(x = 6)$

$= 0.217 + 0.133 + 0.267 + 0.083 = 0.70$

$P(x \;\; 3) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)$

$= 0.050 + 0.083 + 0167 + 0.217 = 0.517$

3. Fire Protection The fire chief for a small volunteer fire department has compiled data on the number of false alarms called in each day for the past 360 days. Table 14.11 presents a frequency distribution summarizing the findings.

(a) Construct the probability distribution for this study.*

(5) Draw a histogram for the distribution.

(c) What is the probability that fewer than four false alarms will be called in on any given day? .Three or more?

Solution:

Same as Q - 2

4. Quality Control Production runs for a particular product are made in lot sizes of 100 units. Each unit is inspected to ensure that it is not defective in any way. The number of defective units per run seems to be random A quality control engineer has gathered data on the number of defective units for each of the last 50 production runs. Table 14.12 presents a frequency distribution summarizing the findings.

(a) Construct the probability distribution for this study

(b) Draw a histogram for the distribution.

(c) What is the probability that a production run will result in fewer than 10 defective units? More than 10?

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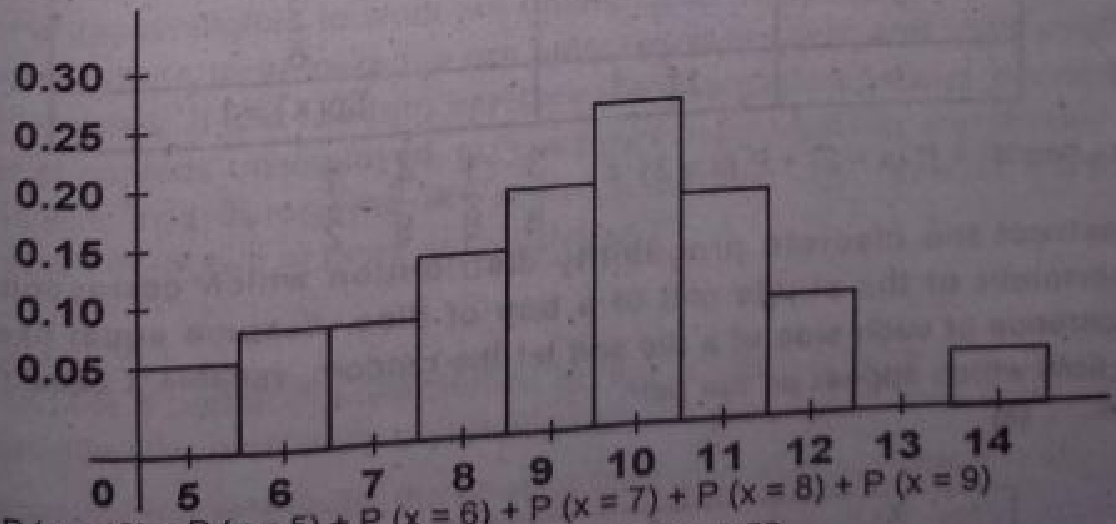
Solution:

X	f	P(x)
5	3	$\frac{3}{50} = 0.06$
6	4	$\frac{4}{50} = 0.08$
7	4	$\frac{4}{50} = 0.08$
8	6	$\frac{6}{50} = 0.12$
9	9	$\frac{9}{50} = 0.18$
10	11	$\frac{11}{50} = 0.22$
11	8	$\frac{8}{50} = 0.16$
12	4	$\frac{4}{50} = 0.08$
13	0	$\frac{0}{50} = 0$
14	1	$\frac{1}{50} = 0.02$
-	$\Sigma f = 50$	$\Sigma p(x) = 1$

(b)



(c)

$P(x < 10) = P(x = 5) + P(x = 6) + P(x = 7) + P(x = 8) + P(x = 9)$

$= 0.06 + 0.08 + 0.08 + 0.12 + 0.18 = 0.52$

$P(x > 10) = P(x = 11) + P(x = 12) + P(x = 13) + P(x = 14)$

$= 0.16 + 0.08 + 0 + 0.02 = 0.26$

5. **Drunken Driving Local police have instituted a roadblock program for checking the sobriety of drivers. Cars are selected at random and drivers checked for signs of excessive drinking. If it is suspected that a driver baa had too much to drink, standard tests of sobriety are administered. In reviewing the success of the program, a police lieutenant has compiled data on 150 roadblock efforts. The lieutenant is interested in the number of "hits," or drivers found to be legally drunk for each roadblock effort. Table 14.13 summarizes the findings.**
   - (a) **Construct the probability distribution for this study.**
   - (b) **Draw a histogram for the distribution.**
   - (c) **What is the probability a roadblock effort will identify any drunken drivers? Five or more?**

Solution:

Same as Q - 4

6. **Construct the discrete probability distribution which corresponds to the experiment of tossing a fair coin three times. Let the random variable X equal the number of heads occurring in three tosses. What is the probability of two or more heads?**

Solution:

X	f	P(x)
0	1	$\frac{1}{8}$
1	3	$\frac{3}{8}$
2	3	$\frac{3}{8}$
3	1	$\frac{3}{8}$
-	$\Sigma f = 1$	$\Sigma p(x) = 1$

P (x $\geq$ heads) = P (x = 2) + P (x = 3) = $\frac{3}{8}+\frac{1}{8}=\frac{4}{8}=\frac{1}{2}$

7. **Construct the discrete probability distribution which corresponds to the experiment of the single roll of a boy of dice. Assume equal likelihood of occurrence of each side of a die and let the random, variable X equal the sum of the dots which appear on the pair.**

Solution: (a)

X	f	P(x)

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2	1	$\frac{1}{36}$
3	2	$\frac{2}{36}$
4	3	$\frac{3}{36}$
5	4	$\frac{4}{36}$
6	5	$\frac{5}{36}$
7	6	$\frac{6}{36}$
8	5	$\frac{5}{36}$
9	4	$\frac{4}{36}$
10	3	$\frac{3}{36}$
11	2	$\frac{2}{36}$
12	1	$\frac{1}{36}$
-	$\Sigma f = 36$	$\Sigma p(x) = 1$

8. **Unemployment Unemployment statistics within a western state indicate that 6 percent of those eligible to work are unemployed. Suppose that an experiment is conducted where three persons are selected at random and their employment status is noted. If the random variable for this experiment is denned as the number of persons unemployed, (a) construct the probability distribution for this experiment, and determine the probability that (b) none of the three is unemployed or (c) two or more are employed.**

Solution:

Same as Q - 7

9. **Table 14.14 is ft probability distribution for the random variable X.**
   (a) Determine the probability distribution for the random variable X^2.
   (b) Determine the probability distribution for the random variable X + 1.

Solution:

(a)

x^2	P(X)
$1^2 = 1$	.15
$2^2 = 4$	.20
$3^2 = 9$	.30
$4^2 = 16$	.25
$5^2 = 25$	.15

(b)

X + 1	P(X)
1+1 = 2	.15
2+1 = 3	.20
3+1 = 4	.30
4+1 = 5	.25
5+1 = 6	.15

10. Given that a random variable can assume values of 0, 1, 2, and 3, which of the following cases satisfy the conditions for being probability distributions?

(a) $P(X=0) = \frac{1}{6}, P(X=1) = \frac{1}{3}, P(X=2) = 0, P(X=3) = \frac{1}{2}$

(b) $P(X=0) = .2, P(X=1) = .3, P(X=2)\ .2, P(X=3) = .1$

(c) $P(X=0) = .1, P(X=1) = .25, P(X=2) = .15, P(X=3) = .2.$
$P(X=4) = .3$

(d) $P(X=0) = .18, P(X=1) = .23, P(X=2) = .26, P(X=3) = .33$

Solution:

(a) We have to prove that

$\Sigma P(X) = 1$

L.H.S $= \Sigma P(X) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= \frac{1}{6} + \frac{1}{3} + 0 + \frac{1}{2} = 1 = \text{R.H.S}$

Hence it is a probability distribution.

(b) We have to prove that

$\Sigma P(X) = 1$

L.H.S $= \Sigma P(X) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= 0.2 + 0.3 + 0.2 + 0.1 = 0.8 \neq 1$

Hence it is not a probability distribution.

(c) We have to prove that

$\Sigma P(X) = 1$

L.H.S $= \Sigma P(X) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= 0.1 + 0.25 + 0.15 + 0.2 + 0.3 = 1 = \text{R.H.S}$

Hence it is a probability distribution.

(d) We have to prove that

$\Sigma P(X) = 1$

L.H.S $= \Sigma P(X) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= 0.18 + 0.23 + 0.26 + 0.33 = 1 = \text{R.H.S}$

Hence it is a probability distribution.

Solved Section 5.1

For the following data sets, compute:

(a) Mean (b) Median (c) Mode

(d) Range (e) Standard deviation

1. **20, 40, 60, 80, 100, 120, 140, 160, 180, 200.**

Solution:

(a) Mean $= \dfrac{20 + 40 + 60 + 80 + 100 + 120 + 140 + 160 + 180 + 200}{10}$

$= \dfrac{1100}{10} = 110$

(b) Median $= \left(\dfrac{n+1}{2}\right)^{th}$ item $= \left(\dfrac{10+1}{2}\right)^{th}$ item $= 5.5^{th}$ item

So, median $= \dfrac{100+120}{2} = 110$

(c) No mode

(d) $X_m = 200, X_0 = 20$

Range $= X_m = X_0 = 200 - 20 = 180$

(e) Standard deviation

x	x^2
20	400
40	1600
60	3600
80	6400
100	10000
120	14400
140	19600
160	25600
180	32400
200	40000
$\Sigma X = 110$	$\Sigma x^2 = 154000$

$S = \sqrt{\dfrac{\Sigma X^2}{n} - \left(\dfrac{\Sigma X}{n}\right)^2} = \sqrt{\dfrac{154000}{10} - \left(\dfrac{1100}{10}\right)^2}$

$= \sqrt{15400 - 12100} = \sqrt{3300} = 57.44$

2. **5, 10, 40, 20, 35, 20, 50, 0, 5, 15, 25, 30, 20, 40, 45**

Solution:

0, 5, 5, 10, 15, 20, 20, 20, 25, 30, 35, 40, 40, 45, 50

(a) Mean: $\frac{0+5+5+10\ 15+20+20+20+25+30+35+40+40+45+50}{15}$

Mean $= \frac{360}{15} = 24$

(b) Median $= \left(\frac{n+1}{2}\right)^{th}$ item $= \left(\frac{15+1}{2}\right)^{th}$ item $=$ 8th item

So, Median = 20

(c) Mode = 20

(d) Range $= X_m - X_0 = 50 - 0 = 50$

x	x^2
0	0
5	25
5	25
10	100
15	225
20	400
20	400
20	400
25	625
30	900
35	1225
40	1600
40	1600
45	2025
50	2025
$\Sigma x = 360$	$\Sigma x^2 = 154000$

$$S = \sqrt{\frac{\Sigma X^2}{n} - \left(\frac{\Sigma X}{n}\right)^2} = \sqrt{\frac{12050}{15} - \left(\frac{360}{15}\right)^2}$$

$$= \sqrt{803.33 - 576} = \sqrt{227.33} = 15.08$$

3. 30, 36, 28, 18, 42, 10, 20, 52

Solution:

10, 18, 20, 28, 30, 36, 42, 52

(a) Mean $= = \frac{10+18+20+28+30+36+42+52}{8} = \frac{236}{8} = 29.5$

(b) Median $= \left(\frac{n+1}{2}\right)^{th}$ item $= \left(\frac{8+1}{2}\right)^{th}$ item $= 4.5^{th}$ item

So, Median = $\frac{28+30}{2}=\frac{58}{2}=29$

(c) No mode

(d) Range = $X_m - X_0 = 52 - 10 = 42$

x	x^2
10	100
18	324
20	400
28	784
30	900
36	1296
42	1764
52	2704
$\Sigma x = 236$	$\Sigma x^2 = 8272$

$$S=\sqrt{\frac{\Sigma X^2}{n}-\left(\frac{\Sigma X}{n}\right)^2}=\sqrt{\frac{8272}{8}-\left(\frac{236}{8}\right)^2}$$

$$=\sqrt{1034-870.25}=\sqrt{163.75}=12.80$$

Q. 4, 5, 6

Solution:

Same as Q - 1, 2, 3

7. **Determine the mean, median, and mode for the following frequency distribution.**

Solution:

X	F	FX	C.B	C.F
20	8	160	15-25	8
30	12	360	25-35	20
40	$16 \rightarrow F_1$	400	35-45	30 median class
50	$16 \rightarrow F_m$	800	45-55	46
80	$6 \rightarrow F_2$	480	55-85	52
-	$\Sigma F = 52$	$\Sigma FX = 2200$	-	-

Mean = $\frac{\Sigma FX}{\Sigma F}=\frac{2200}{52}=42.31$

Median = $\ell=\frac{h}{f}\left(\frac{n}{2}-C\right)$, $\frac{n}{2}=\frac{52}{2}=26$

$\ell=35, h=10, f=10,\qquad c=20$

Median = $35+\frac{10}{10}(26-20)=35+1(6)=35+6=41$

$$\text{Mode} = \ell + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} + h$$

$$\ell = 45, F_m = 16, F_1 = 10, F_2 = 6, h = 10$$

$$\text{Mode} = 45 + \frac{(16-10)}{(16-10)+(16-6)} \times 10 = 45 + \frac{6}{6+10} x = 45 + \frac{6}{16} \times 10$$

$$= 45 + \frac{60}{16} = 45 + 3.75 = 48.75$$

8. Determine the mean, median, and mode for the following frequency distribution.

Solution:

Same as Q - 7

9. Forest Fire control table 14.18 summarizes data from an experiment in which the department of environmental management documented the number of forest fires reported each day over a 60 day period. Determine the mean, median, and mode for this data and interpret the meaning of each.

Solution:

X	F	FX
0	10	0
1	8	8
2	6	16
3	6	18
4	5	20
5	9	45
6	7	42
7	5	35
8	4	32
–	$\Sigma x = 60$	$\Sigma FX = 216$

$$\text{Mean} = \frac{\Sigma FX}{\Sigma F} = \frac{216}{60} = 3.6$$

Median = 4 (Middle value)

Mode = 0 (Value of X against highest frequency)

10. Absenteeism An employer has been concerned about absenteeism in her firm. Union - management relations have been strained in recent months due to an inability to reach agreement on a new contract for the 50 employees in the union. Table 14.19 summarizes data the employer has gathered on daily absenteeism over the past 30 workdays. Determine the mean, median, and mode for this data and interpret the meaning of each.

Solution:

Same as Q - 9

11. Table 14.20 presents a discrete probability distribution associated with the daily

demand for a product.

(a) Determine the mean daily demand.

(b) What is the standard deviation of daily demand?

Solution:

X	P(X)	XP(X)	X^2 P(X)
10	.08	0.8	8
20	.24	4.8	96
30	.28	8.4	252
40	.30	12.0	480
50	.10	5.0	250
-	**1.00**	**31**	**1086**

Mean = $\Sigma XP(X) = 31$

$$S = \sqrt{\Sigma X^2P(X) - (\Sigma XP(X))^2} = \sqrt{1086 - (31)^2} = \sqrt{125} = 11.18$$

12. A manufactured product consists of five electrical components. Each of the components has a limited lifetime. The company has tested the product to determine the reliability of the components. Table 14.21 presents a probability distribution where the random variable X indicates the number of components which fail during the first 100 hours of operation.

(a) What is the mean number of components which fail during the first 100 hours of operation?

(b) What is the standard deviation of the random variable X?

(c) If the product will continue to operate if no more than two components fail, what percentage of the manufactured parts will continue to operate during the first 100 hours?

Solution:

Same as Q - 11

13. Computer the respective means and standard deviations for the two distributions in table 14.22.

Solution:

x_1	$P(x_1)$	x_2	$P(x_2)$	$x_1P(x_1)$	$X_1^2P(X_1)$	$X_2P(X_2)$	$X_2^2P(X_2)$
500	1	0	0.050	500	250000	0	0
–	–	100	0.125	–	–	12.5	1250
–	–	300	0.200	–	–	60	18000
–	–	500	0.250	–	–	125	62500
–	–	700	0.200	–	–	140	98000
–	–	900	0.125	–	–	1125	101250
–	–	1000	0.050	–	–	50	50000
–	–		1.000	500	250000	500	331000
–	–	–					

$\overline{X_1} = \Sigma X_1 P(X_1) = 500$

$S_1 = \sqrt{\Sigma X_1^2 P(X_1) - [\Sigma X_1 P(X_1)]^2} = \sqrt{250000 - (500)^2}$

$= \sqrt{250000 - 250000} = \sqrt{0} = 0$

$\overline{X_2} = \Sigma X_2 P(X_2) = 500$

$S_2 = \sqrt{\Sigma X_2^2 P(X_2) - [\Sigma X_2 P(X_2)]^2} = \sqrt{331000 - (500)^2}$

$= \sqrt{331000 - 250000} = \sqrt{81000} = 284.60$

14. For Exercise 2 in Sec. 14.1, compute (a) the mean number of major snowstorms per year and (b) the standard deviation for the probability distribution.

Solution:

X	F	FX	FX^2
0	3	3	3
1	5	15	15
2	10	20	40
3	13	39	117
4	8	32	128
5	16	80	400
6	5	30	180
–	$\Sigma x = 60$	$\Sigma Fx = 219$	$\Sigma Fx^2 = 883$

$\text{Mean} = \frac{\Sigma FX}{\Sigma F} = \frac{219}{60} = 3.65$

$S = \sqrt{\frac{\Sigma FX^2}{\Sigma F} - \left(\frac{\Sigma FX}{\Sigma F}\right)^2} = \sqrt{\frac{883}{60} - \left(\frac{219}{60}\right)^2} = \sqrt{14.72 - 13.32} = 1.18$

15. For Exercise 3 in Sec. 14.1, compute (a) the mean number of false alarms per day and (b) the standard deviation.

Solution:

Same as Q - 14

16. For Exercise 3 in Sec. 14.1, compute (a) the mean number of defective units per production run and (b) the standard deviation.

Solution:

X	F	FX	FX^2

Scanned with CamScanner

5	3	15	75
6	4	24	144
7	4	28	196
8	6	48	384
9	9	81	729
10	11	110	1100
11	8	88	968
12	4	48	576
13	0	0	0
14	1	14	196
–	$\Sigma f = 50$	$\Sigma fX = 456$	$\Sigma fx^2 = 4368$

Mean = $\frac{\Sigma FX}{\Sigma F} = \frac{456}{50} = 9.12$

$$S = \sqrt{\frac{\Sigma FX^2}{\Sigma F} - \left(\frac{\Sigma FX}{\Sigma F}\right)^2} = \sqrt{\frac{4368}{50} - \left(\frac{456}{50}\right)^2}$$

$$= \sqrt{87.36 - 83.17} = \sqrt{4.18} = 2.05$$

17. **For Exercise 3 in Sec. 14.1, compute (a) the mean number of drunken drivers identified per roadblock and (b) the standard deviation.**

Solution:

Same as Q - 16

18. **Given a data set consisting of n items $x_1, x_2, \ldots, x_n$ with means x, show that the sum of deviations from the mean equals zero.**

Solution:

Same as Q - 16

Solved Section 5.3

1. **Determine which of the following random variables are not variables in a Bernoulli process.**

(a) X = the number of heads in the toss of a coin 20 times

(b) X = the heights of 10 students selected at random

(c) X = the number of 6's which appear in five rolls of a pair of dice

(d) X = scores earned by 100 different students on a standardized test

(e) X = the closing price of a stock for 10 randomly selected days

(f) X = the number of arrivals per hour at an emergency room observed for 20 randomly selected hours of operation

(g) X = the number of false alarms in a sample of 10 fire alarms where the probability that any alarm is a false alarm equals .18

Solution:

(a) Bernoulli Variable
(b) Bernoulli Variable
(c) Bernoulli Variable
(d) Not Bernoulli Variable
(e) Not Bernoulli Variable
(f) Not Bernoulli Variable
(g) Bernoulli Variable

2. A fair coin is to be flipped 4 times. What is the probability that exactly two heads will OCCUF? Pour heads? Two or more heads?

Solution:

$$N = 4, P = \frac{1}{2}, \quad q = \frac{1}{2}$$

$$P(x=2) = ?, P(x=4) = ? P(x \geq 2) = ?$$

$$P(x=2) = C_2^4\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = (6)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) = \frac{3}{8} = 0.375$$

$$P(x=4) = C_4^4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0 = (1)\left(\frac{1}{16}\right)(1) = \frac{1}{16} = 0.0625$$

$$P(x \geq 2) = P(x=2) + P(x=3) + P(x=4)$$

$$= C_2^4\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 + C_3^4\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3 C_3^4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0$$

$$= (6)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) + (4)\left(\frac{1}{8}\right)\left(\frac{1}{2}\right) + (1)\left(\frac{1}{16}\right)(1)$$

$$= \frac{6}{16} + \frac{4}{16} + \frac{1}{16} = \frac{11}{16} = 0.6875$$

3. A fair die will be rolled 4 times. What is the probability that exactly two is will occur? Fewer than four Is?

Solution:

$$n = 4, P = \frac{1}{6}, q = 1 - P = 1 - \frac{1}{6} = \frac{5}{6}$$

$$P(x=1) = C_1^4\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^3 = (4)\left(\frac{1}{6}\right)\left(\frac{125}{216}\right) = \frac{500}{1296} = 0.39$$

$$P(x<1) = P(x=0) = C_0^4\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^4 = (1)(1)\left(\frac{625}{1296}\right) = \frac{625}{1296} = 0.48$$

4. Drunken Driving A state has determined that of all traffic accidents in which a fatality occurs, 70 percent involve situations m which at least one driver has been drinking. If a sample of four fatal accidents is selected at random, construct the binomial distribution where the random variable X equals the number of accidents m which at least one driver was drinking.

Solution:

$P = 70\% = 0.70, q = 1 - P = 1 - 0.70 = 0.30, n = 4 \quad x = 0,1,2,3,4$

$P(X = x) = C_x^n P^x q^{n-x} = C_x^4 (0.70)^x (0.30)^{4-x}$ where $x = 0,1,2,3,4$

5. **Couch Potatoes It has been determined that 80 percent of all American households have at least one television set. If five residences are selected at random, construct the binomial distribution where the random variable X equals the number of residences having at least one television.**

Solution:

$P = 80\% = 0.80$, $q = 1 - P = 1 - 0.80 = 0.20$, $n = 5$

$x = 0, 1, 2, 3, 4, 5.$ $\quad P(X = x) = C_x^n P^x q^{n-x}$

$= C_x^5 (0.80)^x (0.20)^{5-x}$ where $x = 0, 1, 2, 3, 4, 5.$

6. **A firm which conducts consumer surveys by mail has found that 30 percent of those families receiving a questionnaire will return it. In a survey of 10 families, what is the probability that exactly five families will return the questionnaire? Exactly 6 families? Ten families?**

Solution:

$P = 30\% = 0.30, q = 1 - P = 1 - 0.30 = 0.70, n = 10$

$P(x = 5) = C_5^{10} (0.30)^5 (0.70)^5 = 0.10$

$P(x = 6) = C_6^{10} (0.30)^6 (0.70)^4 = 0.04$

$P(x = 10) = C_{10}^{10} (0.30)^{10} (0.70)^0 = 0.000059$

7. **A student takes a true-false examination which consists of 10 questions. The student knows nothing about the subject and chooses answers at random. Assuming independence between questions and a probability of -9 of answering any question correctly, what is the probability that the student will pass the test (assume that passing means getting seven or more correct)? If the test contains 20 questions, does the probability of passing change (14 or more correct)?**

Solution:

$n = 10$, $P = 0.90$, $q = 1 - P = 1 - 0.90 = 0.10$

$P(x \geq 7) = P(x = 7) + P(x = 8) + P(x = 9) + P(x = 10)$

$= C_7^{10} (0.90)^7 (0.10)^3 + C_8^{10} (0.90)^8 (0.10)^2 + C_9^{10} (0.90)^9 (0.10)^1 + C_{10}^{10} (0.90)^{10} (0.10)^1$

$= 0.0574 + 0.1937 + 0.3874 + 0.3487 = 0.9872$

$n = 20$, $P = 0.90$, $q = 0.10$

$P(x \geq 14) = P(x = 14) + P(x = 15) + P(x = 16) + P(x = 17) + P(x = 18) + P(x = 19) + P(x = 20)$

$= C_{14}^{20} (0.90)^{14} (0.10)^6 + C_{15}^{20} (0.90)^{15} (0.10)^5 \ldots\ldots$

$\ldots\ldots + C_{16}^{20} (0.90)^{16} (0.10)^4 + C_{20}^{20} (0.90)^{20} (0.10)^0$

$= 0.0089 + 0.0319 + 0.0898 + 0.1901 + 0.2852 + 0.2702 + 0.1216 = 0.9977$

8. Immunization a particular influenza vaccine has been found to be 95 percent effective in providing immunity. In a random sample of five vaccinated people who have been exposed to this strain of influenza, what is the probability that none of the five will come down with the disease?

Solution:

P = 95% = 0.95, q = 1 - P = 1 - 0.95 = 0.05, n = 5

$P(x = 0) = C^5_0 (0.95)^0 (0.05)^5 = (1)(1)(0.00000313) = 0.000000313$

9. An urn contain 4 red balls, 2 green balls, and 4 blue balls. If 10 balls are selected at random with replacement between .each draw, what is the probability the exactly 4 red balls will be selected? What is the probability that exactly 2 green balls will be selected?

Solution:

Same as above

10. A manufacturing process produces defective parts randomly at a rate of 8 percent. In a sample of 10 parts, what is the probability that fewer than 2 will be defective?

Solution:

P = 8% = 0.08, q = 1 - P = 1 - 0.08 = 0.92 , n = 10

$P(x < 2) = P(x = 0) + P(x = 1)$

$= C^{10}_0 (0.08)^0 (0.92)^{10} + C^{10}_1 (0.08)^1 (0.91)^9 = 0.4344 + 0.3777 = 0.8121$

11. In Exercise 10, what is the mean- number of defective parts espected to equal? What is the interpretation of this value? What is the standard deviation for this distribution?

Solution:

Same as above

12. In a local hospital 48 percent of all babies born are males. On a particular day five babies are born. What is the probability that four or more of the babies are males? What is the mean of this distribution for n = 5? What is the standard deviation?

Solution:

Same as above

13. Political Poll For an upcoming U,S. senatorial election, opinion polls indicate that 50 percent of the population support the Democratic candidate, 40 percent support the Republican candidate, and 10 percent are undecided. If a sample of five persons is selected at random what is the probability that at least four persons will be supportive of the Democratic candidate? Fewer than two persons will support the Democratic candidate?

Solution:

Same as above

14. Cigarette Smoking A local hospital has been conducting an experimental program to assist persons to stop cigarette smoking. Upon completion of the program,

participants realize a 60 percent success rate. If a sample of four past participants is selected at random, what is the probability that all four will have stopped smoking? At least three will have stopped?

Solution:

Same as above

15. Economic Recession An opinion poll reveals that 80 percent of the persons in one New England state believe that the area is suffering an economic recession. If a sample of six persons is selected at random in that New England State; what is the probability that exactly half of the persons will believe that a recession exists?

Solution:

Same as above

Solved Section 5.4

1. Given a normal distribution where $\mu = 50$ and $\mu = 8$, determine the z values corresponding to each of the fallowing values of the random variable: (a) 56, (6) 42, (c) 66, (d) 36, and (e) 75.

Solution:

$\mu = 50, \quad \sigma = 8$

(a) $x = 56 \quad z = \frac{x-\mu}{\sigma} = \frac{56-50}{8} = \frac{6}{8} = 0.75$

(b) $x = 42 \quad z = \frac{x-\mu}{\sigma} = \frac{42-50}{8} = \frac{8}{8} = -1$

(c) $x = 66 \quad z = \frac{x-\mu}{\sigma} = \frac{66-50}{8} = \frac{16}{8} = 2$

(d) $x = 36 \quad z = \frac{x-\mu}{\sigma} = \frac{36-50}{8} = -\frac{14}{8} = -1.75$

(e) $x = 75 \quad z = \frac{x-\mu}{\sigma} = \frac{75-50}{8} = \frac{25}{8} = 3.125$

2. Given a normal distribution where $\mu = 300$ and $\sigma = 60$; determine the z values corresponding to the following values of the random variable: (a) 320, (b) 160, (c) 365, (d) 430, and (e) 130,

Solution:

Same as Q - 1

3. Given a normal distribution where $\mu = 0.72$ and $\sigma = 0.08$, determine the z values corresponding to each of the following values of the random variable; (a) 0.84, (b) 0.62, (c) 0.50, (d) 0.90, and (e) 0.48.

Solution:

Same as Q - 1

4. **Given a normal distribution where $\mu = 18$ and $\sigma = 4.0$, determine the z values corresponding to each of the following values of the random variable: (a) 25, (b) 12.5, (c) 22.5, (d) 17.2, and (e) 19.8.**

Solution:

Same as Q - 1

5. **For the standard normal distribution determine:**

(a) P (z > 2.4) (b) P (z < 1.2)
(c) P (0.8 < z < 3.0) (d) P (- 2.3 $\leq$ z $\leq$ 2.8)

Solution:

$\mu = 0, \sigma = 1$

(a) P (z > 2.4) = 0.5 - 0.4918 = 0.0082
(b) P (z < 1.2) = 0.5 + 0.3849 = 0.8849
(c) P (0.8 < z < 3.0) = P (0 < z < 3.0) - P (0 < z < 0.8)
= 0.49865 - 0.2881 = 0.2106
(d) P (-2.3 z 2.8) = P (-2.3 z 0) + P (0 z 2.8)
= 0.4893 + 0.4974 = 0.9867

6. **For the standard normal distribution determine:**

(a) P (2 > - 1.6) (b) P (z < + 1.3)
(c) P (-1.7 < z < 0.3) (d) P (- 1.4 $\leq$ z $\leq$ 0.9)

Solution:

Same as Q - 5

7. **For the standard normal distribution determine:**

(a) P (z > 0.25) (6) P (z $\leq$ - 0.4)
(c) P (- 1.5 < z < 0.6) (d) P (- 1.3 $\leq$ z $\leq$ 0.45)

Solution:

Same as Q - 5

8. **For the standard normal distribution determine:**

(a) P (0.8 < z < 1.35) (b) P (- 1.35 < z < - 1.25)
(c) P (- 0.7 $\leq$ z $\leq$ - 0.25) (d) P (- 0.45 < z < 0.05)

Solution:

Same as Q - 5

9. **Given a random variable X which is normally distributed with a mean of 15 and standard deviation of 2.5, determine:**

(a) P (X $\geq$ 11.8) (b) P (X $\leq$ 17.8)
(c) P (9.6 $\leq$ X $\leq$ 16.1) (d) P (8.6 $\leq$ X $\leq$ 10.9)

Solution:

$\mu = 15, \sigma = 2.5$

(a) P (x $\geq$ 11.8) = (z $\geq$ - 1.4) = P (-1.4 $\leq$ z $\leq$ 0) + P (0 $\leq$ z $\leq \infty$)
= 0.4192 + 0.5 = 0.9192
(b) P (x $\leq$ 17.8) = P (z $\leq$ 1.12) = P (- $\infty \leq$ z $\leq$ 0) + P (0 $\leq$ z $\leq$ 1.12)

$= 0.5 + 0.3686 = 0.8686$

(c) $P(9.6 \leq X \leq 16.1) = P(-2.16 \leq z \leq 0.44)+P(-2.16 \leq z \leq 0)+P(0 \leq z \leq 0.44)$
$= 0.4846 + 0.1700 = 0.6566$

(d) $P(8.6 \leq X \leq 10.9) = P(-2.56 \leq z \leq -1.64) = P(-2.56 \leq z \leq 0) - P(-1.64 \leq z \leq 0)$
$= 0.4948 - 0.4495 = 0.0453$

10. **Given a random variable X which is normally distributed with a mean of 75 and standard deviation of 5, determine:**

(a) $P(X \geq 80)$ (b) $P(X < 78.5)$

(c) $P(66 \leq X \leq 72.5)$ (d) $P(80 < X < 88.6)$

Solution:

Same as Q - 9

11. **Given a random variable X which is normally distributed with a mean of 300 and standard deviation of 20, determine;**

(a) $P(X \geq 256)$ (b) $P(275 \leq X \leq 345)$

(c) $P(316 \leq X \leq 346)$ (d) $P(270 \leq X \leq 295)$

Solution:

Same as Q - 9

12. **Given a random variable X which is normally distributed with a mean of 160 and standard deviation of 8, determine:**

(a) $P(X \leq 150)$ (b) $P(148 \leq X \leq 154)$

(c) $(162 \leq 184)$ (d) $P(154 \leq X \leq 172)$

Solution:

Same as Q - 9

13. **Birth Weights The weights of newborn babies at a particular hospital have been observed to be normally distributed with a mean of 7.4 pounds and a standard deviation of 0.4 pound. What is the probability that a baby born in this hospital will weigh more than 8 pounds? Leas than 7 pounds?**

Solution:

Same as Q - 9

14. **The annual income of workers in one state is normally distributed with a mean of $17,500 and a standard deviation of $2,000. If a worker is chosen at random, what is the probability that the worker earns more than $16,000? Less than $12,000? Between $15,009 and $20,000?**

Solution:

Same as Q - 9

15. **A manufacturer has conducted a study of the lifetime of a particular type of light bulb. The study concluded that the lifetime, measured in hours; is a random variable with a normal distribution. The mean lifetime is 650 hours with a standard deviation of 100 hours. What is the probability that a bulb selected at random would have a lifetime between 500 and 800 hours? Greater than 900**

hours?

Solution:

Same as Q - 9

16. Grades on a national aptitude test have been found to be normally distributor? with a mean of 480 and a standard deviation of 75. What is the probability that a student selected at random will score between 450 and 540? Greater than 600?

Solution:

Same as Q - 9

17. In a large city the number of calls for police service during a 24 hour period seems to be random. The number of calls has been found to be normally distributed with a wean of 225 and a standard deviation of 30. What is the probability that for a randomly selected day the number of calls will be fewer than 300? More than 180?

Solution:

Same as Q - 9

18. Annual sales (in dollars) per salesperson for a copy machine manufacturer are normally distributed with a mean of $480,000 and standard deviation of $40,000. If a salesperson is selected at random, what is the probability his or her annual sales (a) exceed $600,000, (&) are between $400,000 and $500,000, (c) are leas than $450,000, or (d) are between $540,000 and $600,000?

Solution:

Same as Q - 9

19. Physical B Fitness A national physical fitness test has been administered. One element of the test measured the number of push-ups a person could do. For high school seniors, the number of push-ups was normally distributed with a mean of 12.6 and a standard deviation of 5.0. If a high school senior is selected at random, what is the probability that a senior could do (a) more than 16 push-ups, (b) more than 20 push-ups, (c) between -10 and 15 push-ups, and (d) fewer than 25 push-ups?

Solution:

Same as Q - 9

20. Seismology A seismologist has gathered data on the frequency of earthquakes around the world which measure 6.0 or greater on the Richter scale. The seismologist estimates that the number of earthquakes per year is normally distributed with a mean of 24 and standard deviation of 4.0. In any given year, what is the probability that there will be (a) more than 30 earthquakes, (b) fewer than 18 earthquakes, (c) more than 16 earth-quakes, and (d) between 20 and 25 earthquakes?

Solution:

Same as Q - 9

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Chapter - 6

DIFFERENTIATION

Solved Section 6.2

For the following exercises, find the indicated limits.

1. $\lim_{x\to 0}(3x^2 - 5x + 3)$

Solution:

$\lim_{x\to 0}(3x^2 - 5x + 3)$

$= 3\lim_{x\to 0} x^2 - 5\lim_{x\to 0}x + 3$

$= 3(0)^2 - 5(0) + 3$

$= 0 - 0 + 3$

$= 3$

2. $\lim_{x\to 2}(2x^3 - 10x)$

Solution:

$\lim_{x\to 2}(2x^3 - 10x)$

$= 2\lim_{x\to 2} x^3 - 10\lim_{x\to 2} x$

$= 2(2)^3 - 10(2)$

$= 2(8) - 20$

$= 16 - 20$

$= -4$

3. $\lim_{x\to 2}\left(\frac{x^3}{3} - 7x^2\right)$

Solution:

$\lim_{x\to 2}\left(\frac{x^3}{3} - 7x^2\right)$

$= \frac{1}{3}\lim_{x\to 2} x^3 - 7\lim_{x\to 2} x^2$

$= \frac{1}{3}(2)^3 - 7(2)^2$

$= \frac{8}{3} - 288$

$= -\frac{76}{3}$

4. $\lim_{x\to 3}\frac{2x-8}{x+4}$

Solution:

$\lim_{x\to 3}\frac{2x-8}{x+4}$

$= \frac{\lim_{x\to 3}(2x-8)}{\lim_{x\to 3}(x+4)}$

$= \frac{2(3)-8}{3+4}$

$= \frac{6-8}{3+4} = -\frac{2}{7}$

5. $\lim_{x\to 2}\frac{2x^2+3}{x^2+4x-2}$

Solution:

$\lim_{x\to 2}\frac{2x^2+3}{x^2+4x-2}$

$= \frac{\lim_{x\to 2}(2x^2+3)}{\lim_{x\to 2}(x^2+4x-2)}$

$= \frac{2(2)^2+3}{(2)^2+4(2)-2}$

$= \frac{8+3}{4+8-2} = \frac{11}{10}$

6. $\lim_{x \to -4} 250$

Solution:

$$\lim_{x \to -4} 250 = 250$$

7. $\lim_{x \to 0} 175$

Solution:

$$\lim_{x \to 0} 175 = 175$$

8. $\lim_{x \to -10} \frac{x^2+25}{x}$

Solution:

$$\lim_{x \to -10} \frac{x^2+25}{x}$$

$$= \frac{\lim_{x \to -10}(x^2+25)}{\lim_{x \to -10}(x)}$$

$$= \frac{(-10)^2+25}{-10}$$

$$= \frac{100+25}{-10}$$

$$= -\frac{125}{10}$$

$$= -\frac{25}{2}$$

9. $\lim_{x \to 1}(3x^3+2x^2+x)$

Solution:

$$\lim_{x \to 1}(3x^3+2x^2+x)$$

$$= \frac{\lim_{x \to 1}}{3} x^3 + 2\lim_{x \to 1} x^2 + \lim_{x \to 1} x$$

$$= 3(1)^3 + 2(1)^2 + 1$$

$$= 3+2+1 = 6$$

10. $\lim_{x \to 2}(-x^4+8x^2)$

Solution:

$$\lim_{x \to 2}(-x^4+8x^2)$$

$$= -\lim_{x \to 2} x^4 + 8\lim_{x \to 2} x^2$$

$$= -(2)^4 + 8(2)^2$$

$$= -16+32 = 16$$

11. $\lim_{x \to -2}(-x^3+5x^2+10)$

Solution:

$$\lim_{x \to -2}(-x^3+5x^2+10)$$

$$= -\lim_{x \to -2} x^3 + 5\lim_{x \to -2} x^2 + 10$$

$$= -(-2)^3 + 5(-2)^2 + 10$$

$$= -(-8) + 5(4) + 10$$

$$= 8+20+10 = 38$$

12. $\lim_{x \to 2}(5x^3+10x^2)$

Solution:

$$\lim_{x \to 2}(5x^3+10x^2)$$

$$= 5\lim_{x \to 2} x^3 + 10\lim_{x \to 2} x^2$$

$$= 5(2)^3 + 10(2)^2$$

$$= 5(8) + 10(4)$$

$$= 40+40 = 80$$

13. $\lim_{x \to 5}\left(\frac{4x-20}{3x^2-7x+5}\right)$

Solution:

$$\lim_{x \to 5}\left(\frac{4x-20}{3x^2-7x+5}\right)$$

$$= \frac{\lim_{x \to 5} 4(x-5)}{\lim_{x \to 5}(3x^2-7x+5)}$$

$$= \frac{4(5-5)}{3(5)^2-7(5)+5}$$

$$= \frac{4(0)}{75-35+5}$$

$$= \frac{0}{45} = 0$$

14. $\lim_{x\to -2}\left(\frac{10+x^2}{5-8x+2x^2}\right)$

Solution:

$$\lim_{x\to -2}\left(\frac{10+x^2}{5-8x+2x^2}\right)$$

$$=\frac{\lim_{x\to -2}(10+x^2)}{\lim_{x\to -2}(5-8x+2x^2)}$$

$$=\frac{10+(-2)^2}{5-8(-2)+2(-2)^2}$$

$$=\frac{10+(-2)^2}{5-8(-2)+2(-2)^2}$$

$$=\frac{10+4}{5+16+8}$$

$$=\frac{14}{29}$$

15. $\lim_{x\to -3}(6x^3+2x)(5x-10)$

Solution:

$$\lim_{x\to -3}(6x^3+2x)(5x-10)$$

$$=\left[6(-3)^3+2(-3)\right]\left[5(-3)-10\right]$$

$$=\left[6(-27)-6\right]\left[-15-10\right]$$

$$=(-162-6)(-25)$$

$$=(-168)(-25)=4200$$

16. $\lim_{x\to 5}\left[\left(\frac{x+3}{x+6}\right)(x^2-12)\right]$

Solution:

$$\lim_{x\to 5}\left[\left(\frac{x+3}{x+6}\right)(x^2-12)\right]$$

$$=\frac{\lim_{x\to 5}(x+3)}{\lim_{x\to 5}(x+6)}\lim_{x\to 5}(x^2-12)$$

$$=\frac{(5+3)}{(5+6)}\left[(5)^2-12\right]$$

$$=\frac{8}{11}(25-12)=\frac{8}{11}(13)$$

$$=\frac{104}{11}=9.45$$

17. $\lim_{x\to -4}\frac{x^2+8x-14}{2x+7}$

Solution:

$$\lim_{x\to -4}\frac{x^2+8x-14}{2x+7}$$

$$=\frac{\lim_{x\to -4}(x^2+8x-14)}{\lim_{x\to -4}(2x+7)}$$

$$=\frac{(-4)^2+8(-4)-14}{2(-4)+7}$$

$$=\frac{16-32-14}{-8+7}=\frac{-30}{-1}=30$$

18. $\lim_{x\to 1}\frac{x^2+3x-24}{x-3}$

Solution:

$$\lim_{x\to 1}\frac{x^2+3x-24}{x-3}$$

$$=\frac{\lim_{x\to 1}(x^2+3x-24)}{\lim_{x\to 1}(x-3)}$$

$$=\frac{(1)^2+3(1)-24}{1-3}$$

$$=\frac{1+3-24}{-2}=\frac{4-24}{-2}$$

$$=\frac{-20}{-2}=10$$

19. $\lim_{x\to -4}\frac{x^2-16}{x+4}$

Solution:

$$\lim_{x \to -4} \frac{x^2 - 16}{x + 4}$$

$$= \lim_{x \to -4} \frac{(x)^2 - (4)^2}{x + 4}$$

$$= \lim_{x \to -4} \frac{(x+4)(x-4)}{x+4}$$

$$= \lim_{x \to -4} (x - 4)$$

$$= -4 - 4 = -8$$

20. $= \lim_{x \to -9} \frac{81 - x^2}{9 + x}$

Solution:

$$= \lim_{x \to -9} \frac{81 - x^2}{9 + x}$$

$$= \lim_{x \to -9} \frac{(9)^2 - (x)^2}{(9 + x)}$$

$$= \lim_{x \to -9} \frac{(9 - x) - (9 + x)}{(9 + x)}$$

$$= \lim_{x \to -9} (9 - x) = 9 - (-9) = 9 + 9 = 18$$

21. $\lim_{x \to c} (4x^3 - 5x^2 + 10)$

Solution:

$$\lim_{x \to c} (4x^3 - 5x^2 + 10)$$

$$= 4c^3 - 5c^2 + 10$$

22. $\lim_{x \to -d} (x^2 - 2x + 3)$

Solution:

$$\lim_{x \to -d} (x^2 - 2x + 3)$$

$$= (-d)^2 - 2(-d) + 3 = d^2 + 2d + 3$$

For the following exercises, find the indicated limit and comment on the existence of any asymptotes.

23. $\lim_{x \to \infty} \frac{4}{x^2}$

Solution:

$$\lim_{x \to \infty} \frac{4}{x^2}$$

$$= \lim_{x \to \infty} \frac{4/x^2}{x^2/x^2} = \lim_{x \to \infty} \frac{4/x^2}{1}$$

$$= \frac{\lim_{x \to \infty} \left(4/x^2\right)}{\lim_{x \to \infty} (1)}$$

$$= \frac{4/\infty}{1} = \frac{0}{1} = 0 \quad \text{(Say 'a')}$$

For horizontal asymptote:

$$\lim_{x \to \infty} f(x) = a$$

$$\Rightarrow y = 0 \qquad \therefore a = 0$$

For vertical asymptote:

$$\lim_{x \to \infty} f(x) = \infty$$

$$\Rightarrow \frac{4}{x^2} = \infty$$

(Means numerator is not zero but denominator is zero).

$$\Rightarrow \quad x^2 = 0$$

$$x = 0$$

24. $\lim_{x \to \infty} \frac{5x - 3}{x + 10}$

Solution:

$$\lim_{x \to \infty} \frac{5x - 3}{x + 10}$$

$$= \lim_{x \to \infty} \frac{5x/x - 3/x}{x/x + 10/x}$$

$$= \lim_{x \to \infty} \frac{5 - 3/x}{1 + 10/x}$$

$$= \frac{5 - 3/\infty}{1 + 10/\infty} = \frac{5 - 0}{1 + 0} = \frac{5}{1} = 5 \quad \text{(Say a)}$$

For horizontal asymptote:

$$\lim_{x\to\infty} f(x) = a$$

$\Rightarrow \quad y = 5 \qquad \therefore\ a = 5$

For vertical asymptote:

$$\lim_{x\to\infty} f(x) = \infty$$

$$\Rightarrow \quad \frac{5x-3}{x+10} = \infty$$

$$\Rightarrow \quad x+10 = 0$$

$$x = -10$$

25. $\lim_{x\to\infty} \frac{-3x}{5x+100}$

Solution:

$$\lim_{x\to\infty} \frac{-3x}{5x+100}$$

$$= \lim_{x\to\infty} \frac{-3\frac{x}{x}}{5\frac{x}{x}+\frac{100}{x}}$$

$$= \lim_{x\to\infty} \frac{-3x}{5x+\frac{100}{x}} = \frac{-3}{5+\frac{100}{x}}$$

$$= \frac{-3}{5+\frac{100}{\infty}} = \frac{-3}{5+0} = -\frac{3}{5} \text{ (say a)}$$

For horizontal asymptote:

$$\lim_{x\to\infty} f(x) = a$$

$$\Rightarrow \quad y = -\frac{3}{5}$$

For vertical asymptote:

$$\lim_{x\to\infty} f(x) = \infty$$

$$\Rightarrow \quad \frac{-3x}{5x+100} = \infty$$

$$\Rightarrow \quad 5x + 100 = 0$$

$$5x = -100$$

$$x = -20$$

26. $\lim_{x\to\infty} \frac{8x+10}{-4x}$

Solution:

$$\lim_{x\to-\infty} \frac{8x+10}{-4x}$$

$$= \lim_{x\to-\infty} \frac{8\frac{x}{x}+\frac{10}{x}}{-4\frac{x}{x}}$$

$$= \lim_{x\to-\infty} \frac{8+\frac{10}{x}}{-4} = \frac{8+\frac{10}{\infty}}{-4}$$

$$= \frac{8+\frac{10}{\infty}}{-4} = \frac{8}{-4} = -2 \qquad \text{(Say a)}$$

For horizontal asymptote:

$$\lim_{x\to-\infty} f(x) = a$$

$$\Rightarrow \quad y = -2$$

For vertical asymptote:

$$\lim_{x\to-\infty} f(x) = \infty$$

$$\frac{8x+10}{-4x} = \infty$$

$$\Rightarrow \quad -4x = 0 = x = 0$$

27. $\lim_{x\to-\infty} 2x$

Solution:

$$\lim_{x\to-\infty} 2x = 2(-\infty) = \infty$$

No horizontal or vertical asymptotes.

28. $\lim_{x\to\infty} \frac{x^2+3}{x-4}$

Solution:

$$\lim_{x\to\infty} \frac{x^2+3}{x-4}$$

$$= \lim_{x\to\infty} \frac{\frac{x^2}{x^2}-\frac{3}{x^2}}{\frac{x}{x^2}-\frac{4}{x^2}}$$

$$= \lim_{x\to\infty} \frac{1-\frac{3}{x^2}}{\frac{1}{x}-\frac{4}{x^2}} = \frac{1-\frac{3}{\infty}}{\frac{1}{\infty}-\frac{4}{\infty}}$$

$$= \frac{1-3/\infty}{1/\infty - 4/\infty} = \frac{1-0}{0-0} = \frac{1}{0} = \infty$$

For horizontal asymptotes:

No Horizontal asymptotes.

For vertical asymptotes:

$$\lim_{x\to\infty} \frac{x^2+3}{x-4} = \infty$$

$$\frac{x^2+3}{x-4} = \infty$$

$$\Rightarrow \quad x-4=0$$

$$x=4$$

29. $\lim_{x\to\infty} \frac{-8x}{4x+1000}$

Solution:

$$\lim_{x\to\infty} \frac{-8x}{4x+1000}$$

$$= \lim_{x\to\infty} \frac{-8x/x}{4x/x + 1000/x}$$

$$= \lim_{x\to\infty} \frac{-8}{4+1000/x}$$

$$= \frac{-8}{4+1000/\infty}$$

$$= -\frac{8}{4} = -2 \quad \text{(Say a)}$$

For horizontal asymptotes:

$$\lim_{x\to\infty} f(x) = a$$

$$\Rightarrow \quad y = -2$$

For vertical asymptotes:

$$\lim_{x\to\infty} f(x) = \infty$$

$$\frac{-8x}{4x+1000} = \infty$$

$$\Rightarrow \quad 4x+1000=0$$

$$4x = -1000$$

$$x = -250$$

30. $\lim_{x\to-\infty} \frac{5x+10,000}{2x-5000}$

Solution:

$$\lim_{x\to-\infty} \frac{5x+10,000}{2x-5000}$$

$$= \lim_{x\to-\infty} \frac{5x/x + 10,000/x}{2x/x - 5000/x}$$

$$= \lim_{x\to-\infty} \frac{5+1000/x}{2-5000/x} = \frac{5+1000/\infty}{2-5000/\infty}$$

$$= \frac{5+0}{2-0} = \frac{5}{2} \quad \text{(Say a)}$$

For horizontal asymptotes:

$$\lim_{x\to\infty} f(x) = a$$

$$\Rightarrow \quad y = \frac{5}{2}$$

For vertical asymptote:

$$\lim_{x\to\infty} f(x) = \infty$$

$$\frac{5x+10000}{2x-5000} = \infty$$

$$\Rightarrow \quad 2x - 5000 = 0$$

$$2x = 5000$$

$$x = 2500$$

31. $\lim_{x\to-\infty} \frac{100-3x^3}{-x^3}$

Solution:

$$\lim_{x\to-\infty} \frac{100-3x^3}{-x^3}$$

$$= \lim_{x\to-\infty} \frac{100/x^3 - 3x^3/x^3}{-x^3/x^3}$$

$$= \lim_{x\to-\infty} \frac{100/x^3 - 3}{-1} = \frac{100/\infty - 3}{-1}$$

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$$=\frac{0-3}{-1}=\frac{-3}{-1}=3 \quad \text{(Say a)}$$

For horizontal asymptote:

$$\lim_{x\to\infty} f(x)=a$$

$$\Rightarrow \quad y=3$$

For vertical asymptote:

$$\lim_{x\to\infty} f(x)=\infty$$

$$=\frac{100-3x^3}{-x^3}=\infty$$

$$-x^3=0$$

$$\Rightarrow \quad x=0$$

32. $\lim_{x\to\infty}\frac{3x^3-500}{5000-x^3}$

Solution:

$$\lim_{x\to\infty}\frac{3x^3-500}{5000-x^3}$$

$$=\lim_{x\to\infty}\frac{3x^3/x^3-500/x^3}{5000/x^3-x^3/x^3}$$

$$=\lim_{x\to\infty}\frac{3-500/x^3}{5000/x^3-1}$$

$$=\frac{3-\frac{500}{\infty}}{\frac{5000}{\infty}-1}$$

$$=\frac{3-0}{0-1}=\frac{3}{-1}=-3 \quad \textbf{(say a)}$$

For horizontal asymptote:

$$\lim_{x\to\infty} f(x)=a$$

$$\Rightarrow \quad y=-3$$

For vertical asymptote:

$$\lim_{x\to\infty} f(x)=\infty$$

$$\frac{3x^3-500}{5000-x^3}=\infty$$

$$\Rightarrow \quad 5000-x^3=0$$

$$x^3=5000$$

$$x=(5000)^{\frac{1}{3}}$$

x = 17 (approx).

In the following exercises, determine whether there are any discontinuities and, if so, where they occur.

33. **$f(x)=3x^2+2x+1$**

Solution:

$$\lim_{x\to a} f(x)=\lim_{x\to a}(3x^2+2x+1)$$

$$=3a^2+2a+1=f(a)$$

Hence, f (x) is defined for any real number.

34. **$f(x)=\frac{1}{x+3}$**

Solution:

$$f(x)=\frac{1}{x+3}$$

f (x) is not defined when

$x+3=0$

$x=-3$

When $x=-3$ then $f(x)\to\infty$

Hence, f (x) is discontinuous at $x=-3$

35. **$f(x)=\frac{x^4}{5}$**

Solution:

$$\lim_{x\to a} f(x)=\frac{x^4}{5}$$

$$=\lim_{x\to a}\left(\frac{x^4}{5}\right)$$

$$=\frac{a^4}{5}$$

$$=f(a)$$

Hence f (x) is defined for any real number.

36. $f(x)\frac{3x^2}{x+5}$

Solution:

$$f(x)\frac{3x^2}{x+5}$$

f (x) is not defined when

$x + 5 = 0$

$x = -5$

When x = - then f (x) $\to \infty$

Hence, f (x) is discontinuous at x = -5

37. $f(x) = \frac{1}{8-2x}$

Solution:

$$f(x) = \frac{1}{8-2x}$$

F (x) is not defined when

$8 - 2x = 0$

$-2x = -8$

$2x = 8$

$x = 4$

When x = 4 then f (x) $\to \infty$

Hence, f (x) is discontinuous at x = 4.

38. **f (x) = | x |**

Solution:

$$\lim_{x\to a} f(x) = \lim_{x\to a} |x| = |a| = f(a)$$

Hence, f (x) is defined for any real number.

39. $f(x) = \frac{2x+3}{x^2+4x-21}$

Solution:

f (x) is not defined when

$x^2 + 4x - 21 = 0$

$x^2 + 7x - 3x - 21 = 0$

$x(x + 7) - 3(x + 7) = 0$

$(x - 3)(x + 7) = 0$

Either x - 3 = 0 or x + 7 = 0

x = 3 x = - 7

When x = 3 or x = - 7 then f (x) $\to \infty$

Hence, f (x) is discontinuous at x = 3 or x = - 7

40. $f(x) = \frac{x-2}{x^2-8x}$

Solution:

$$f(x) = \frac{x-2}{x^2-8x}$$

F (x) is not defined when

$x^2 - 8x = 0$

$x(x - 8) = 0$

Either x = 0 or x - 8 = 0, x = 8

When x = 0 or x = 8 then f (x) $\to \infty$

Hence, f x) is discontinuous at x = 0 or x = 8

41. $f(x) = \frac{4x-3}{x^3-x^2-6x}$

Solution:

$$f(x) = \frac{4x-3}{x^3-x^2-6x}$$

F (x) is not defined when

$x^3 - x^2 - 6x = 0$

$x(x^2 - x - 6) = 0$

$x[x^2 - 3x + 2x - 6] = 0$

$x[x(x-3) + 2(x-3)] = 0$

$x(x+2)(x-3) = 0$

Either x = 0 or x + 2 = 0 or x - 3 = 0

x = 0, x = -2, x = 3

When x = 0 or x = -2 or x = 3 then f (x) $\to \infty$

Hence, f (x) is discontinuous at

x = 0 or x = - 2 or x = 3.

42. $f(x) = \frac{5/x}{2x^2+7x-15}$

Solution:

f (x) is not defined when

$2x^2 + 7x - 1 = 0$

$2x^2 + 10x - 3x - 1 = 0$

$2x(x+5) - 3(x+5) = 0$

$(2x-3)(x+5) = 0$

Either $2x - 3 = 0$ or $x + 5 = 0$

$2x = 3$, $x = -5$, $x = \frac{3}{2}$

When x = or x = - 5 then $f(x) \to \infty$

Hence, f (x) is discontinuous at x = or x = - 5

43. $f(x) = \frac{20}{x^2 - 3x - 10}$

Solution:

f (x) is not defined when

$x^2 - 3x - 10 = 0$

$x^2 - 5x + 2x - 10 = 0$

$x(x-5) + 2(x-5) = 0$

$(x+2)(x-5) = 0$

Either $x + 2 = 0$ or $x - 5 = 0$

$x = -2$ $x = 5$

When x = - 2 or x = 5 then $f(x) \to \infty$

Hence, f (x) is discontinuous at x = -2 or x =

44. $f(x) = \frac{4/x}{18 + 3x - x^2}$

Solution:

f (x) is not defined when

$18 + 3x - x^2 = 0$

$-x^2 + 3x + 18 = 0$

$-(x^2 - 3x - 18) = 0$

$\Rightarrow$ $x2 - 3x - 18 = 0$

$x2 - 6x + 3x - 18 = 0$

$x(x-6) + 3(x-6) = 0$

$(x+3)(x-6) = 0$

Either $x + 3 = 0$ or $x - 6 = 0$

$x = -3$ $x = 6$

When x = -3 or x = 6 then f (x) ?

Hence, f (x) is discontinuous at

x = - 3 or x = 6.

45. $f(x) = \frac{10/(5-x)}{4 - x^2}$

Solution:

f (x) is not defined when

$4 - x^2 = 0$

$x^2 = 4$

$x = \pm 2$

x = - 2 or x = + 2

When x = - 2 or x = + 2 then $f(x) \to \infty$

Hence, f (x) is discontinuous at

x = - 2 or x = + 2.

46. $f(x) = \frac{5/(3-x)}{x^2 - 16}$

Solution:

$f(x) = \frac{5/(3-x)}{x^2 - 16}$

f (x) is not defined when

$x2 - 16 = 0$

$x2 = 16$

$x = \pm 4$

x = - 4 or x = + 4

When x = - 4 or x = + 4 then $f(x) \to \infty$

Hence, f (x) is discontinuous at

x = - 4 or x = + 4.

47. $f(x) = \frac{3x - 5}{x^4 - 27x}$

Solution:

$f(x) = \frac{3x - 5}{x^4 - 27x}$

f (x) is not defined when

$x^4 - 27x = 0$

$x(x^3 - 27) = 0$

Either $x = 0$ or $x^3 - 27 = 0$

$x^3 = 27$

$(x)^3 = (3)^3$

$x = 3$

When x = 0 or x = 3 then $f(x) \to \infty$

Hence f (x) is discontinuous at x = 0 or x = 3.

48. $f(x) \dfrac{3/(x^2-1)}{2/(x^2-4)}$

Solution:

$$f(x) \dfrac{3/(x^2-1)}{2/(x^2-4)}$$

$$= \frac{3}{(x^2-1)} \times \frac{x^2-4}{2}$$

$$= \frac{3(x^2-4)}{(x^2-1)}$$

f (x) is not defined when

$2(x^2-1) = 0$

$x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$

When x = - 1 or x = + 1 then $f(x) \to \infty$

Hence, f (x) is discontinuous at

x = - 1 or x = + 1.

Solved Section 6.3

For each of the following functions, determine the average rate of change in the value of y in moving from x = - 1 to x = 2

1. **$y = f(x) = 3x^2$**

Solution:

$y = f(x) = 3x^2$

$y_1 = f(-1) = 3(-1)^2 = 3(1) = 3$

$y_2 = f(1) = 3(2)^2 = 3(4) = 12$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12-3}{2-(-1)}$$

$$= \frac{9}{2+1} = \frac{9}{3} = 3$$

2. **$y = f(x) = 5x^3$**

Solution:

$y = f(x) = 5x^3$

$y_1 = f(-1) = 5(-1)^3 = 5(-1) = -5$

$y_2 = f(2) = 5(2)^3 = 5(8) = 40$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{40-(-5)}{2+1}$$

$$= \frac{45}{3} = 15$$

3. **$y = f(x) = x^2 - 2x + 3$**

Solution:

$y = f(x) = x^2 - 2x + 3$

$y_1 = f(-1) = (-1)^2 - 2(-1) + 3 = 6$

$y_2 = f(2) = (2)^2 - 2(2) + 3$

$= 4 - 4 + 3 = 3$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3-6}{2-(-1)} = \frac{-3}{2+1} = \frac{-3}{3} = -1$$

4. **$y = f(x) = x^3 - 2x^2 + x + 2$**

Solution:

$y = f(x) = x^3 - 2x^2 + x + 2$

$y_1 = f(-1) = (-1)^3 - 2(-1)^2 + (-1) + 2$

$= -1 - 2 - 1 + 2 = -2$

$y_2 = f(2) = (2)^3 - 2(2)^2 + (2) + 2$

$= 8 - 8 + 2 + 2 = 4$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-2)}{2-(-1)}$$

$$=\frac{4+2}{2+1}=\frac{6}{3}=2$$

$\frac{\Delta y}{\Delta x} = \frac{y_2 - y}{x_2 - x_1}$

5. **$y = f(x) = \frac{x^2}{x+4}$**

Solution:

$$y = f(x) = \frac{x^2}{x+4}$$

$$y_1 = f(-1) = \frac{(-1)^2}{-1+4} = \frac{1}{3}$$

$$y_2 = f(2) = \frac{(2)^2}{2+4} = \frac{4}{6} = \frac{2}{3}$$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{\frac{2}{3} - \frac{1}{3}}{2-(-1)} = \frac{\frac{1}{3}}{2+1} = \frac{1}{3x3} = 9$$

6. **$y = f(x) = \frac{x^3}{2}$**

Solution:

$$y = f(x) = \frac{x^3}{2}$$

$$y_1 = f(-1) = \frac{(-1)^3}{2} = \frac{-1}{2}$$

$$y_2 = f(2) = \frac{(2)^3}{2} = \frac{8}{2}$$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{\frac{8}{2} + \frac{1}{2}}{2+1} = \frac{\frac{9}{2}}{3} = \frac{9}{2+3} = \frac{3}{2}$$

7. **$y = f(x) = 2x^2 + 6x + 3$**

Solution:

$$y = f(x) = 2x^2 + 6x + 3$$

$$y_1 = f(-1) = 2(-1)^2 + 6(-1) + 3$$
$$= 2 - 6 + 3 = -1$$

$$y_2 = f(2) = 2(2)^2 + 6(2) + 3$$
$$= 8 + 12 + 3 = 23$$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{23-(-1)}{2-(-1)} = \frac{24}{3} = 8$$

8. **$y = f(x) = 2x^2 - 8x + 10$**

Solution:

$$y = f(x) = 2x^2 - 8x + 10$$

$$y_1 = f(-1) = 2(-1)^2 - 8(-1) + 10$$
$$= 2 + 8 + 10 = 20$$

$$y_2 = f(2) = 2(2)^2 - 8(2) + 10$$
$$= 8 - 16 + 10 = 21$$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2-20}{2-(-1)} = \frac{-18}{3} = -6$$

9. **$y = f(x) = 4x^2 - 2x$**

Solution:

$$y = f(x) = 4x^2 - 2x$$

$$y_1 = f(-1) = 4(-1)^2 - 2(-1) = 4 + 2 = 6$$

$$y_2 = f(2) = 4(2)^2 - 2(2) = 16 - 4 = 12$$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{12-6}{2-(-1)} = \frac{6}{3} = 2$$

10. **$yf(x) = -x^2 + 2x + 4$**

Solution:

$$yf(x) = -x^2 + 2x + 4$$

$y_1 = f(-1) = -(-1)^2 + 2(-1) + 4$

$= -1 - 2 + 4 = 1$

$y_2 = f(2) = -(2)^2 + 2(2) + 4$

$= -4 + 4 + 4 = 4$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4-1}{2-(-1)} = \frac{3}{3} = 1$$

11. $y = f(x) = -5x^2$

Solution:

$y = f(x) = -5x^2$

$y_1 = f(-1) = -5(-1)^2 = -5$

$y_2 = f(2) = -5(2)^2 = -20$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-20-(-5)}{2-(-1)} = \frac{-20+5}{2+1} = \frac{-15}{3} = -5$$

12. $y = f(x) = 3x^3 + 4x - 5$

Solution:

$y = f(x) = 3x^3 + 4x - 5$

$y_1 = f(-1) = 3(-1)^3 + 4(-1) - 5$

$= -3 - 4 - 5 = -12$

$y_2 = f(2) = 3(2)^3 + 4(2) - 5$

$= 24 + 8 - 5 = 27$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{27-(-12)}{2-(-1)} = \frac{27+12}{2+1} = \frac{39}{3} = 13$$

13. $y = f(x) = x^4$

Solution:

$y = f(x) = x^4$

$y_1 = f(-1) = (-1)^4 = 1$

$y_2 = f(2) = (2)^4 = 16$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{16-1}{2-(-1)} = \frac{15}{3} = 5$$

14. $y = f(x) = x^4 - 10$

Solution:

$y = f(x) = x^4 - 10$

$y_1 = f(-1) = (-1)^4 - 10 = 1 - 10 = -9$

$y_2 = f(2) = (2)^4 - 10 = 16 - 10 = 6$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6-(-9)}{2-(-1)} = \frac{6+9}{2+1} = \frac{15}{3} = 5$$

Q.15 to Q. 21 are not import for paper point of view. For exercise 22 - 39, (a) determine the general expression for the difference quotient and (b) use the difference quotient to compute the slope of the secant line connecting points at x = 1 and x = 2.

22. $y = f(x) = 4x^2 + 3$

Solution:

$y = f(x) = 4x^2 + 3$

(a) $$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{x\Delta}$$

$$= \frac{\left[4(x+\Delta x)^2 + 3\right] - \left[4x^2 + 3\right]}{\Delta x}$$

$$= \frac{\left[4\left(x^2 + 2x\Delta x + (\Delta x)^2 + 3\right) - \left(4x^2 + 3\right)\right.}{\Delta x}$$

$$=\frac{4x^2+8x\Delta x+4(\Delta x)^2+3-4x^2-3}{\Delta x}$$

$$=\frac{8x\Delta x+4(\Delta x)^2}{\Delta x}$$

$$=8x+4\Delta x$$

(b) $y=f(x)=4x^2+3$

$$y_1=f(1)=4(1)^2+3=4+3=7$$

$$y_2=f(3)=4(3)^2+3=36+3=41$$

$$\frac{\Delta y}{\Delta x}=\frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{41-7}{3-1}=\frac{12}{2}=\frac{34}{2}=17$$

23. $y=f(x)=x^2+3x$

Solution:

$$\frac{\Delta y}{\Delta x}=\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$=\frac{\left[(x+\Delta x)^2+3(x+\Delta x)\right]-(x^2+3x)}{\Delta x}$$

$$=\frac{x^2+2x\Delta x+(\Delta x)^2+3x+3\Delta x-x^2-3x}{\Delta x}$$

$$=\frac{2x\Delta x+(\Delta x)^2+3\Delta x}{\Delta x}$$

$$=\frac{\Delta x+(2x+\Delta x+3)}{\Delta x}$$

$$=2x+\Delta x+3$$

(b) $y=f(x)=x^2+3x$

$$y_1=f(1)=(1)^2+3(1)=1+3=4$$

$$y_2=f(3)=(3)^2+3(3)=9+9=18$$

$$\frac{\Delta y}{\Delta x}=\frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{18-4}{3-1}=\frac{14}{2}=7$$

24. $y=f(x)=10x^2+20x$

25. $y=f(x)=5$

26. $y=f(x)=-3x^2+8x+10$

27. $y=f(x)=5x^2+20x$

28. $y=f(x)=\frac{x^2}{2}+2x$

29. $y=f(x)=\frac{x^2}{3}+5x$

30. $y=f(x)=ax^2-bx$

31. $y=f(x)=mx^2-n$

32. $y=f(x)=x^2$

33. $y=f(x)=-2x^3$

34. $y=f(x)=1/x$

35. $y=f(x)=5/x$

36. $y=f(x)=2x^2-10$

37. $y=f(x)=-5x^2$

38. $y=f(x)=-4/x$

39. $y=f(x)=-6/x$

Solution:

Above questions are same as Q.22 and Q.23

Solved Section 6.4

In exercises 1 - 24, (a) determine the derivative of f using the limit approach, and (b) determine the slope at x = 1 and x = - 2.

1. $f(x)=4x+6$

Solution:

$$\frac{\Delta y}{\Delta x}=\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$=\frac{\left[4(x+\Delta x)+6\right]-(4x+6)}{\Delta x}$$

$$=\frac{4x+4\Delta x+6-4x-6}{\Delta x}=\frac{4\Delta x}{\Delta x}$$

$$=4$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}(4)$$

$$\frac{dy}{dx}=4$$

2. f (x) = - 20

Solution:

$$\frac{\Delta y}{\Delta x}=\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$=\frac{(-20)-(-20)}{\Delta x}$$

$$=\frac{-20+20}{\Delta x}=\frac{0}{\Delta x}=0$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}(0)$$

$$\frac{dy}{dx}=0$$

3. f (x) = 8 x2

Solution:

$$\frac{\Delta y}{\Delta x}=\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$=\frac{8(x+\Delta x)^2-8x^2}{\Delta x}$$

$$=\frac{8x^2+16x\Delta x+8(\Delta x)^2-8x^2}{\Delta x}$$

$$=\frac{16x\Delta x+8(\Delta x)^2}{\Delta x}$$

$$=\frac{\Delta x(16x+8\Delta x)}{\Delta x}$$

$$=16x+8\Delta x$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}(16x+8\Delta x)$$

$$\frac{dy}{dx}=16x+8(0)=16x$$

4. f (x) $5x^2$

Solution:

$$\frac{\Delta y}{\Delta x}=\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$=\frac{5(x+\Delta x)^2-5x^2}{\Delta x}$$

$$=\frac{5\left(x^2+2x\Delta x+(\Delta x)^2\right)-5x^2}{\Delta x}$$

$$=\frac{5x^2+10x\Delta x+5(\Delta x)^2-5x^2}{\Delta x}$$

$$=\frac{10x\Delta+5(\Delta x)^2}{\Delta x}$$

$$=\frac{\Delta x(10x+5\Delta x)}{\Delta x}$$

$$=10x+5\Delta x$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}(10x+5\Delta x)$$

$$\frac{dy}{dx}=10x+5(0)=10x$$

5. f (x) = $3x^2$ - 5x

Solution:

$$\frac{\Delta y}{\Delta x}=\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$=\frac{\left[3(x+\Delta x)-5(x+\Delta x)\right]-(3x^2-5x)}{\Delta x}$$

$$=\frac{3\left(x^2+2x\Delta x+(\Delta x)^2\right)-5x-5\Delta x-3x^2+5x}{\Delta x}$$

$$=\frac{3x^2+6x\Delta x+3(\Delta x)^2-5x-5\Delta x-3x^2+5x}{\Delta x}$$

$$=\frac{6x\Delta x+3(\Delta x)^2-5\Delta x}{\Delta x}$$

$$=6x+3\Delta x-5$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}(6x+3\Delta x-5)$$

$$\frac{dy}{dx}=6x+3(0)-5=6x-5$$

6. $f(x) = 10x^2 - 8x$

Solution:

$$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \frac{\left[10(x+\Delta x)^2 - 8(x+\Delta x) - (10x^2 - 8x)\right]}{\Delta x}$$

$$= \frac{10(x+\Delta x)^2 + (\Delta x)^2 - 8x - 8\Delta x - 10x^2 + 8x}{\Delta x}$$

$$= \frac{10x^2 + 2x\Delta x + 10(\Delta x)^2 - 8x - 8\Delta x - 10x^2 + 8x}{\Delta x}$$

$$= \frac{\Delta x(20x + 10\Delta x - 8)}{\Delta x}$$

$= 20x + 10\Delta x - 8$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0}(20x + 10\Delta x - 8)$$

$$\frac{dy}{dx} = 20x + 10(0) - 8 = 20x - 8$$

7. $f(x) = x2 - 3x + 5$

Solution:

$$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \frac{\left[(x+\Delta x)^2 - 3(x+\Delta x) + 5 - (x^2 - 3x + 5)\right]}{\Delta x}$$

$$= \frac{x^2 + 2x(\Delta x) + (\Delta x)^2 - 3x - 3\Delta x + 5 - x^2 + 3x - 5}{\Delta x}$$

$$= \frac{2x\Delta x + (\Delta x)^2 - 3\Delta x}{\Delta x}$$

$$= \frac{\Delta x(2x + \Delta x - 3)}{\Delta x} = 2x + \Delta - 3$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0}(2x + \Delta x - 3)$$

$$\frac{dy}{dx} = 2x + 0 - 3 = 2x - 3$$

8. $f(x) = 15x^2 + 2x + 8$

Solution:

$$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \frac{\left[15(x+\Delta x)^2 + 2(x+\Delta x) + 8\right] - (15x^2 + 2x + 8)}{\Delta x}$$

$$= \frac{15\left(x^2 + 2x\Delta x + (\Delta x)^2\right) + 2x + 2\Delta x + 8 - 15x^2 2x - 8}{\Delta x}$$

$$= \frac{15\left(x^2 + 2x\Delta x + (\Delta x)^2\right) + 2x + 2\Delta x + 8 - 15x^2 - 2x - 8}{\Delta x}$$

$$= \frac{15x^2 + 30x\Delta x + (\Delta x)^2 + 2x + 2x + 2\Delta x + 8 - 15x^2 - 2x - 8}{\Delta x}$$

$$= \frac{30x\Delta x + (\Delta x)^2 + 2\Delta x}{\Delta x}$$

$$= \frac{\Delta x(30x + \Delta x + 2)}{\Delta x} = 30x + \Delta x + 2$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0}(30x + \Delta x + 2)$$

$$\frac{dy}{dx} = 30x + 0 + 2 = 30x + 2$$

9. $f(x) = -6x^2 + 3x - 1$

10. $f(x) = -3x^2 + 10x$

11. $f(x) = 20x^2 - 10$

Solution:

Above questions are same as Q - 1 to Q - 8

12. $f(x) = \frac{x^2}{2} + 6x$

Solution:

$$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \frac{\left[(x+\Delta x)^2 + 6(x+\Delta x)\right] - \left(\frac{x^2}{2} + 6x\right)}{\Delta x}$$

$$=\frac{x^2+2x\Delta x+(\Delta x)^2+12x+12\Delta x-x^2-12x}{\Delta x}$$

$$=\frac{(\Delta x)^2+2x\Delta x+12\Delta x}{2\Delta x}$$

$$=\frac{\Delta x(\Delta x+2x+12\Delta x)}{2\Delta x}$$

$$=\frac{\Delta x+2x+12}{2}$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\left(\frac{\Delta x+2x+12}{2}\right)$$

$$\frac{dy}{dx}=\frac{0+2x+12}{2}$$

$$=\frac{2(x+6)}{2}=x+6$$

13. f (x) = ax + b

Solution:

$$\frac{\Delta y}{\Delta x}=\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$=\frac{[a(x+\Delta x)+b]-(ax+b)}{\Delta x}$$

$$=\frac{ax+a\Delta x+b-ax+b}{\Delta x}$$

$$=\frac{a\Delta x}{\Delta x}=a$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}a$$

$$\frac{dy}{dx}=a$$

14. f (x) = ax^2+bx

Solution:

$$\frac{\Delta y}{\Delta x}=\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$=\frac{[-a(x+\Delta x)^2+b(x+\Delta x)]-(-ax^2+bx)}{\Delta x}$$

$$=\frac{-a\left(x^2+2x\Delta x+(\Delta x)^2+bx+b\Delta x+ax^2-bx\right)}{\Delta x}$$

$$=\frac{-20x\Delta x-(\Delta x)^2+b\Delta x}{\Delta x}$$

$$=\frac{\Delta x(-20x-\Delta x+b)}{\Delta x}=-20x-\Delta x+b$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}(-20x-\Delta x+b)$$

$$\frac{dy}{dx}=-20x-0+b=-20x-b$$

15. f (x) = $-\frac{2}{x}$

Solution:

$$\frac{\Delta y}{\Delta x}=\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$=\frac{\frac{-2-}{x+\Delta x}-\left(-\frac{2}{x}\right)}{\Delta x}$$

$$=\frac{-\frac{2}{x+\Delta x}+\frac{2}{x}}{\Delta x}$$

$$=\frac{\frac{-2x+2(x+\Delta x)}{x(x+\Delta x)}}{\Delta x}$$

$$=\frac{-2x+2x+2\Delta x}{\Delta x.x(x+\Delta x)}$$

$$=\frac{2\Delta x}{\Delta x.x(x+\Delta x)}$$

$$=\frac{2}{x(x+\Delta x)}$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{2}{x(x+\Delta x)}$$

$$\frac{dy}{dx}=\frac{2}{x(x+0)}$$

$= \frac{2}{x.x} = \frac{2}{x^2}$

16. $f(x) = \frac{4}{x}$

17. $f(x) = \frac{a}{x}$

18. $f(x) = \frac{5}{x^2}$

19. $f(x) = \frac{-3}{x}$

20. $f(x) = 3x^3$

21. $f(x) = \frac{x^3}{2}$

22. $f(x) = \frac{4}{x^2}$

23. $f(x) = x^3 + 3x^2$

24. $f(x) = x^4$

Solution:

All above questions are same as Q - 15.

Solved Section 6.5

For Exercise 1 - 38, Find f' (x).

1. **$f(x) = 140$**

Solution:

$f(x) = 140$

$f(x) = 0$

2. **$f(x) = -55$**

Solution:

$f(x) = -55$

$f(x) = 0$

3. **$f(x) = 0.55$**

Solution:

$f(x) = 0.55$

$f(x) = 0$

4. **$f(x) = 4x^0$**

Solution:

$f(x) = 4x^0$

$f(x) = 4(0.x^{0-1})$

$= 4(0) = 0$

5. **$f(x) = x^3 - 4x$**

Solution:

$f(x) = x^3 - 4x$

$f(x) = 3x^{3-1} - 4(1)$

$= 3x^2 - 4$

6. **$f(x) = -\frac{3x}{4} + 9$**

Solution:

$f(x) = -\frac{3x}{4} + 9$

$f(x) = -\frac{3x}{4}(1) + 0 = -\frac{3}{4}$

7. **$f(x) = 2x^5$**

Solution:

$f(x) = 2x^5$

$f'(x) = 2(2x^4) = 10x^4$

8. **$f(x) = -\frac{x}{3}$**

Solution:

$f(x) = -\frac{x}{3}$

$f'(x) = \quad (1) = -\frac{1}{3}$

9. **$f(x) = 5\sqrt{x^3}$**

Solution:

$f(x) = 5\sqrt{x^3}$

$= (x^3)^{\frac{1}{5}} = x^{3/5}$

$f'(x) = \frac{3}{5}x^{\frac{3}{5}-1}$

10. **$f(x) = 4\sqrt{x^3}$**

Solution:

$f(x) = 4\sqrt{x^3}$

$= (x^3)^{\frac{1}{4}} = x^{\frac{3}{4}}$

$$f'(x) = \frac{3}{4}x^{-\frac{1}{4}} = \frac{3}{4x^{\frac{1}{4}}}$$

11. $f(x) = \sqrt{x^5}$

Solution:

$$f(x) = \sqrt{x^5}$$

$$= (x^5)^{\frac{1}{2}} = x^{\frac{5}{2}}$$

$$f'(x) = \frac{5}{2}x^{\frac{5}{2}-1} = \frac{5}{2}x^{\frac{5}{2}-1} = \frac{5}{2}x^{\frac{3}{2}}$$

12. $f(x) = 3\sqrt{x^7}$

Solution:

$$f(x) = 3\sqrt{x^7}$$

$$f(x) = (x^7)^{\frac{1}{3}} = x^{\frac{7}{3}}$$

$$f'(x) = \frac{7}{3}x^{\frac{7}{3}-1} = \frac{7}{3}x^{\frac{4}{3}}$$

13. $f(x) = x^{10}$

Solution:

$$f(x) = x^{10}$$

$$f(x) = 10x^9$$

14. $f(x) = x^{\frac{5}{3}}$

Solution:

$$f(x) = x^{\frac{5}{3}}$$

$$f'(x) = \frac{5}{3}x^{\frac{5}{3}-1} = \frac{5}{3}x^{-\frac{2}{3}} = \frac{1}{\frac{5}{3}x^{\frac{2}{3}}}$$

15. $f(x) = \frac{x^6}{3} - 2x$

Solution:

$$f(x) = \frac{x^6}{3} - 2x$$

$$f'(x) = \frac{6x^5}{3} - 2(1) = 2x^5 - 2$$

16. $f(x) = \frac{x^4}{2} - 3x^2 + 10$

Solution:

$$f(x) = \frac{x^4}{2} - 3x^2 + 10$$

$$f'(x) = \frac{4x^3}{2} - 3(2x) + 0$$

$$= 2x^3 - 6x + 10$$

17. $f(x) = \frac{x^3}{2} - 100$

Solution:

$$f(x) = \frac{x^3}{2} - 100$$

$$f'(x) = \frac{3x^2}{2} - 0 = \frac{3x^2}{2}$$

18. $f(x) = x^2 - \sqrt{x}$

Solution:

$$f(x) = x^2 - \sqrt{x}$$

$$f(x) = x^2 - x^{\frac{1}{2}}$$

$$f'(x) = 2x - \frac{1}{2}x^{\frac{1}{2}-1} = 2x - \frac{1}{2}x^{-\frac{1}{2}}$$

$$= 2x - \frac{1}{2x^{\frac{1}{2}}} = 2x - \frac{1}{2\sqrt{x}}$$

19. $f(x) = \frac{5}{x^2}$

Solution:

$$f(x) = \frac{5}{x^2} = 5x^{-2}$$

$$f'(x) = 5(-2x^{-2-1}) = 5(-2x^{-3})$$

$$= -10x^{-3} = -\frac{10}{x^3}$$

20. $f(x) = \frac{3}{5x^3}$

Solution:

$$f(x)=\frac{3}{5x^3}$$

$$=\frac{3}{5}x^{-3}=\frac{3}{5}\left(-3x^{-4}\right)$$

$$=-\frac{9}{5}x^{-4}=-\frac{9}{5x^4}$$

21. $f(x)=-\frac{10}{x^4}$

Solution:

$$f(x)=-\frac{10}{x^4}$$

$$=-10x^{-4}$$

$$f'(x)=-10\left(-4x^{-5}\right)$$

$$=40x^{-5}=\frac{40}{x^5}$$

22. $f(x)=\frac{\sqrt{2}}{x^3}$

Solution:

$$f(x)=\frac{\sqrt{2}}{x^3}$$

$$f(x)=\frac{\sqrt{2}}{x^3}=\sqrt{2}x^{-3}$$

$$f'(x)=\sqrt{2}\left(-3x^{-4}\right)=-3\sqrt{2}x^{-4}$$

$$=-\frac{3\sqrt{2}}{x^4}$$

23. $f(x)=x-\frac{1}{\sqrt{x}}$

Solution:

$$f(x)=x-\frac{1}{\sqrt{x}}$$

$$=x-x^{-\frac{1}{2}}$$

$$f'(x)=1-\left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1}=1+\frac{1}{2}x^{-\frac{3}{2}}$$

$$=1+\frac{1}{2x^{\frac{3}{2}}}$$

24. $f(x)=\frac{1}{6\sqrt{}}$

Solution:

$$f(x)=\frac{1}{6\sqrt{x^5}}$$

$$=\frac{1}{x^{\frac{5}{6}}}=\frac{1}{x^{-5/6}}$$

$$f'(x)=-\frac{5}{6}x^{-\frac{5}{6}-1}=-\frac{5}{6}x^{-\frac{11}{6}}$$

$$=-\frac{5}{6x^{\frac{11}{6}}}$$

25. $f(x)=\frac{2}{\sqrt[3]{x}}$

Solution:

$$f(x)=\frac{2}{\sqrt[3]{x}}$$

$$=\frac{2}{x^{\frac{1}{3}}}=2x^{-\frac{1}{3}}$$

$$f'(x)=2\left(-\frac{1}{3}x^{-\frac{1}{3}-1}\right)=-\frac{2}{3}x^{-\frac{4}{3}}$$

$$=-\frac{2}{3x^{\frac{4}{3}}}$$

26. $f(x)=\frac{1}{6\sqrt[3]{x}}$

Solution:

$$f(x)=\frac{1}{6\sqrt[3]{x}}=\frac{1}{6(x)^{\frac{1}{3}}}$$

$$=\frac{1}{6}(x)^{-\frac{1}{3}}$$

$$f'(x)=\frac{1}{6}\left(-\frac{1}{3}x^{-\frac{4}{3}}\right)$$

$$= -\frac{1}{18}x^{-\frac{4}{3}} = -\frac{1}{18x^{\frac{4}{3}}}$$

27. $f(x) = (x^3 - 2x)(x^5 + 6x^2)$

Solution:

$$f(x) = (x^3 - 2x)(x^5 + 6x^2)$$

$$f'(x) = (x^3 - 2x)\{5x^4 + 6(2x)\}$$

$$+(x^5 + 6x^2)\{3x^2 - 2(1)\}$$

$$= (x^3 - 2x)(5x^4 + 12x)$$

$$+(x^2 + 6x^2)(3x^2 - 2)$$

$$= 5x^7 + 12x^4 - 10x^5 - 24x^2$$

$$+3x^4 - 2x^2 + 18x^4 - 12x^2$$

$$= 5x^7 - 10x^5 + 12x^4 + 3x^4$$

$$= 18x^4 - 24x^2 - 2x^2 - 12x^2$$

$$= 5x^7 - 10x^5 + 33x^4 - 38x^2$$

28. $f(x) = \left(\frac{x^2}{2} - 10\right)(x^3 - 2x^2 + 1)$

Solution:

$$f(x) = \left(\frac{x^2}{2} - 10\right)(x^3 - 2x^2 + 1)$$

$$f'(x) = \left(\frac{x^2}{2} - 10\right)\{3x^2 - 4x + 0\}$$

$$+(x^3 - 2x^2 + 1)\left\{\frac{2x}{2} - 0\right\}$$

$$= \left(\frac{x^2}{2} - 10\right)(3x^2 - 4x)$$

$$+(x^3 - 2x^2 + 1)(x)$$

$$= \frac{3}{2}x^4 - 2x^3 - 30x^2 + 40x + x^4 - 2x^3 + x$$

$$= \frac{3}{2}x^4 + x^4 - 2x^3 - 2x^3 - 30x^2 + x$$

$$= \left(\frac{3}{2} + 1\right)x^4 - 4x^3 - 30x^2 + x$$

$$= \frac{5}{2}x^4 - 4x^3 - 30x^2 + x$$

29. $f(x) = (x^3 - x + 3)(x^6 - 10x^4)$

30. $f(x) = (2 - x - 3x^4)(10 + x - 4x^3)$

31. $f(x) = (6x^2 - 2x + 1)\left(x^3/4 + 5\right)$

32. $f(x) = \left[\left(x + 3/2\right)\right]\left[x^2 - 4x + 9\right]$

Solution:

All above questions are same as Q. 28

33. $f(x) = \frac{x}{1 - x^2}$

Solution:

$$f(x) = \frac{x}{1 - x^2}$$

$$f'(x) = \frac{(1 - x^2)\{1\} - (x)\{0 - 2x\}}{(1 - x^2)^2}$$

$$= \frac{(1 - x^2) - (x)(-2x)}{(1 - x^2)^2}$$

$$= \frac{1 - x^2 + 2x^2}{(1 - x^2)^2} = \frac{1 + x^2}{(1 - x^2)^2}$$

34. $f(x) = \frac{4x}{6x^2 - 5}$

Solution:

$$f(x) = \frac{4x}{6x^2 - 5}$$

$$f'(x) =$$

$$\frac{(6x^2 - 5)\{4(1)\} - (4x)\{6(2x) - 0\}}{(6x^2 - 5)^2}$$

$$= \frac{4(6x^2 - 5) - (12x)(4x)}{(6x^2 - 5)^2}$$

$$=\frac{24x^2-20-48x^2}{(6x^2-5)^2}$$

$$=\frac{-24x^2-20}{(6x^2-5)^2}$$

35. $f(x)=\frac{10-x}{x^2+2}$

36. $f(x)=\frac{3x^5}{x^2-2x+1}$

37. $f(x)=\frac{1}{4x^5-3x^2+1}$

38. $f(x)=\frac{-x^3+1}{x^5-20}$

Solution:

All above questions are same as Q. 33, 34.

For exercise 39-48, (a) find f (2) and (b) determine values of x for which f (x) = 0

39. f (x) = 10 x - 5

Solution:

$f'(x) = 10x - 5$

$f'(x) = 10(1) - 0$

$f'(x) = 10$

(a) $f'(x) = 10$

(b) $f'(x) = 0, 10 = 0$

No value.

40. f (x) = 8x² - 12 x + 1

Solution:

$f'(x) = 8x^2 - 12x + 1$

$f'(x) = 8(2x) - 12(1) + 0$

$f'(x) = 16x - 12$

(a) $f'(x) = 16(2) - 12 = 32 - 12 = 20$

(b) $f'(x) = 0$

$16x - 12 = 0$

$16x = 12$

$x = \frac{12}{16}$

$x = \frac{3}{4}$

41. $f(x) = \frac{x^3}{3} - 6x + 8$

Solution:

$f(x) = \frac{x^3}{3} - 6x + 8$

$f'(x) = \frac{3x^2}{3} - 6(1) + 0$

$f'(x) = x^2 - 6$

(a) $f'(2) = (2)^2 - 6 = 4 - 6 = -2$

(b) $f'(x) = 0$

$x^2 - 6 = 0$

$x^2 = 6$

$x = \pm\sqrt{6}$

$x = \pm 2.449$

42. $f(x) = \frac{16x^4}{4} - x$

43. $f(x) = a_2x^2 + a_1x + a_0$

44. $f(x) = -\frac{1}{x}$

45. $f(x) = \frac{x^2}{1-x^2}$

46. $f(x) = x^2 - 16x + 5$

47. $f(x) = -\frac{x^3}{3} + 9x$

Solution:

All above questions are same as Q. 39 & 40.

48. $f(x) = -\frac{x}{x-5}$

Solution:

$f(x) = -\frac{x}{x-5}$

$$f'(x) = -\frac{(x-5)\{1\}-(x)\{1-0\}}{(x-5)^2}$$

$$= -\frac{x-5-x}{(x-5)^2} = -\frac{-5}{(x-5)^2}$$

$$= -\frac{-5}{(x-5)^2} = \frac{5}{(x-5)^2}$$

(a) $f(2) = \frac{5}{(2-5)^2} = \frac{5}{(-3)^2} = \frac{5}{9}$

(b) $f(x) = 0 \quad \frac{5}{(x-5)^2} = 0 \Rightarrow 5 = 0$

No value.

Solved Section 6.6

In Exercise 1 - 38, determine f (x)

1. $f(x) = (1-4x^3)^5$

Solution:

$f(x) = (1-4x^3)^5$

$$f'(x) = 5(1-4x^3)^4 \frac{d}{dx}(1-4x^3)$$

$$= 5(1-4x^3)^4\{0-4(3x^2)\}$$

$$= 5(1-4x^3)^4(-12x^2)$$

$$= -60x^2(1-4x^3)^4$$

2. $f(x) = (7x^2-3x+1)^3$

Solution:

$f(x) = (7x^2-3x+1)^3$

$$f'(x) = 3(7x^2-3x+1)^2 \frac{d}{dx}(7x^2-3x+1)$$

$$= 3(7x^2-3x+1)^2\{7(2x)-3(1)+0\}$$

$$= 3(7x^2-3x+1)^2(12x-3)$$

3. $f(x) = (x^3-2x+5)^4$

4. $f(x) = (5x^3+1)^4$

Solution:

Same as Q. 1 & 2.

5. $f(x) = \sqrt{1-5x^3}$

Solution:

$f(x) = \sqrt{1-5x^3}$

$$f'(x) = \frac{1}{2}(1-5x^3)^{\frac{1}{2}-1}\frac{d}{dx}(1-5x^3)$$

$$= \frac{1}{2}(1-5x^3)^{-\frac{1}{2}}\{0-5(3x^2)\}$$

$$= \frac{1}{2}(1-5x^3)^{-\frac{1}{2}}(-15x^2)$$

$$= \frac{-15x^2}{2\sqrt{1-5x^3}}$$

6. $f(x) = \sqrt[3]{x^2-2x+5}$

Solution:

$f(x) = \sqrt[3]{x^2-2x+5}$

$f(x) = (x^2-2x+5)^{\frac{1}{3}}$

$$f'(x) = \frac{1}{3}(x^2-2x+5)^{\frac{1}{3}-1}\frac{d}{dx}(x^2-2x+5)$$

$$= \frac{1}{3}(x^2-2x+5)^{\frac{-2}{3}}\{2x-2(1)+0\}$$

$$= \frac{1}{3}(x^2-2x+5)^{\frac{-2}{3}}(2x-2)$$

$$= \frac{2x-2}{3(x^2-2x+5)^{\frac{2}{3}}}$$

7. $f(x) = \frac{1}{\sqrt{x^2-1}}$

Solution:

$$f(x) = \frac{1}{\sqrt{x^2-1}} = (x^2-1)^{-\frac{1}{2}}$$

$$f'(x) = -\frac{1}{2}\left(x^2-1\right)^{-\frac{1}{2}-1}\frac{d}{dx}\left(x^2-1\right)$$

$$= -\frac{1}{2}\left(x^2-1\right)^{-\frac{3}{2}}\{2x-0\}$$

$$= -\frac{1}{2}\left(x^2-1\right)^{-\frac{3}{2}}(2x)$$

$$= -\frac{x}{\left(x^1-1\right)^{\frac{3}{2}}} = -\frac{x}{\sqrt{\left(x^2-1\right)^3}}$$

8. **f (x) =** $\sqrt{\frac{1}{x^2+9}}$

Solution:

$$f(x) = \sqrt{\frac{1}{x^2+9}}$$

$$= \frac{1}{\sqrt{x^2+9}} = \left(x^2+9\right)^{-\frac{1}{2}}$$

$$f'(x) = -\frac{1}{2}\left(x^2+9\right)^{-\frac{1}{2}-1}\frac{d}{dx}\left(x^2+9\right)$$

$$= -\frac{1}{2}\left(x^2+9\right)^{-\frac{3}{2}}\{2x+0\}$$

$$= -\frac{1}{2}\left(x^2+9\right)^{-\frac{3}{2}}(2x) = -\frac{x}{\left(x^2+9\right)^{\frac{3}{2}}}$$

9. **f (x) =** e^x

Solution:

$$f(x) = e^x$$

$$f'(x) = e^x\frac{d}{dx}(x) = e^x(1) = e^x$$

10. **f (x) =** e^{x^2}

Solution:

$$f(x) = e^{x^2}$$

$$f'(x) = e^{x^2}\frac{d}{dx}\left(x^2\right)e^{x^2}(2x) = 2xe^{x^2}$$

11. **f (x) =** $10e^{x^2}$

Solution:

$$f(x) = 10e^{x^2}$$

$$f'(x) = 10e^{x^2}\frac{d}{dx}\left(x^2\right) = 10e^{x^2}(2x)$$

$$= 20xe^{x^2}$$

12. **f (x) = -** $5e^{\frac{x}{2}}$

Solution:

$$f(x) = -5e^{\frac{x}{2}}$$

$$f'(x) = -5e^{\frac{x}{2}}\frac{d}{dx}\left(\frac{x}{2}\right)$$

$$-5e^{\frac{x}{2}}\left(\frac{1}{2}\right) = -5e^{\frac{x}{2}}$$

13. **f (x) =** $\left(5e^x\right)^3$

14. **f (x) =** $3x^2e^x$

15. **f (x) =** $4xe^{x^3}$

16. **f (x) =** $\left(e^x-5\right)^4$

17. **f (x) =** $e^{x^2}-2x+5$

18. **f (x) =** $10e^{x^3-2x}$

19. **f (x) =** $\frac{e^x}{x}$

20. **f (x) =** $\frac{2x^2}{e^x}$

21. **f (x) =** $\left(e^x\right)^3$

22. **f (x) =** $\sqrt{e^{2x}}$

Solution:

All above questions are same as Q 9 to Q.12.

23. **f (x) = ln(5x)**

Solution:

$$f(x) = \ln(5x)$$

$$f'(x) = \frac{1}{5x}\frac{d}{dx}(5x)$$

$$=\frac{1}{5x}(5)=\frac{1}{x}$$

24. f (x) = $\ln\left(\frac{x}{2}\right)$

Solution:

$$f(x)=\ln\left(\frac{x}{2}\right)$$

$$f'(x)=\frac{1}{\frac{x}{2}}\frac{d}{dx}\left(\frac{x}{2}\right)=\frac{2}{x}\left(\frac{1}{2}\right)=\frac{1}{x}$$

25. f (x) = $\ln\left(x^2-3\right)$

Solution:

$$f(x)=\ln\left(x^2-3\right)$$

$$f'(x)=\frac{1}{\left(x^2-3\right)}\frac{d}{dx}\left(x^2-3\right)$$

$$=\frac{1}{\left(x^2-3\right)}\{2x-0\}=\frac{2x}{x^2-3}$$

26. f (x) = $\ln\left(x^3-2x^2+5\right)$

Solution:

$$f(x)=\ln\left(x^3-2x^2+5\right)$$

$$f'(x)=\frac{1}{\left(x^3-2x^2+5\right)}\frac{d}{dx}\left(x^3-2x^2+5\right)$$

$$=\frac{1}{\left(x^3-2x^2+5\right)}\{3x^2-2(2x)+0\}$$

$$=\frac{1}{\left(x^3-2x^2+5\right)}\left(3x^2-4x\right)$$

$$=\frac{3x^2-4x}{\left(x^3-2x^2+5\right)}$$

27. f (x) = $x^2\ln x$

Solution:

$$f(x)=x^2\ln x$$

$$f'(x)=x^2\left\{\frac{d}{dx}(\ln x)+\ln x\left\{\frac{d}{dx}\left(x^2\right)\right\}\right.$$

$$=x^2\left\{\frac{1}{x}\right\}\ln x\{2x\}=x+2x\ln x$$

28. f (x) = $(x+3)\ln x^2$

Solution:

$$f(x)=(x+3)\ln x^2$$

$$f'(x)=(x+3)\left\{\frac{d}{dx}\left(\ln x^2\right)\right\}+\ln x^2$$

$$=\left\{\frac{d}{dx}(x+3)\right\}$$

$$=(x+3)\left\{\frac{1}{x^2}(2x)\right\}+\ln x^2\{(1+0)\}$$

$$=(x+3)\left\{\frac{2}{x}\right\}+\ln x^2\{1\}$$

$$=\frac{2}{x}(x+3)+\ln x^2$$

29. f (x) = $\frac{10x}{\ln x}$

Solution:

$$f(x)=\frac{10x}{\ln x}$$

$$f'(x)=\frac{\ln x\{10(1)\}-(10x)\left\{\frac{1}{x}\right\}}{(\ln x)^2}$$

$$=\frac{10\ln x-10}{(\ln x)^2}$$

30. f (x) = $(\ln x)^3$

Solution:

$$f(x)=(\ln x)^3$$

$$f'(x)=3(\ln x)^2.\frac{1}{x}=\frac{3}{x}(\ln x)^2$$

31. f (x) = $\frac{x-1}{\ln 3x}$

32. f (x) = $(5x = \ln x)^5$

33. f (x) = $\dfrac{\ln x}{e^{x^2}}$

34. f (x) = $\dfrac{e^{x^2}+1}{\ln(x+1)}$

Solution:

All above questions are same as Q.30

35. f (x) = $\sqrt{(x-1)^5(6x-5)}$

Solution:

$$f(x) = (x-1)^5(6x-5)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(x-1)^5(6x-5)^{\frac{1}{2}}$$

$$\left[(x-1)^5\{6(1)-0\}+(6x-5)\{5(x-1)^4(1-0)\}\right]$$

$$= \frac{1}{\sqrt[2]{(x-1)^5(6x-5)}}$$

$$\left[6(x-1)^5+5(6x-5)(x-1)^4\right]$$

36. f (x) = $\sqrt[3]{\left(\dfrac{x^2}{5x-1}\right)^2}$

Solution:

$$f(x) = \sqrt[3]{\left(\frac{x^2}{5x-1}\right)^2} = \left(\frac{x^2}{5x-1}\right)^{\frac{2}{3}}$$

$$f'(x) = \frac{2}{3}\left(\frac{x^2}{5x-1}\right)^{-\frac{1}{3}}\left\{\frac{d}{dx}\left(\frac{x^2}{5x-1}\right)\right\}$$

$$= \frac{2}{3}\left(\frac{x^2}{5x-1}\right)^{-\frac{1}{3}}$$

$$\left\{\frac{(5x-1)\{2x\}-(x^2)\{5(1)-0\}}{(5x-1)^2}\right\}$$

$$= \frac{2}{3}\left(\frac{x^2}{5x-1}\right)^{-\frac{1}{3}}\left\{\frac{2x(5x-1)-5x^2}{(5x-1)^2}\right\}$$

$$= \frac{2}{3}\left(\frac{x^{-\frac{2}{3}}}{(5x-1)^{-\frac{2}{3}}}\right)\left\{\frac{10x^2-2x-5x^2}{(5x-1)^2}\right\}$$

$$= \frac{2x^{-\frac{2}{3}}(5x^2-2x)}{3(5x-1)^{2-\frac{2}{3}}} = \frac{2x^{-\frac{2}{3}}(5x^2-2x)}{3(5x-1)^{\frac{4}{3}}}$$

$$= \frac{2x^{-\frac{2}{3}}x(5x-2)}{3(5x-1)^{\frac{4}{3}}} = \frac{2x^{1-\frac{2}{3}}(5x-2)}{3(5x-1)^{\frac{4}{3}}}$$

$$= \frac{2x^{\frac{1}{3}}(5x-2)}{3(5x-1)^{\frac{4}{3}}}$$

37. **f (x) = (6x - 2)** $\sqrt{x^2-5x+3}$

38. **f (x) =** $\sqrt[3]{\dfrac{(1-x^3)^5x^3}{(9-2x^2)}}$

Solution:

Above questions are same as Q.36.

In the following exercise, find $\dfrac{dy}{dx}$.

39. **y = f (u) = 5u + 3 and**
u = g (x) = - 3x + 10

Solution:

y = 5u + 3 and u = - 3x + 10

$$\frac{dy}{du} = 5(1)+0 = 5$$

$$\frac{du}{dx} = -3(1)+0 = -3$$

$$\text{Now } \frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = (5)(-3) = -15$$

40. **y = f (u) = u² - 5 and u = g (x) = 10 x - 3**

41. **y = f (u) = u² - 2 u and u = g (x) = x²**

42. $y = f(u) = u^3$ and $u = g(x) = x^2 + 3x + 1$

43. $y = f(u) = u^4 - u^2 + 1$ and $u = g(x) = x2 - 3$

44. $y = f(u) = 10 - 5u^3$ and $u = g(x) = -x + x^3$

Solution:

All above questions are same as Q.39.

45. $y = f(u) = \sqrt{U}$ and$= \frac{x^2}{2}$

Solution:

$$\frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}} = \frac{1}{2\sqrt{4}} \quad \frac{du}{dx} = \frac{2x}{2} = x$$

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = \frac{1}{2\sqrt{u}}.x$$

$$= \frac{1}{2\sqrt{\frac{x^2}{2}}}.x \qquad \because u = \frac{x^2}{2}$$

$$= \frac{1}{2.\frac{x}{\sqrt{2}}} = \frac{\sqrt{2}}{2x}.x = \frac{\sqrt{2}}{2}$$

46. $y = f(u) = \sqrt{u^1 - 1}$ and $x = g(u) = x^4$

47. $y = f(u) = (u-3)^5$ and $x = g(u) = x^2 - 2x$

48. $y = f(u) = \sqrt{u^3}$ and $x = g(u) = \sqrt{x}$

49. $y = f(u) = e^4$ and $x = g(u) = 2x^2 - 5x$

50. $y = f(u) = e^{u2}$ and $x = g$ ~~(u)~~ $= x^2 - 5$

51. $y = f(u) = \ln(5u-3)$ and $x = g(u) = 4x^2 - 3x^2$

52. $y = f(u) = 10 \ln(15 - u^3)$ and $u = g(x) = x^2 - 2x + 5$

Solution:

All above questions are same as Q.45.

In Exercise 53-64, (a) find f (2), and (b) determine values of x for which f (x) = 0

53. $f(x) = (5x^2 - 10)^8$

Solution:

$$f(x) = (5x^2 - 10)^8$$

$$f'(x) = 8(5x^2 - 10)^7 \frac{d}{dx}(5x^2 - 10)$$

$$= 8(5x^2 - 10)^7 \{5(2x) - 0\}$$

$$= 8(5x^2 - 10)^7 (10x) = 80x(5x^2 - 10)$$

$$= 80x(5x^2 - 10)^7$$

(a) $f'(x) = 80(2)\left[5(2)^2 - 10\right]^7$

$$= 160(20 - 10)^7$$

$$= 160(20 - 10)^7 = (160)(10)^7$$

(b) $f'(x) = 0$

$$80x(5x^2 - 10) = 0$$

Either $80x = 0$ or

$$80x(5x^2 - 10) = 0$$

$x = 0$, $5x^2 - 10 = 0$ $5x^2 = 10$

$x^2 = 2$, $x = \pm\sqrt{2}$

54. $f(x) = (2x - 8)^5$

Solution:

$$f(x) = (2x - 8)^5$$

$$f'(x) = 5(2x - 8)^4 \frac{d}{dx}(2x - 8)$$

$$= 5(2x - 8)^4 \{2(1) - 0\}$$

$$= 5(2x - 8)^4 (2) = 10(2x - 8)^4$$

(a) $f'(x) = 10\left[2(2) - 8\right]^4$

$$= 10(4 - 8)^5 = 10(-4)^4$$

$$= (10)(256) = 2560$$

(b) f' (x) = 0

$10(2x-8)^4=0$

$\Rightarrow \quad (2x-8)^4=0$

$\Rightarrow \quad 2x - 8 = 0, 2x = 8, x = 4$

55. $f(x) = \sqrt{x^2+21}$

Solution:

$f(x) = \sqrt{x^2+21}$

$f'(x) = \frac{1}{2}(x^2+21)^{-\frac{1}{2}}\{2x+0\}$

$= \frac{1}{2}(x^2+21)^{-\frac{1}{2}}(2x) = \frac{x}{\sqrt{x^2+21}}$

(a) $f'(2) = \frac{2}{\sqrt{(2)^2+21}} = \frac{2}{\sqrt{4+21}}$

$\frac{2}{\sqrt{25}} = \frac{2}{5}$

(b) $f'(x) = 0 \; \frac{x}{\sqrt{2^2+21}} = 0$

Solution:

$\Rightarrow \quad x = 0$

57. $f(x) = e^{-x}$

Solution:

$f(x) = e^{-x}$

$f'(x) = e^{-x}$ (-1) e^{-x}

(a) $f'(2) = -e^{-2} = \frac{1}{e^{-2}}$

(b) f' (x) = 0

$-e^{-x} = 0 \Rightarrow e^{-x} = 0$

Taking 'ln' on both sides.

$Ln(e^{-x}) = \ln(0)$

x lne = 0, - x (1) = 0

x = 0 x = 0 ∴ lne = 1

58. $f(x) = e^x$

59. $f(x) = xe^{-x}$

60 $f(x) = xe^x$

Same as Q-57

61. **f (x) = lnx -x**

Solution:

$f(x) = \ln x - x$

$f(x) = \frac{1}{x} - 1$

(a) $f'(2) = \frac{1}{2} - 1 = \frac{1-2}{2} = -\frac{1}{2}$

(b) $f'(x) = 0$

$\frac{1}{x} - 1 = 0 \qquad \frac{1-x}{x} = 0$

$\Rightarrow 1-x=0, -x=-1$

62. **f (x) =ln 30 x - 3x**

63. **f (x) = ln 2x - 2x**

64. $f(x) = \ln x - \frac{x^2}{2}$

Same as Q - 61

Solved Section 6.7

1. **The function h = f (t) 2.5t3, where 0 ≤ t ≤ 30, describe the height h (in hundreds of feet) of a rocket t seconds after it has been launched.**

(a) What is the average velocity during the time interval 0 ≤ t ≤ 10?

(b) What is the instantaneous velocity at t = 10? At t = 20?

Solution:

$h = f(t) = 2.5t^3$

Where $0 \le t \le 30$,

(a) $\Delta h = f(10) - f(0)$

$= 2.5(10)^3 - 2.5(0)^3$

$= 2.5(1000) - 2.5(0)$

$= 2500 \frac{ft}{s}$

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(b) $f(t) = 2.5t^3$

$f'(t) = 2.5(3t^2) = 7.5t^2$

At $t = 10$

$f'(10) = 7.5(10)^2$

$= 7.5(100) = 750 \frac{ft}{s}$

At $t = 20$

$f'(20) = 7.5(20)^2 = 7.5(400)$

$= 3000 \frac{ft}{s}$

2. An object is launched from ground level with an initial velocity of 256 feet per second. The function which describes the height h of the ball is: h=f(t)=356t - 16t²

Solution:

$h = f(t) = 256t - 16t^2$

(a) $\Delta h = f(1) - f(0)$

$= \left[256(1) - 16(1)^2\right]$

$- \left[256(0) - 16(0)^2\right]$

$= [256 - 16] - [0 - 0]$

$= 240 \frac{ft}{s}$

(b) $f(t) = 256t - 16t^2$

$f'(x) = 256(1) - 16(2t)$

$= 256 - 32t$

Put $f'(t) = 0$, $256 - 32t = 0$

$256 - 32t = 0$

$-32t = -256$

$t = \frac{256}{32}$ $t = 8$ see

(c) Put $f(t) = 0$

$256t - 16t^2 = 0$

$16t(16 - t) = 0$

Either $16t = 0$ or $16 - t = 0$

$t = 0$ $t = 16$

Now $f'(t) = 256 - 32t$

And $f'(x) = 256 - 32t$

$f'(0) = 256 - 32(0)$

$f'(16) = 256 - 32(16)$

$f'(0) = 256 - 32(0)$

$f'(16) = 256 - 32(16)$

$= 256 - 0$ $= 256 - 512$

$= 256 \frac{ft}{s}$ $= -256 \frac{ft}{s}$

3. A ball is dropped from the roof of a building which is 256 feet high. The height of the ball is described by the function. H = f(t)=-16t² + 256

Where h equals the height in feet and t equals time measured in seconds from when the ball was dropped.

(a) What is the average velocity during the time interval

$1 \le t \le 2$?

(b) What is the instantaneous velocity at t = 3?

(c) What is the velocity of the ball at the instant it hits the ground?

Solution:

$h = f(t) = -16t^2 + 256$

(a) $\Delta h = f(2) - f(1)$

Solution:

$= \left[-16(2)^2 + 256\right]$

$- \left[-16(1)^2 + 256\right]$

$= [-64 + 256] - [-16 + 256]$

$= 192 - (+240)$

$= 192 - 240 = -48 \frac{ft}{s}$

(b) $f(t) = -16t^3 + 256$

Solution:

$f(t) = -16(2t) + 0$

$f(t) = -32t$

$f(3) = -32(3) = -96\ \frac{ft}{s}$

(c) **Put f (t) = 0**

Solution:

$-16t^2 + 256 = 0$

$-16t^2 = -256$

$t^2 =$

$t^2 = 16$

$t^x = 4$

Now $f'(t) = -32t$

$f'(4) = -32(4)$

$= -128\ \frac{ft}{s}$

4. Epidemic Control An epidemic is spreading through a large western state. Health officials estimate that the number of persons who will be afflicted by the disease is function of time since the disease was first detected. Specifically, the function is

$n = f(t) = 300t^3 - 20t^2$

Where n equals the number of persons and $0 \le t \le 60$, measured in days.

(a) **How many persons are expected to have caught the disease after 10 days? After 30 days?**

(b) **What is the average rate at which the disease is expected to spread between t = 10 and t = 30?**

(c) **What is the instantaneous rate at which the disease is expected to be spreading at t = 20?**

Solution:

$n = f(t) = 300t^3 - 20t^2$

(a) $f(10) = 300(10)^3 - 20(10)^2$

$= 300(1000) - 20(100)$

$= 300000 - 2000 = 298000$

$f(30) = 300(30)^3 - 20(30)^2$

$= 300(27000) - 20(900)$

$= 8100000 - 18000$

$= 8082000$

(b) $\frac{\Delta n}{\Delta t} = \frac{f(30) - f(10)}{30 - 10}$

$= \frac{808200 - 29800}{20}$

$= \frac{7784000}{20} = 38920$

(c) $f(t) = 300t^3 - 20t^2$

$f(t) = 300(3t^2) - 20(2t)$

$= 900t^2 - 40t$

$f'(20) = 900(20)^2 - 40(20)$

$= 900(400) - 800$

$= 360000 - 800$

$= 360000 - 800 = 359200$

5. **Population Growth The population of a country is estimated by the function $P = 125e\ 0.035^t$ where P equals the population (in millions) and t equals time measured in years since 1990.**

(a) **What is the population expected to equal in the year 2000?**

(b) **Determine the expression for the instantaneous rate of change in the population.**

(c) **What is the instantaneous rate of change in the population expected to equal in the year 2000?**

6. Investment Appreciation A rare piece of artwork has been appreciating in value over recent years. The function. $V = 1.5e^{0.08t}$

estimates the value V of the artwork (measured in millions of dollars) as a function of time t, measured in years since 1986.

(a) What is the value estimated to equal in the year 1996? In the year 2000?

(b) Determine the general expression for the instantaneous rate of change in the value of the artwork.

(c) At what rate is the value of the artwork expected to be increasing in the year 2000?

7. Endangered Species The population of a rare species of wildlife is declining. The function. $P = 75,000e^{-0.025t}$

estimates the population P of the species as a function of time, measured in years since 1980.

(a) What is the population expected to equal in the year 1996.

(b) Determine the general expression for the instantaneous rate of change in the population.

(c) At what rate is the population estimated to be declining in the year 1996?

8. Asset Depreciation the value of a particular asset is estimated by the function.

$V = 240,000e^{-0.04t}$

where V is the value of the asset, measured in years.

(a) What is the value of the asset expected to equal when 4 years old?

(b) Determine the general expression for the instantaneous rate of change in the value of the asset.

(c) What is the rate of change expected to equal when the asset is 10 years old?

Solution:

All above questions are same as Q.1 to Q. 4.

Solved Section 6.8

For exercise 1 - 30, (a) find f′ (x), (b) evaluate f′ (1) and f″ (1) and (c) verbalize the meaning of f′ (1) and f″ (1).

1. $f(x) = 15$

$f(x) = 15$

(a) $f'(x) = 0$

$f''(x) = 0$

(b) $f'(1) = 0$

$f''(1) = 0$

2. $f(x) = 24 - 10x$

Solution:

$f(x) = 24 - 10x$

(a) $f'(x) = 0 - 10(1) = -10$

$f''(x) = 0$

(b) $f'(1) = -10, \; f''(1) = 0$

3. $f(x) = 4x^2 - x + 5$

Solution:

$f(x) = 4x^2 - x + 5$

(a) $f'(x) = 4(2x) - 1 + 0 \quad = 8x - 1$

$f''(x) = 8(1) - 0 = 8$

$f''(x) = 8$

(b) $f(x) = 4(2x) - 1 + 0 \quad = 8x - 1$

$f''(1) = 8(1) - 1 = 8 - 1 = 7$

$f''(1) = 8$

4. $f(x) = x^2 - 15x + 10$

Solution:

$f(x) = x^2 - 15x + 10$

(a) $f'(x) = 2x - 15(1) + 0 = 2x - 15$

$f''(x) = 2(1) - 0 = 2$

(b) $f'(1) = 2x - 15 = 2 - 15 = -13$

$f''(1) = 2$

5. $f(x) = 5x^3$

Solution:

$f(x) = 5x^3$

(a) $f'(x) = 5(3x^2) = 15x^2$

$f''(1) = 15(2x) = 30x$

(b) $f'(1) = 15(1)^2 = 15$

$f''(1) = 30(1) = 30$

6. $f(x) = 7x^3 - 2x^2 + 5x + 1$

Solution:

$f(x) = 7x^3 - 2x^2 + 5x + 1$

(a) $f''(x) = 7(3x^2) - 2(2x) + 5(1) + 0$

$= 21x2 - 4x + 5$

$f''(x) = 21(2x) - 4(1) + 0 = 42x - 4$

(b) $f''(1) = 21(1)2 - 4(1) + 5$

$= 21 - 4 + 5 = 22$

$f''(1) = 42(1) - 4 = 42\text{-}4 = 38$

7. $f(x) = \dfrac{x^4}{4} - \dfrac{x^3}{3} + 10x$

8. $f(x) = \dfrac{x^5}{5} - \dfrac{x^3}{3} + 100$

9. $f(x) = \dfrac{1}{x}$

10. $f(x) = (x^2 - 5)^5$

11. $f(x) = 3x^5 - 2x^3$

12. $f(x) = 5x^4 - 10x^2$

13. $f(x) = \dfrac{2}{x^2}$

14. $f(x) = -\dfrac{4}{x^3}$

15. $f(x) = (x-5)^4$

16. $f(x) = (5-2x)^3$

Solution:

All above questions are same as Q.1 to Q. 6.

17. $f(x) = \sqrt{x+1}$

Solution:

$f(x) = \sqrt{x+1} = (x+1)^{\frac{1}{2}}$

(a) $f(x) = \dfrac{1}{2}(x+1)^{\frac{1}{2}}\{1+0\} = \dfrac{1}{2}(x+1)^{-\frac{1}{2}}$

$= \dfrac{1}{2(x+1)^{\frac{1}{2}}} = \dfrac{1}{2\sqrt{x+1}}$

$f''(x) = \dfrac{1}{2}\left(-\dfrac{1}{2}\right)(x+1)^{-\frac{3}{2}}\{1+0\}$

$= \dfrac{1}{4}(x+1)^{-\frac{3}{2}} = -\dfrac{1}{4(x+1)^{\frac{3}{2}}}$

$= -\dfrac{1}{4\sqrt{(x+1)^3}}$

(b) $f(1) = \dfrac{1}{2\sqrt{(1+1)^{\frac{1}{2}}}} = \dfrac{1}{2\sqrt{2}}$

Solution:

$f''(1) = \dfrac{1}{4\sqrt{(1+1)^{\frac{3}{2}}}} = -\dfrac{1}{4(2)^{\frac{3}{2}}}$

$= -\dfrac{1}{4\sqrt{8}}$

18. $f(x) = \sqrt{10-2x}$

Solution:

Same as Q - 17

19. $f(x) = e^{2x}$

Solution:

$f(x) = e^{2x}$

(a) $f'(x) = e^{2x}(2) = 2e^{2x}$

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$f''(x) = 2\ e^{2x} = 4\ e^{2x}$

(b) $f'(1) = 2^{e2}(1) = 2^{e2} = 14.778$

$f''(1) = 4\ e^{2(1)}(1) = 4e^2 = 29.556$

20. $f(x) = e^{10-2x}$

21. $f(x) = e^{x4}$

22. $f(x) = e^{x^2/x}$

23. $f(x) = \ln 2x$

Solution:

$f(x) = \ln 2x$

(a) $f'(x) = \frac{1}{2x}(2) = \frac{1}{x}$

$f''(x) = -\frac{1}{x^2}$

(b) $f'(1) = \frac{1}{1} = 1$

$f''(1) = -\frac{1}{(1)^2} = 1$

24. $f(x) = \ln 4x$

25. $f(x) = \ln\left(x^2 - 5\right)$

26. $f(x) = \ln\left(x^3 + 4\right)$

Solution:

Same as Q.23.

27. $f(x) = \frac{x}{1-x^2}$

Solution:

$f(x) = \frac{x}{1-x^2}$

(a) $f'(x) = \frac{(1-x^2)\{1\} - (x)\{0-2x\}}{(1-x^2)^2}$

$= \frac{(1-x^2)+2x^2}{(1-x^2)^2} = \frac{1-x^2+2x^2}{(1-x^2)^2}$

$= \frac{1+x^2}{(1-x^2)^2}$

$f''(x) =$

$= \frac{(1-x^2)^2\{0+2x\} - (1+x^2)\left\{2(1-x^2)^1(0-2x)\right\}}{(1-x^2)^4}$

$= \frac{(1-x^2)(2x) - (1+x^2)\left\{-4x(1-x^2)\right\}}{(1-x^2)^4}$

$= \frac{2x(1-x^2) + 4x(1+x^2)(1-x^2)}{(1-x^2)^4}$

$= \frac{(1-x^2)\left[2x + 4x(1+x^2)\right]}{(1-x^2)^4}$

$= \frac{2x+4x+4x^3}{(1-x^2)^3} = \frac{6x+4x^3}{(1-x^2)^3}$

(b) $f'(1) = \frac{1+(1)^2}{\left[1-(1)^2\right]^2} = \frac{1+1}{(1-1)^2} = \frac{2}{0} = \infty$

$f''(1) = \frac{6(1)+4(1)^3}{\left(1-(1)^2\right)^3} = \frac{6+4}{(1-1)^3} = \frac{10}{0} = \infty$

28. $f(x) = \frac{2x}{x^2+1}$

Solution:

Same as Q - 27

Solution:

$f(x) = e^x \ln x$

(a) $f'(x) = e^x . \frac{1}{x} + e^x . \ln x$

$f''(x) = \left\{e^x\left(-\frac{1}{x^2}\right) + \frac{1}{x}.e^x\right\}$

$+ \left\{e^x . \frac{1}{x} + \ln x . e^x\right\}$

$$=-\frac{e^x}{x^2}+\frac{e^x}{x}+\frac{e^x}{x}+e^x \ln x$$

$$=-\frac{e^x}{x^2}+2\frac{e^x}{x}+e^x \ln x$$

(b) $f'(1)=e^1.\frac{1}{1}+e^1.\ln(1)$

$$=e+e(0)=e=2.718$$

$$f''(1)=-\frac{e^1}{(1)^2}+2\frac{e^1}{(1)}+e^{(1)}\ln(1)$$

$$=-e+2e+e(0)=e=2.718$$

$$f(x)=\frac{2x}{x^2+1}$$

30. $f(x)= e^{2x}\ln x$

Solution:

Same as Q - 29.

31. The height of a felling object dropped from a height of 1,000 feet is described by the function.

$h = 1,000 - 16t^2$

where h I measured in feet and t is measured in seconds.

(a) What is the velocity at t 4?

(b) What is the acceleration at t = 4?

Solution:

$h = 1000 - 16 t^2$

(a) $h(t) = 1000 - 16t^2$

$h(4) = 1000 - 16 (4)2$

$= 1000 - 256 = 744$

(b) $h(t) = 1000 - 16 t^2$

$h'(t) = 0 - 16 (2t) = - 32 t$

$h'(4) = - 32 (4) = - 128$

32. The height of a falling object dropped from a height of 1,200 feet in described by the function.

$h = 1,200 - 16t^2$

where h is measured in feet and t is measured in seconds.

(a) What is the velocity at t = 3? At t = 5?

(b) What is the acceleration at t = 3? At t = 5?

33. A ball thrown upward from the roof of a building which is 600 feet high will be at a height of h feet after t seconds, as described by the function.

$h = f(t) = - 16t^2 + 50\ 50t + 600$

(a) WQhat is the height of the ball after 3 seconds?

(b) What is the velocity of the ball after 3 seconds? (A negative sign implies a downward direction.)

(c) What is the acceleration of the ball at t = 0? At t=5?

34. A ball thrown upward from the roof of a building which is 750 feet high will be at a height of h feet after t seconds, as described by the function.

$h = f(t) = - 16t^2 + 50 + 750$

(a) What is the height of the ball after 5 seconds?

(b) What is the velocity of the ball after 5 seconds? (A negative sign implies a downward direction).

(c) What is the acceleration of the ball at t = 0? At t = 5?

Solution:

All above questions are same as Q-31. In exercise 35-50, find all higher - order derivates.

35. $f(x) = 16x^3 - 4x^2$

Solution:

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$f(x) = 16x^3 - 4x^2$

$f'(x) = 16(3x^2) - 4(2x)$

$= 48x^2 - 8x$

$f''(x) = 48(2x) - 8(1)$

$= 96x - 8$

$f'''(x) = 96(1) - 0$

$f''''(x) = 0$

36. f (x) = 2500

Solution:

$f(x) = 2500$

$f'(x) = 0$

37. f (x) = mx + b

Solution:

$f(x) = mx + b$

$f'(x) = m(1) + 0 = m$

$f''(x) = 0$

38. $f(x) = -\frac{x}{4} + 10$

Solution:

$f(x) = -\frac{x}{4} + 10$

$f'(x) = -\frac{1}{4} + 0 = -\frac{1}{4}$

$f''(x) = 0$

39. $f(x) = x^5 - 5x^4 - 30x^2$

Solution:

$f(x) = x^5 - 5x^4 - 30x^2$

$f'(x) = 5x^4 - 5(4x^3) - 30(2x)$

$= 5x^4 - 20x^3 - 60x$

$f''(x) = 5(4x^3) - 20(3x^2) - 60(1)$

$= 20x^3 - 60x^2 - 60$

$f'''(x) = 20(3x^2) - 60(2x) - 0$

$= 60x^2 - 120x$

$f''''(x) = 60(2x) - 120(1)$

$= 120x - 120$

$f'''''(x) = 120(1) - 0 = 120$

$f''''''(x) = 0$

40 $f(x) = (x - 10)^3$

Solution:

$f(x) = (x - 10)^3$

$f'(x) = 3(x - 10)^2 \{1 - 0\}$

$= 3(x - 10)^2$

$f''(x) = 3(2)(x - 10)^1 \{1 - 0\}$

$= 6(x - 10)$

$f'''(x) = 6[1 - 0] = 6(1) = 6$

$f''''(x) = 0$

41. $f(x) = a_3 x_3 + a_3 x_2 + a_1 x + a_0$

Solution:

$f(x) = a_3 x_3 + a_3 x_2 + a_1 x + a_0$

$f'(x) = a_3(3x^2) + a_2(2x) + a_1(1) + 0$

$= 3a_3x^2 + 2a_2 x + a_1$

$f''(x) = 3a_3(2x) + 2a_2(1) + 0$

$= 6a_3 x + 2a_2$

$f'''(x) = 6a_3(1) + 0 = 6a_3$

$f''''(x) = 0$

42. $f(x) = (a_1x + b_1)(a_2x + b_2)$

43. $f(x) = 10x^4 - 2x^3 + 3$

44. $f(x) = -x^5 + 3x^2$

45. $f(x) = (ax + b)^3$

46. $f(x) = (cx - d)^4$

47. $f(x) = a_4 x_4 + a_3 x_3$

48. $f(x) = \frac{x^4}{4} - \frac{x^3}{3}$

49. $f(x) = e^x$

50. $f(x) = e^{-x}$

Solution:

All above questions are same as Q - 35, Q - 51.

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Chapter - 7

OPTIMIZATION: METHODOLOGY

Solved Section 7.1

For each of the following functions, (a) determine whether f is increasing or decreasing at x=1. Determine the values of x for which f is (b) an increasing function, (c) a decreasing function, and (d) neither increasing nor decreasing.

1. $f(x) = 20 - 4x$

Solution:

$f(x) = 20 - 4x$

$f'(x) = 0 - 4(1) = -4$

As $f'(x) < 0$, so f will be a decreasing function at x=1

2. $f(x) = 15x + 16$

Solution:

Same as Q - 1

3. $f(x) = x^2 - 3x + 20$

Solution:

$f(x) = x^2 - 3x + 20$

$f'(x) = 2x - 3(1) + 0 = 2x - 3$

(a) $f'(1) = 2(1) - 3 = 2 - 3 = -1 < 0$

f is a decreasing function at x=1.

Now $f(x) = x^2 - 3x + 20$

$f'(x) = 2x - 3(1) + 20 = 2x - 3$

(b) f will be an increasing function, when

$f'(x) > 0$

$2x - 3 > 0$

$2x > 3$

$x > \frac{3}{2}$

(c) f will be a decreasing function, when.

$f'(x) < 0$

$2x - 3 < 0$

$2x < 3$

$x < \frac{3}{2}$

(d) f will be neither increasing nor decreasing, when.

$f(x) = 0$

$2x - 3 = 0$

$2x = 3$

$x = \frac{3}{2}$

4. $f(x) = 3x^2 + 12x + 9$

Solution:

Same as Q - 3

5. $f(x) = \frac{x^3}{3} + \frac{x^2}{2}$

Solution:

(a) $f(x) = \frac{x^3}{3} + \frac{x^2}{2}$

$f'(x) = \frac{3x^2}{3} + \frac{2x}{2} = x^2 + x$

$f'(1) = (1)^2 + (1) = 1 + 1 = 2 > 0$

f is an increasing function at x = 1.

$f(x) = \frac{x^3}{3} + \frac{x^2}{2}$

$f(x) = \frac{3x^2}{3} + \frac{2x}{2} = x^2 + x$

(b) f will be an increasing function, when

$f'(x) > 0$

$x^2 + x > 0$

$x(x+1) > 0$

$x > 0$ or $x + 1 > 0$

$x > -1$

(c) f will be a decreasing function, when.

$f'(x) < 0$

$x^2 + x < 0$

$x(x+1) < 0$

$x < 0$ or $x + 1 < 0$

$x < -1$

(d) f will be neither increasing nor decreasing, when.

$f'(x) = 0$

$x^2 + x = 0$

$x(x+1) = 0$

$x = 0$ or $x + 1 = 0$

$x = -1$

6. f (x) =

Solution:

Same as Q - 5

7. $f(x) = x^4 + 2x^2$

Solution:

(a) $f(x) = x^4 + 2x^2$

$f'(x) = 4x^3 + 2(2x) = 4x^3 + 4x$

$f'(1) = 4(1)^3 + 4(1) = 4+4 = 8$

f is increasing function at x=1

$f'(x) = x^4 + 2x^2$

$f(x) = 4x^3 + 2(2x) = 4x^3 + 4x$

(b) f is an increasing function, when

$f'(x) > 0$

$4x^3 + 4x > 0$

$4x(x^2+1) > 0$

$4x > 0$ or $x^2 + 1 > 0$

$x > 0$ $x^2 > -1$

Not possible

(c) f is an increasing function, when

$f'(x) < 0$

$4x^3 + 4x < 0$

$4x(x^2+1) < 0$

$4x < 0$ or $x^2 + 1 < 0$

$x < 0$ $x^2 < -1$

Not possible

(d) f is an increasing function, when

$f'(x) = 0$

$4x^3 + 4x = 0$

$4x(x^2+1) = 0$

$4x = 0$ or $x^2 + 1 = 0$

$x = 0$ $x^2 = -1$

Not possible

8. $f(x) = 3x^5$

9. $f(x) = (x+3)^{3/2}$

10. $f(x) = \dfrac{3x^2}{x^2-1}$

Solution:

Same as above

11. $f(x) = -x^2 + 4x + 15$

Solution:

(a) $f(x) = -x^2 + 4x + 15$

$f'(x) = -2x + 4(1) + 0$

$= -2x + 4$

$f'(x) = -2(1) + 4 = -2+4 = 2 > 0$

f is increasing function at x = 1

Now $f(x) = -x^2 + 4x + 15$

$f'(x) = -2x + 4(1) + 0$

$= -2x + 4$

(b) f will be an increasing function, when

Solution:

$f'(x) > 0$

$-2x + 4 > 0$

$-2x > -4$

$2x < 4$

$x < 2$

(c) f will be an increasing function, when

Solution:

$f'(x) < 0$

$-2x + 4 < 0$

$-2x < -4$

$2x > 4$

$x > 2$

(d) f will be neither increasing function, mor decreaning, when

Solution:

$f'(x) = 0$

$-2x + 4 = 0$

$-2x = -4$

$2x = \frac{-4}{-2}$

$x = 2$

Q-12 to Q-20

Solution:

Same as Q - 11

For each of the following function, use f (x) to determine the concavity conditions at x = - 2 and x = 1.

21. $f(x) - 3x^2 + 2x - 3$

Solution:

$f(x) = -3x^2 + 2x - 3$

$f'(x) = -3(2x) + 2(1) - 0$

$= -6x + 2$

$f''(x) = -6(1) + 0 = -6$

Put x = - 2 in f″ (x), we get

$f''(x) = -6 < 0$

The graph of f is concave down at x = - 2. put x = 1 in f″ (x), we get

$f''(x) = -6 < 0$

The graph of f is concave down at x = 1.

22. $f(x) = x^3 + 12x + 1$

Solution:

$f(x) = x^3 + 12x + 1$

$f'(x) = 3x^2 + 12(1) + 0$

$= 3x^2 + 12$

$f''(x) = 3(2x) + 0 = 6x$

Put x = - 2 in f″ (x), we get

$f''(-2) = 6(-2) = -12 < 0$

The graph of f is concave down at x = - 2.

Put x = 1 in f″ (x), we get

$f''(1) = 6(1) = 6 > 0$

The graph of f is concave up at x = 1.

23. $f(x) = x^2 - 4x + 9$

24. $f(x) = -x^2 + 5x$

25. $f(x) = \sqrt{x^2 + 10}$

26. $f(x) = (x + 1)^6$

27. $f(x) = x2 + 3x^3$

28. $f(x) = \frac{x^2}{1+x}$

Solution:

Same as Q - 22

29. $f(x) = 5x^3 \ 4x^2 + + 10x$

Solution:

$f(x) = 5x^3 \ 4x^2 + + 10x$

$f'(x) = 5(3x^2) \ 4(2x) + 10(1)$

$= 15x^2 \ 8x + 10$

$f''(x) = 15(2x) \ 8(1) + 0$

$= 30x \ 8$

Put x = 2 in f″ (x), we get

$f''(2) = 30(2) \ 8 = 60 \ 8$

$= 68 < 0$

The graph of f is concave down at x = 2.

Put x = 1 in f″ (x), we get

$f(1) = 30(1) \ 8 = 30 \ 8$

$= 22 > 0$

The graph of f is concave up at x = 1.

30. $f(x) = x^4 + 2x^3 = 10x^2$

Solution:

$f(x) = x^4 + 2x^3 = 10x^2$

$f(x) = 4x^3 + 2(3x^2) - 10(2x)$

$= 4x^3 + 6x^2 - 20x$

$f(x) = 4(3x^2) + 6(2x) \ 20(1)$

$= 12x^2 + 12x \ 20$

Put x = 2 in f (x), we get

$f(2) = 12(-2)^2 + 12(-2) - 20$

$= 48 - 24 - 20 = 48 - 40$

$= 4 > 0$

The graph of f is concave up at x = - 2.

Put x = 1 is f″ (x), we get

$f''(1) = 12(1)^2 + 12(1) - 20$

$= 12 + 12 \ 20 = 24 - 20$

$= 4 > 0$

The graph of f is concave up at x = 1.

31. $f(x) = \frac{x^3}{3} - \frac{x^2}{2} + 10x$

32. $f(x) = \frac{5x^3}{3} + \frac{3x^2}{2} - 5x + 25$

33. $f(x) = \left(x^2 - 1\right)^3$

34. $f(x) = (20 \quad 3x)^5$

35. $f(x) = (3x^2 + 2)^4$

36. $f(x) = (2x - 8)^5$

37. $f(x) = \sqrt{4x - 10}$

38. $f(x) = \frac{x^3}{1 - x}$

Solution:

Same as Q - 30

39. $F(x) = e^x$

Solution:

$f(x) = e^x$

$f'(x) = e^x$

$f''(x) = e^x$

Put x = - 2 in f" (x), we get

$f(-2) = e^{-2} = 0.1353 > 0$

The graph of f is concave up at x = - 2.

Put x = 1 in f" (x), we get

$f(1) = e^1 = 2.7183 > 0$

The graph of f is concave up at x = 1

40. $f(x) = e^{-x}$

Solution:

Same as Q - 39

41. $f(x) = \ln x$

Solution:

$f(x) = \ln x$

$f'(x) = \frac{1}{x}$

$f''(x) = \frac{1}{x^2}$

Put x = - 2 in f" (x), we get

$$f''(-2) = -\frac{1}{(-2)^2} = -\frac{1}{4} = -0.25 < 0$$

Thus graph of f is concave down.

Put x = 1 in f (x), we get

$$f''(1) = -\frac{1}{(1)^2} = \frac{-1}{1} = -1 < 0$$

The graph of f is concave down.

If a > 0, b > 0, and c > 0, determine the value of x for which f is (a) increasing (b) decreasing, (c) concave up, (d) concave down, if

43. $f(x) = ax + b$

Solution:

$f(x) = ax + b$

$f'(x) = a(1) + 0$

$f'(x) = a$

$f''(x) = 0$

As $f'(x) > 0$ as $a > 0$

So f is an increasing function.

As $f''(x) = 0$

So, no conclusion can be drawn about the concavity.

44. $f(x) = b \quad ax$

Solution:

Same as Q - 43

45. $f(x) = ax^2 + bx + c$

Solution:

$f(x) = ax^2 + bx + c$

$f'(x) = 2ax + b(1) + 0 = 2ax + b$

$f''(x) = 2a(1) + 0 = 2a$

Put $f'(x) = 0$

$2ax + b = 0$

$2ax = -b$

$\Rightarrow \quad x = -\frac{b}{2a}$

Put 'x' in f' (x), we get

$$f'(x) = 2a\left(-\frac{b}{2a}\right) + b$$

$$= -b + b = 0$$

As $f'(x) = 0$.

So, f is neither increasing nor decreasing function.

As $f''(x) = 2a > 0$ as $a > 0$

So, f is concave up.

46. $f(x) = -ax^2 - bx - c$

47. $f(x) = ax^3$

48. $f(x) = ax^4$

Solution:

Same as Q - 45

For each of the following function, identify the location of any inflection points.

49. $f(x) = x^r - 9x^2$

Solution:

$f(x) = x^r - 9x^2$

$f'(x) = 3x^2 - 9(2x)$

$= 3x^2 - 18x$

$f''(x) = 3(2x) - 18(1)$

$= 6x - 18$

Put $f''(x) = 0$

$6x - 18 = 0$

$6x = 18$

$x = \frac{18}{6}$

$x = 3$

For $x = 3$, $f''(x)$ is evaluated to the left and right of

$x = 3$ at $x = 2.9$

and $x = 3.1$

$f''(2.9) = 6(2.9) - 18$

$= 17.4 - 18$

$= -0.6$

$f''(3.1) = 6(3.1) - 18$

$= 18.6 - 18 = 0.6$

Since the sign of $f''(x)$ changes, an inflection point exists at $x = 3$.

50. $f(x) = x^3 + 24x^2$

51. $f(x) = \frac{x^4}{12} - \frac{x^3}{3}$

52. $f(x) = (3 - x)^4$

53. $f(x) = (x - 5)^3$

54. $f(x) = \frac{x^5}{20} - \frac{x^3}{6}$

55. $f(x) = -10x^4 + 100$

56. $f(x) = (x-1)/x$

Solution:

Same as Q - 49

57. $f(x) = x^3 + 6x^2 - 18$

Solution:

$f(x) = x^3 + 6x^2 - 18$

$f'(x) = 3x^2 + 6(2x) - 0$

$= 3x^2 + 12x$

$f''(x) = 3(2x) + 12(1)$

$= 6x + 12$

Put $f''(x) = 0$

$6x + 12 = 0$

$6x = -12$

$x = -2$

For $x = -2$, $f''(x)$ is evaluated to the left and right of $x = 1 - 2$ at $x = -2.1$ and $x = -1.9$.

$f''(-2.1) = 6(-2.1) + 12$

$= -12.6 + 12$

$= -0.6$

$f''(-1.9) = 6(-1.9) + 62$

$= -11.4 + 12 = 0.6$

Since the sign of $f''(x)$ changes an inflection point exists at $x = -2$.

58. $f(x) = x^3 - 30x^2$

59. $f(x) = \frac{x^4}{12} + \frac{x^3}{6} - 3x^2$

60. $f(x) = \frac{x^4}{12} + \frac{7x^3}{6} - 5x^2$

61. $f(x) = \frac{x^4}{4} - \frac{9x^2}{2} + 100$

62. $f(x) = \frac{x^6}{30} - \frac{4x^2}{2}$

63. $f(x) = (3x-12)^{5/2}$

64. $f(x) = (2x-8)^{7/2}$

65. $f(x) = (x-5)^5$

66. $f(x) = (x+2)^4$

67. $f(x) = e^x$

68. $f(x) = e^{-x}$

69. $f(x) = \ln x$

70. $f(x) = -\ln x$

71. to Q - 76.

Note:

(Not important for papers points of view)

Solved Section 7.2

For each of the following functions determine the location of all critical points and determine their nature.

1. $f(x) = 3x^2 - 48x + 100$

Solution:

$f(x) = 3x^2 - 48x + 100$

$f'(x) = 3(2x) - 48(1) + 0$

$= 6x - 18$

$f''(x) = 6(1) - 0 = 6$

Put $f''(x) = 0$

$6x - 48 = 0$

$6x = 48$

$x = 8$

Put $x = 8$ in f (x), we get

$f(8) = 3(8)^2 - 48(8) + 100$

$= 3(64) - 384 + 100$

$= 192 - 384 + 100 = -92$

The critical point is located at (8,- 92) Now put (8, - 92) in f" (x), we get $f''(x) = 6 > 0$

The graph of f is concave up with x = 8 and a relative minimum occurs at (8, - 92).

2. $f(x) = \frac{x^3}{x} - 5x^2 + 16x + 100$

Solution:

Same as Q - 1.

3. $f(x) = -10x^3 + 5$

4. $f(x) = x^2 - 8x + 4$

Solution:

Same as Q - 1.

5. $f(x) = \frac{x^3}{3} - 2.5x^2 + 4x$

Solution:

$f(x) = \frac{x^3}{3} - 2.5x^2 + 4x$

$f'(x) = \frac{x^3}{3} - 2.5(2) + 4(1)$

$= x^2 - 5x + 4$

$f''(x) = 2x - 5(1) + 0 = 2x - 5$

Put $f'(x) = 0$

$x^2 - 5x + 4 = 0$

$x^2 - 4x - x + 4 = 0$

$x(x-4) - 1(x-4) = 0$

$(x-1)(x-4) = 0$

$\Rightarrow$ $x - 1 = 0$ or $x - 4 = 0$

$x = 1$ $\quad$ $x = 4$

Put x = 1 in f (x), we get

$f(1) = \frac{(1)^3}{3} - 2.5(1)^2 + 4(1)$

$= \frac{1}{3} - 2.5 + 4 = \frac{11}{6} = 1\frac{11}{6}$

Put x= 4 in f (x), we get

$f(x) = \frac{(4)^3}{3} - 2.5(4)^2 + 4(4)$

$= \frac{64}{3} - 40 + 16$

$= \frac{64}{3} - 24 = -\frac{8}{3}\left(4, -\frac{8}{3}\right)$

The critical points are located

$\left(1, \frac{11}{6}\right)$ and $\left(4, -\frac{8}{3}\right)$

Now put $\left(1, \frac{11}{6}\right)$ in f" (x), we get

$f''(1) = 2(1) - 5 = 2 - 5$

$= -3 < 0$

The graph of f is concave down at x = 1 and a relative maximum at $\left(1, \frac{11}{6}\right)$

Now put $\left(4, -\frac{8}{3}\right)$ in f" (x), we get

$f''(4) = 2(4) - 5 = 8 - 5 = 3 > 0$

The graph of f is concave up at x = 4 and a relative minimum at $\left(4, -\frac{8}{3}\right)$

Q - 6 to Q - 20

Solution:

Same as Q - 5

21. $f(x) = -2x^2 + \frac{x^4}{4}$

Solution:

$f(x) = -2x^2 + \frac{x^4}{4}$

$f'(x) = -2(2x) + \frac{4x^3}{4}$

$= -4x + 4x^3$

$f''(x) = -4(1) + 4(3x^2)$

$= -4 + 12x^2$

Put f' (x) = 0

$= -4x + 4x^3 = 0$

$= 4x^3 - 4x = 0$

$= 4x(x^2 - 1) = 0$

$= 4x(x - 1)(x + 1) = 0$

$= 4x = 0,\ x - 1 = 0,\ x + 1 = 0$

$= x = 0,\quad x = 1,\quad x = -1,$

Put x = 0 in f (x), we get

$f(0) = -2(0)^2 + \frac{(0)^4}{4} = 0(0,0)$

Put x = 1 in f (x), we get

$f(1) = -2(1)^2 + \frac{(1)^4}{4}$

$= -2 + \frac{1}{4} = -\frac{7}{4} \not\in \left(1, -\frac{7}{7}\right)$

The critical points are located at

$(0,0), \left(1, -\frac{7}{4}\right)\left(1, -\frac{7}{7}\right)$

Put (0,0) in f" (x), we get

$f(x) = -4 + 12(0) = -4 < 0$

The graph of f is concave down at x = 0 and a relative maximum at (0,0)

Put $\left(1, -\frac{7}{4}\right)$ in f" (x), we get

$f''(x) = -4 + 12(1) = -4 + 12 = 8 > 0$

The graph of f is concave up when x = 1 and a relative minimum at $\left(1, -\frac{7}{4}\right)$

Put $\left(1, -\frac{7}{7}\right)$ in f" (x), we get

$f''(x) = -4 + 12(-1)$

$= -4 - 12$

$= -16 < 0$

The graph of f is concave down when x = - 1 and a relative minimum at

Q - 22 to Q - 45

Solution:

Same as Q - 21

46. f (x) = lnx - 0.5x

Solution:

$f(x) = \ln x - 0.5x$

$f'(x) = \frac{1}{x} - 0.5(1) = \frac{1}{x} - 0.5$

$f''(x) = - - 0$

$f''(x) = -\frac{1}{x^2}$

Put $f''(x) = 0$

$= \frac{1}{x} - 0.5 = 0$

$= \frac{1}{x} = 0.5$

$= x = \frac{1}{0.5}$

$= x = 2$

Put 'x' in f (x), we get

$f(2) = \ln 2 - 0.5(2)$

$= 0.6931 - 1$

$= -0.31 < 0$

The critical point located at(2, - 0.31)

Put x = 2 in f" (x), we get

$f(x) = -\frac{1}{2^2} = -\frac{1}{4} < 0$

The graph of f is concave down atx = 2 and a relative maximum at (2, - 0.31).

Q. 47 to Q - 58 are same as Q - 46

Q. 59 to Q - 62 are not important for paper point of view.

Solved Section 7.3

This section is not important for paper point of view.

Solved Section 7.4

In the following exercise, determine the location and values of the absolute maximum and absolute minimum for f.

1. $f(x) = 2x^2 - 4x + 5$

Solution:

$f(x) = 2x^2 - 4x + 5$

$f'(x) = 2x^2 - 4(1) + 0 = 4x - 4$

$f''(x) = 4(1) - 0 = 4$

Put $f'(x) = 0$

$4x - 4 = 0$

$4x = 4$

$x = 1$

There is no critical value within the domain of the function $2 \leq x \leq 8$.

2. $f(x) = -x^2 + 8x - 100$, where $-2 \leq x \leq 4$.

Solution:

Same as Q - 1.

3. $f(x) = x^3 - 12x^2$, where $2 \leq x \leq 10$

Solution:

$f(x) = x^3 - 12x^2$

$f'(x) = 3x^3 - 12(2x)$

$= 3x^2 - 24x$

$f''(x) = 3(2x) - 24(1)$

$= 6x - 24$

Put $f'(x) = 0$

$= 3x^2 - 24x = 0$

$= 3x(x - 8) = 0$

$= 3x = 0, x - 8 = 0$

$= x = 0 \quad x = 8$

The only critical value within the domain of the function is x = 8

Put x = 8 in f(x) , we get

$f(8) = (8)^3 - 12(8)^2$

$= 512 - 768 = -256$

Thus, a critical point occurs at (8, - 256).

Put x = 8 in f" (x), we get

$f(x) = 6(8) - 24$

$= 48 - 24$

$= 24 > 0$

Since f" (8) > 0, a relative minimum occurs at (8, - 256). Because f (x) is defined for all real x, no other critical value exist.

The value of f (x) at the endpoints of the domain are.

$f(2) = (2)^3 - 12 (2)^2$

$= 8 - 12 (4) = 8 - 48 = - 40$

$f(10) = (10)^3 - 12 (10)^2$

$= 1000 - 12 (100)$

$= 1000 - 1200$

$= - 200$

Now, comparing f (2), f (8) and f (10), we find the absolutes minimum of - 256 occurs at x = 8. and the absolute maximum of - 40 occurs when x = 2.

Q - 4 to Q - 9 are same as Q - 3.

10. $f(x) = \frac{x^3}{3} - \frac{x^2}{2} - 6x$

Where $0 \leq x \leq 5$

Solution:

$f(x) = \frac{x^3}{3} - \frac{x^2}{2} - 6x$

$f'(x) = \frac{x^3}{3} - \frac{x^2}{2} - 6(1)$

$= x^2 - x - 6$

$f''(x) = 2x - 1 - 0 = 2x - 1$

Put $f'(x) = 0$

$= x^2 - x - 6 = 0$

$= x^2 - 3x + 2x - 6 = 0$

$= x (x - 3) + 2 (x - 3) = 0$

$= (x + 2) (x - 3) = 0$

$= x + 2 = 0 , x - 3 = 0$

$= x = - 2, x = 3$

The only critical value within the domain of the function is x = 3.

Put x = 3 in f (x), we get

$f'(x) = \frac{(3)^3}{3} - \frac{(3)^2}{2} - 6(3)$

$= \frac{27}{3} - \frac{9}{2} - 18$

$= 9 - \frac{9}{2} - 18$

$= -\frac{27}{2}$

Thus, a critical point occurs at $\left(x, -\frac{27}{2}\right)$.

Put x = 3 in f" (x), we get.

$f''(x) = 2 (3) - 1 = 6 - 1 = 5 > 0$

Since f (3) > 0, a relative minimum occurs at $\left(x, -\frac{27}{2}\right)$. Because

f' (x) is defined for all real x, no other critical value exist.

The value of f(x)at the endpoints of the domain are.

$f(0) = \frac{(0)^3}{3} - \frac{(0)^2}{2} - 6(0)$

$= 0 - 0 - 0 = 0$

$f(5) = \frac{(5)^3}{3} - \frac{(5)^2}{2} - 6(5)$

$= \frac{125}{3} - \frac{25}{2} - 30 = -\frac{5}{6}$

Comparing f (0), f (3), f and f (5), we find the absolute maximum of 'O' at x = 0.

Q - 11 to Q - 18

Solution:

Same as Q - 10.

☆☆☆☆☆☆☆

Chapter - 8
OPTIMIZATION:APPLICATIONS

Solved Section 8.1

1. A fern has determined that total revenue is a function of the price charged for its product. Specifically, the total revenue function is

$R = f(p) = 10p^2 + 1,750p$

where p equals the price in dollar.

(a) Determine the price p which results in maximum total revenue.

(b) What is the maximum value for total revenue?

Solution:

$R = f(p) = -10p^2 + 175p$

(a) $f(p) = -10(2p) + 1750$

$(1) = 1750 - 20p$

Put $f'(p) = 0$

$1750 - 20p = 0$

$20p = 1750$

$p = = \frac{1750}{20}$ \$87.50

(b) $f.(p) = 0 - 20(1)$

$= -20 < 0$

There fore, a relative maximum of f occurs at p = 87.50

The maximum value of R is found by Substituting p = 87.50 into f, we get

$f(87.50) = -10(87.50)^2 + 1750(87.50)$

f (87.50) = s\$ 76562.050

2. The demand function for a firm's product is

q = 150,000 75p

where q equals the number of units demanded and p equate the price in dollars.

(a) Determine the price which should be charged to maximize total revenue.

(b) What is the maximum value for total revenue?

(c) How many units are expected to be demanded?

Solution:

Same as Q - 1

3. The annual profit for a firm depends upon the number of units produced. Specifically, the function which describes the relationship between profit P (stated in dollars) and the number of units produced x is

$P = 0.01x^2 + 5,000x \;\; 25,000$

(a) Determine the number of units x which will result in maximum profit.

(b) What is the expected maximum profit?

Solution:

$f(x) = p = -0.01x^2 + 5000x - 25000$

(a) $f(x) = -0.01(2x) + 5000(1) - 0$

$= -0.02x + 5000$

Put $f(x) = 0$

$-0.02x + 5000 = 0$

$-0.02x = -5000$

$$x = \frac{5000}{0.025} = 250,000 \text{ units.}$$

(b) $f''(x) = 0.02(1) + 0$

$= 0.02 < 0$

Therefore, a relative maximum of f occurs at $x = 250,000$

The expected maximum profit is

$p = 0.01(250,000)^2 + 5000(250000)$

$25000 = 624,975,000$

4. Beach Management A community which is located m a resort area is trying to decide on the parking fee to charge at the town-owned beach. There are other beaches in the area, and there is competition for bathers among the different beaches. The town has determined the following function which expresses the average number of cars per day q as a function of the parking fee p stated in cents.

$Q = 6,000 \quad 12p$

(a) Determine the fee which should be charged to maximize daily beach revenues.

(b) What is the maximum daily beach revenue expected to be?

(c) How many cars are expected on an average day?

Solution:

Same as Q - 3.

5. Import Tax Management The United States government is studying the import tax structure for color television sets imported from other countries into the United States. The government is trying to determine the amount of the tax to charge on each TV set. The government realizes that the demand for imported TV sets will be affected by the tax. It estimates that the demand for imported sets D, measured in hundreds of TV sets, will be related to the import tax t, measured in cents, according to the function

(a) Determine the impost tax which will result In maximum tax revenues from importing TV sate.

(b) What is the maximum revenue?

(c) What will the demand for imported color TV sets equal with this tax?

Solution:

Same as Q - 3.

6. A manufacture has determined a cost function which expresses the annual coat of purchasing, owning, and maintaining its raw material inventory as & function of the sizeof each order. The cost function is

$$c = \frac{51,200}{q} + 80q + 750,000$$

(a) Determine the order size q which minimizes annual inventory coat.

(b) What are minimum inventory costs expected to equal?

Solution:

$$c = \frac{51,200}{q} + 80q + 750,000$$

(a) $c = f(q) = 51200\,q^{-1} + 80q + 75000$

$f(q) = 51200(-1)\,q^{-2} + 80q(1) + 0$

$$= -\frac{51,200}{q^2} + 80$$

Put $f'(q) = 0$

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$$-\frac{51,200}{q^2}+80=0$$

$$-\frac{51,200}{q^2}=-80$$

or $$q^2=\frac{51,200}{80}$$

$$q^2 = 640$$

$$q = 25.30 \text{ (unit).}$$

(b) $f''(q) = -51200(-2q^{-3}) + 0$

$$=\frac{102400}{q^3}$$

$$f''(25.30)=\frac{102400}{(25.30)^3}$$

$$=6.32>0$$

thus, the minimum value of f occurs when q = 25.30. This minimum average cost per units is.

$$f(25.30)=\frac{51200}{25.30}+80(25.30)+75000$$

$$= \$\ 79,047.72$$

7. In Exercise 6 assume that the maximum amount of the raw material which can be accepted in any one shipment as 20 tons.

(a) Given this restriction, determine the order size q which minimizes annual inventory coat.

(b) What are the minimum annual inventory costs?

(c) How do these results compare with those in Exercise 6?

Solution:

Same as Q - 6

8. A major distributor of racquetballs is thriving. One of the distributor's major problems is keeping up with the demand for racquetballs. Balls are purchased periodically from a sporting goods manufacturer. The annual cost of purchasing, owning, and maintaining the inventory of racquetballs is described by the function.

$$C=\frac{280,000}{q}+0.15q+2,000,000$$

where q equals the order size (in dozens of racquetballs) and C equals the annual inventory cost.

(a) Determine the order size q which minimizes annual inventory cost.

(b) What are the minimum inventory costs expected to equal?

Solution:

Same as Q - 6

9. The distributor in Exercise 8 has storage facilities to accept up to 1,200 dozens of balls in any one shipment

(a) Determine the order size q which minimizes annual inventory costs.

(b) What are the minimum inventory coats?

(c) How do these results compare with those in Exercise 8?

Solution:

Same as Q - 6

10. The total cot of producing q units of a certain product is described by the function

$C = -5,000,000 + 250g + 0.002g^2$

where C is the total cost stated in dollars.

(a) How many units should be produced in order to minimize the average cost

per unit ?

(b) **What is the minimum average cost per unit?**

(c) **What is the total cost of production at this level of output?**

Solution:

$c = f(q) = 5000000 + 250q + 0.002q^2$

(a) $f'(q) = 0 + 250(1) + 0.002 \quad (2q)$

$= 250 + 0.004q$

Put $f(q) = 0$

$250 + 0.004q = 0$

$0.004q = 250$

$$q = \frac{250}{0.004} = \$62,500$$

(b) $f''(q) = 0 + 0.004(1)$

$= 0.004 > 0$

Thus, the minimum value of f occurs when q = $ 62,500.

This minimum average cost per unit is.

f (62,500) = 50,000,000 + 250 (62500) + 0.002 (62500)= 20,625,125

11. **The total cost of producing q unite of a certain product is described by the function**

$c = 350{,}000 + 7{,}500q + 0.25q^2$

where C is the total cost stated in dollars.

(a) **Determine how many units q should be produced in order to minimize the average cost per unit.**

(b) **What is the minimum average cost per unit?**

(c) **What is the total coat of production at this level of output?**

Solution:

Same as Q - 10.

12. **Re- solve Exercise 11 if the maximum production capacity is 1,000 units.**

Solution:

Same as Q - 10.

13. **Public Utilities A cable TV antenna company has determined that its profitability depends upon the monthly fee it charges its customers. Specifically, the relationship which describes annual profit P (stated in dollars) as a function of the monthly rental fee r (stated in dollars) is**

$P = -50{,}000r^2 + 2{,}750{,}000r - 5{,}000{,}000$

(a) **Determine the monthly rental fee r which will lead to maximum profit.**

(b) **What is the expected maximum profit?**

Solution:

$P = f(r) = -50{,}000r^2 + 2{,}750{,}000r - 5{,}000{,}000$

(a) $f'(r) = -50{,}000(2r) + 2{,}750{,}000(1) - 0$

$= -100{,}000r + 2{,}750{,}000$

Put $f'(r) = 0$

$-100{,}000r + 2{,}750{,}000 = 0$

$100{,}000r = 2{,}750{,}000$

$r = \$27.50$

(b) $f''(r) = -100{,}000(1) + 0$

$= -100{,}000 < 0$

Thus, the minimum value of f occurs when q = 27.50,

This maximum average cost per unit is.

f (27.50) = - 50,000 (27.50)
+ 2,750,000 (27.50)
- 5,000,00 = $ 32,812,500

14. **In Exercise 13 assume that the local public utility commission has restricted the CATV company to a monthly fee not to exceed $20.**

(a) What fee leads to a maximum profit for the company?

(b) What is the effect of the utility commission's ruling on the profitability of the firm?

Solution:

Same as Q - 13.

15. A company estimates that the demand for its product fluctuates with the price it charges- The demand function is

q = 280,000 - 400p

where q equals the number of units demanded and? equals the price in dollars. The total cost of producing q units of the product is estimated by he function

C = 350,000 + 300g + 0.0015q²

(a) Determine how many units 'q should be produced in order to maximize annual profit.

(b) What price should be charged?

(c) What is the annual profit expected to equal?

Solution:

$q = 280,000 - 400\,p$

$c = 350,00 + 300q + 0.0015q^2$

Now

$q = 280,000 - 400\,p$

$400\,p = 280,000 - q$

$$p = \frac{280,000}{400} - \frac{1}{400}q$$

$p = 700 - 0.0025\,q$

We know that

$R = pq$

$= (700 - 0.0025q)q$

$= 700q - 0.0025q^2$

Now

$p = f(q)$

$p = R - C$

$p = (700q - 0.0025q^2) - (350,000 + 300q + 0.0015q^2)$

$p = 700\,q - 0.0025q^2 - 350,000 - 300q - 0.0015q^2$

$f(q)\ p = -0.00\,40q^2 + 400q - 350,000$

(a) $f'(q) = -0.0040\,(2q) + 400\,(1) - 0$

$= -0.0080q + 400$

Put $f'(q) = 0$

$-0.0080q + 400 = 0$

$0.0080q = 400$

$$q = \frac{400}{0.0080}$$

$q = \$\ 50,000$

(b) $p = 700 - 0.0025\,(50,000)$

$= \$\ 575$

(c) $p = -0.0040\,(50,000)^2 + 400\,(50,000) - 350,000$

$= \$\ 9,650,000$

16. Solve the previous exercise, using the marginal approach to profit maximization.

Solution:

Same as Q - 15.

17. If annual capacity is 40,000 units in Exercise 15, how many units q will result in maximum profit? What is the loss in profit attributable to the restricted capacity?

Solution:

Same as Q - 15.

18. An equivalent way of solving Example 2 is to state total revenue as a function of q, the average number of

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riders per hour. Formulate the function R= g (q) and determine the number of riders q which will result in maximum total revenue. Verify that the maximum value of R and the price which should be charged are the same as obtained in Example 2.

Solution:

Same as Q - 15.

19. The total cost and total revenue functions for a product are

$C(q) = 500 + 100q + 0.5q^2$

$B(q) = 500q$

(a) Using the marginal approach, determine the profit-maximizing level of output.

(b) What is the maximum profit?

Solution:

(a) $c(q) = 500 + 100q + 0.5q2$

$\Rightarrow c'(q) = 100 + q \Rightarrow c''(q) - 1$

$R(q) = 500q \Rightarrow R'(q) = 500$

$\Rightarrow R''(q) = 0$

As $c'(q) = R'(q)$

$100 + q = 500$

$q = 500 - 100 = 400$

(b) As. $R(q) < c < 0$

$\Rightarrow 0 < 1$

There is a relative maximum on profit function

When $q = 400.$

Now $R(q) = R(q) - c(q)$

$= 500q - (500 + 100q + 0.5q^2)$

$= 500q - 500 + 100q - 0.5q^2)$

$= -0.5q^2 + 400q - 500$

$= -0.5(400)^2 + 400(400) - 500$

$= \$ 79,500$

20. A firm sells each unit of a product for \$50. The total cost of producing x (thousand) units is described by the function

$C(x) = 10 \quad 2.5x^2 + x^3$

where C (x) is measured in thousands of dollars.

(a) Use the marginal approach to determine the profit-maximizing level of output.

(b) What is total revenue at this level of output? Total cost? Total profit?

Solution:

Same as 19.

21. The profit function for a firm is

$P(q) = \quad 4.5q^2 + 36,0009 \quad 45,000$

(a) Using the marginal approach, determine the profit-maximizing level of output,

(b) What is the maximum profit?

Solution:

$P(q) = -4.5q^2 + 36,000q - 45,000$

(a) Cannot determine using marginal approach

(b) $P(q) = -4.5q^2 + 36,000q - 45,000$

$P(q) = -4.5(2q) + 36,000q(1) - 0$

$= -9q + 36,000$

$P(q) = -9(1) + 0 = -9$

Put $P(q) = 0$

$-9q + 36,000 = 0$

$-9q = -36,000$

$q = 4000$

As $P''(q) < 0$, thus a relative maximum at $q = 4000$, This maximum value is

$P(q) = -4.5(4000)^2 + 36,000(4000) - 45,000$

= $ 71,955,000

22. The total coat and total revenue functions for a product are

$$C(q) = 5,000,000 + 250q + 0.002q^2$$

$$R(q) = 1,250q \quad 0.005q^2$$

(a) Using the marginal approach, determine the profit-maximizing level of output.

(b) What is the maximum profit?

Solution:

Same as Q - 21.

23. The total coat and total revenue functions for a product are

$$C(q) = 40,000 + 25q + 0.002q^3$$

$$R(q) = 75q \quad 0.008q^2$$

(a) Using the marginal approach, determine the profit maximizing level of output.

(b) What is the maximum profit?

Solution:

Same as Q - 21.

24. Portrayed in Fig. 17.10 is a total cost function C(q) and a total revenue function R (q).

Discuss the economic significance of the four levels of output q_1, q_2, q_3, and q_4.

Solution:

Same as Q - 21.

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Q25.(a) Solve the following linear inequalities and represent the solutions on the number line.

i. $|x-1| \le 1$

Solution:

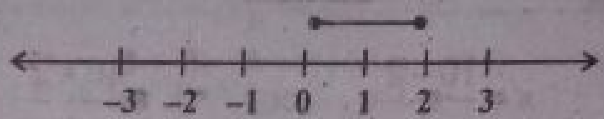
$$|x-1| \le 1$$

By definition of absolute value, we have

$$-1 \le x-1 \le +1$$

$$-1+1 \le x \le +1+1$$

$$0 \le x \le 2$$



ii. $|x^2-1| \ge 1$

Solution:

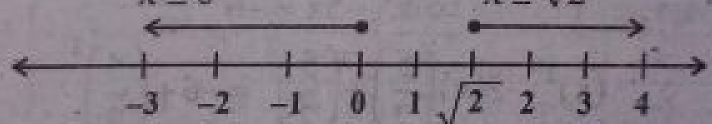
By definition of absolute value, we have

$x^2-1 \le -1$ or $x^2-1 \ge 1$

$x^2 \le -1+1$ $\quad$ $x^2 \ge 1+1$

$x^2 \le 0$ $\quad$ $x^2 \ge 2$

$x \le 0$ $\quad$ $x \ge \sqrt{2}$



(b) **Find the solutions of the following second degree equation by three different methods $2x^2-10x+12=0$**

Solution:

By factorization:

$$2x^2-10x+12=0$$

$$2x^2-6x-4x+12=0$$

$$2x(x-3)-4(x-3)=0$$

$$(2x-4)(x-3)=0$$

$$\Rightarrow 2x-4=0 \quad \text{or} \quad x-3=0$$

$$2x=4 \qquad x=3$$

$$x=2$$

By Quadratic formula:

$$2x^2-10x+12=0$$

Compare it with

$$ax^2+bx+c=0$$

Here

$$a=2,\ b=-10,\ c=12$$

We know that

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(2)(12)}}{2(2)}$$

$$x = \frac{10 \pm \sqrt{100-96}}{4}$$

$$x = \frac{10 \pm \sqrt{4}}{4}$$

$$x = \frac{10 \pm 2}{4}$$

$$x = \frac{10-2}{4} \quad \text{or} \quad x = \frac{10+2}{4}$$

$$x = \frac{8}{4} \qquad\qquad x = \frac{12}{4}$$

$$x = 2 \qquad\qquad x = 3$$

By Completing Square Method:

$$2x^2 - 10x + 12 = 0$$

$$2(x^2 - 5x + 6) = 0$$

$$\Rightarrow x^2 - 5x = -6$$

$$(x)^2 - 2(x)\left(\frac{5}{2}\right) + \left(\frac{5}{2}\right)^2 = -6 + \left(\frac{5}{2}\right)^2$$

$$\left(x - \frac{5}{2}\right)^2 = -6 + \frac{25}{4}$$

$$\left(x - \frac{5}{2}\right)^2 = \frac{-24+25}{4}$$

$$\left(x - \frac{5}{2}\right)^2 = \frac{1}{4}$$

$$\left(x - \frac{5}{2}\right)^2 = \left(\frac{1}{2}\right)^2$$

Taking Square root on both sides, we get

$$\sqrt{\left(x - \frac{5}{2}\right)^2} = \pm\sqrt{\left(\frac{1}{2}\right)^2}$$

$$x - \frac{5}{2} = \pm\frac{1}{2}$$

$$x = \frac{5}{2} \pm \frac{1}{2}$$

$$\Rightarrow x = \frac{5}{2} - \frac{1}{2} \quad \text{or} \quad x = \frac{5}{2} + \frac{1}{2}$$

$$x=\frac{5-1}{2} \quad \text{or} \quad x=\frac{5+1}{2}$$

$$x=\frac{4}{2} \quad \text{or} \quad x=\frac{6}{2}$$

$$x=2 \quad \text{or} \quad x=3$$

Q26.(a) Write down the important characteristics of linear equations.

Ans:

Characteristics of Linear Equations

General Form:

Linear Equation with two Variables

A linear equation involving two variable x and y has the standard form

$$ax + by = c \qquad (2.1)$$

Where a, b, and c are constants and a and b cannot both equal zero.

Linear equations are first-degree equations. Each variable in the equation is raised (implicitly) to the first power. The presence of terms having exponents other than 1 (e.g.,x^2) or of terms involving a product of variables (e.g., 2xy) would exclude an equation from being considered linear.

The following are all examples of linear equations involving two variables:

	Eq. (2.1) Parameters		
	a	b	c
$2x+5y=-5$	2	5	–5
$-x+\frac{1}{2}y=0$	–1	$\frac{1}{2}$	0
$\frac{x}{3}=25$	$\frac{1}{2}$	0	25
$\sqrt{2}u-0.05v=3.76$	$\sqrt{2}$	–0.05	3.76
$2s-4t=-1$	2	–4	$-\frac{1}{2}$

(Note: The names of the variables may be different from x and y.)

The following are examples of equations which are not linear. Can you explain why?

$$2x+3xy-4y=10$$

$$x+y^2=6$$

$$\sqrt{u}+\sqrt{v}=-10$$

$$ax+\frac{b}{y}=c$$

The form of an equation may not always be obvious. Initially, the equation

$$2x=\frac{5x-2y}{4}+10$$

might not appear to be linear. However, multiplying both sides of the equation by 4 and moving all variables to the left side yields 3x + 2y = 40, which is in the form of Eq. (2.1)

Representation Using Linear Equations:

Given a linear equation having the form ax + by = c, the solution set for the equation is the set of all ordered pairs (x,y) which satisfy the equation. Using set notation the solution set S can be specified as

$$S = \{(x,y)\}\,|ax + by = c$$

(b) Write down the equation of line y - 2x + 5 = 0 at the point (3,1) and with slope m = 2 in the point-slope form.

Solution:

The point-slope form is:

$y - y_1 = m(x - x_1)$

Here $(x_1, y_1) = (3,-1)$ and $m = 2$

So,

$$y - 1 = 2(x - 3)$$
$$y - 1 = 2x - 6$$
$$y = 2x - 6 + 1$$
$$y = 2x - 5$$

Q27.(a) What is the general rule for matrices multiplication? Explain with examples.

Solution: General Rule for matrices multiplication:

1- The inner product is defined only if the row and column vectors contain the same number of elements.

2- The inner product results when a row vector is multiplied by a column vector and the resulting product is a scalar quantity.

3- The inner product is computed by multiplying corresponding elements in the two vectors and algebraically summing.

Example:

Consider the multiplication of the following vectors:

$$AB = (5 \; -2)\begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

To find the inner product, the first element in the row vector is multiplied by the first element in the column vector, the resulting product is added to the product of element 2 in the row vector and element 2 in the column vector. For the vectors indicated, the inner product is computed as $a_{11}b_{11} + a_{12}b_{21}$, or

$$(5 \; -2)\begin{pmatrix} 4 \\ 6 \end{pmatrix} = (5)(4) + (-2)(6) = 8$$

(b) Prove that for the identity matrix I given by $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$I^n = I \text{ where } n = 1, 2, 3, ...$$

Solution:

For n = 2

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$$I^2 = I$$

L.H.S $= I^2 = I.I$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= I = \text{R.H.S}$$

For $n = 3$

$$I^3 = I$$

L.H.S $= I^3$

$$= I^2.I$$
$$= (I.I).I$$
$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.I = \begin{pmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{pmatrix}.I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.I$$
$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
$$= I = \text{R.H.S}$$

Similarly, we can prove that

$$I^4 = I$$

And so on

$$I^n = I \quad \text{for} \quad n = 1, 2, 3, \ldots\ldots$$

Q28.(a) Examine the following function for the critical points and determine their nature also;

$$f(x) = x^3 - x^2 - 5x - 1$$

Solution:

$$f(x) = x^3 - x^2 - 5x - 1$$
$$f'(x) = 3x^2 - 2x - 5(1) - 0 = 3x^2 - 2x - 5$$
$$f''(x) = 3(2x) - 2(1) - 0 = 6x - 2$$

Put $f''(x) = 0$

$$6x - 2 = 0$$
$$6x = 2$$
$$x = \frac{2}{6} = \frac{1}{3}$$

Put $x = \frac{1}{3}$ in f(x), we get

$$f\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^2 - 5\left(\frac{1}{3}\right) - 1 = \frac{1}{27} - \frac{1}{9} - \frac{5}{3} - 1$$
$$= \frac{1 - 3 - 45 - 27}{27} = \frac{-74}{27}$$

The Critical point is located at $\left(\frac{1}{3}, \frac{-74}{27}\right)$.

Now put $\left(\frac{1}{3}, \frac{-74}{27}\right)$ in $f''(x)$, we get

$$f''\left(\frac{1}{3}\right)=6\left(\frac{1}{3}\right)-2=2-2=0$$

The graph of f(x) has points of inflextion at $x=\frac{1}{3}$

(b) Explain the difference between the cost function and the Revenue function with the help of examples.

Ans: Revenue Function:

The following applications focus on revenue maximization. Recall that the money which flows into an organization from either selling products or providing services is referred to as revenue. The most fundamental way of computing total revenue from selling a product (or service) is

Total revenue = (price per unit) (quantity sold)

An assumption in this relationship is that the selling price is the same for all units sold.

Example:

The demand for the product of a firm varies with the price that the firm charges for the product. The firm estimates that annual total revenue R (Stated in \$1,000s) is a function of the price p (stated in dollars). Specifically,

$$R=f(p)=-50p^2+500p$$

(a) Determine the price which should be charged in order to maximize total revenue.
(b) what is the maximum value of annual total revenue?

Solution:

(a) From Chap.6 we should recognize that the revenue function is quadratic. It will graph as a parabola which is concave down. Thus the maximum value of R will occur at the vertex. The first derivative of the revenue function is

$$f'(p)=-100p+500$$

If we set $f'(p)$ equal to 0,

$$-100p+500=0$$
$$-100p=-500$$

Or a critical value occurs when

$$P=5$$

Although we know that a relative maximum occurs when p = 5 (because of our knowledge of quadratic functions), let's formally verify this using the second-derivative test:

$$f''(p)=-100 \quad \text{and} \quad f''(5)=-100<0$$

Therefore, a relative maximum of f occurs at p = 5.

(b) The maximum value of R is found by substituting p = 5 into f, or

$$f(5)=-50(5^2)+500(5)$$
$$=-1,250+2,500=1,250$$

Thus, annual total revenue is expected to be maximized at \$1,250 (1,000s) or \$1.25 million when the firm charges 45 per unit.

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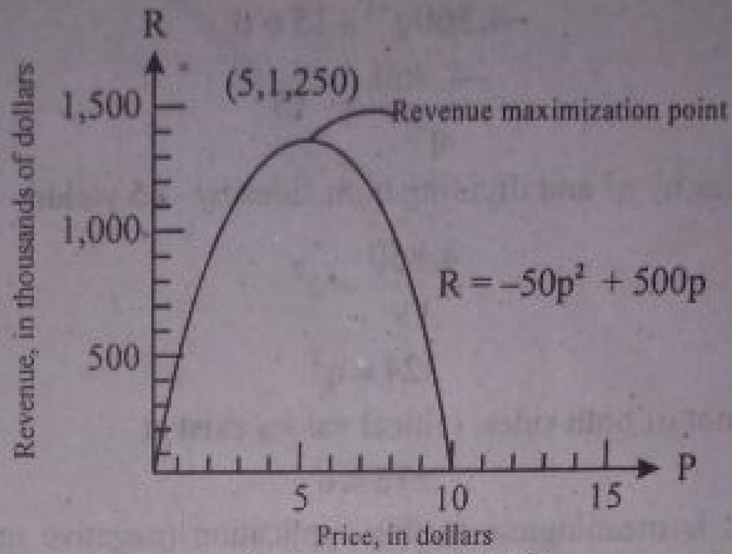


Quadratic revenue function.

Cost Function:

As mentioned earlier, costs represent cash outflows for an organization. Most organizations seek ways to minimize these outflows. This section presents applications which deal with the minimization of some measure of cost.

Example:

(Inventory Management) A common problem in organizations is determining how much of a needed item should be kept on hand. For retailers, the problem may relate to how many units of each product should be kept in stock. For producers, the problem may involve how much of each raw material should be kept available. This problem is identified with an area called inventory control, or inventory management. Concerning the question of how much "inventory" to keep on hand, there may be costs associated with having too little or too much inventory on hand.

A retailer of motorized bicycles has examined cost data and has determined a cost function which expresses the annual cost of purchasing, owning, and maintaining inventory as a function of the size (number of units) of each order it place for the bicycles. The cost function is

$$C = f(q) = \frac{4,860}{q} + 15q + 750,000$$

Where C equals annual inventory cost, stated in dollars, and q equals the number of cycles ordered each time the retailer replenishes the supply.

(a) Determine the order size which minimizes annual inventory cost.

(b) What is minimum annual inventory cost expected to equal?

Solution:

(a) The first derivative is

$$f'(q) = -4,860p^{-2} + 15$$

If f′ is set equal to 0,

$$-4,860q^{-2}+15=0$$

When
$$\frac{-4,860}{q^2}=-15$$

Multiplying both sides by q^2 and dividing both sides by −15 yields

$$\frac{4,860}{15}=q^2$$
$$324=q^2$$

Taking the square root of both sides, critical values exist at

$$\pm 18=q$$

The value q = −18 is meaningless in this application (negative order quantities are not possible). The nature of the only meaningful critical point (q = 18) is checked by finding f'':

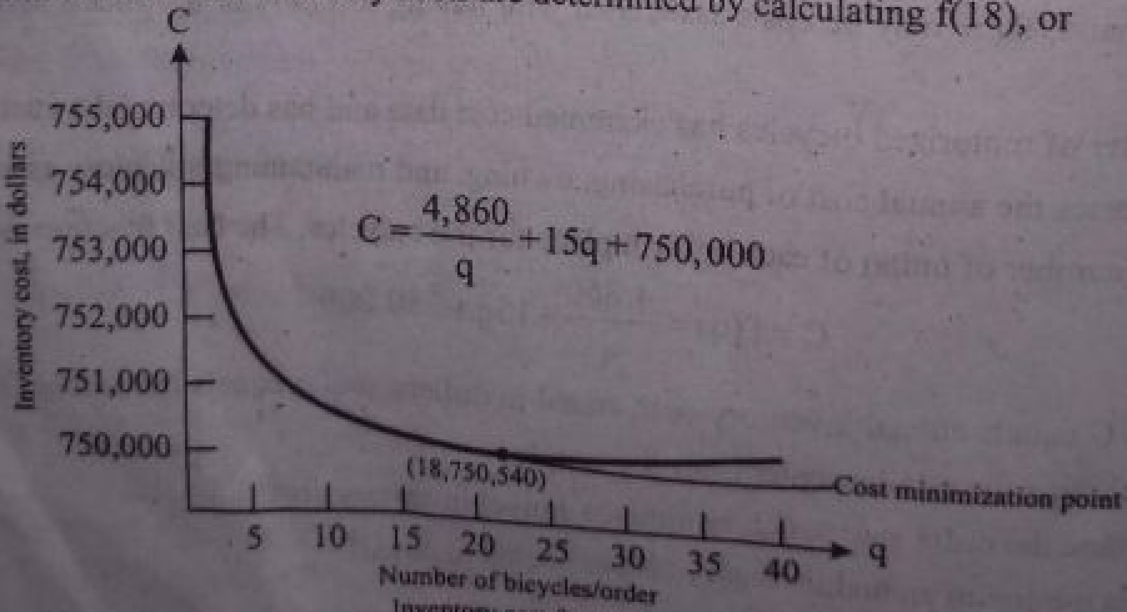
$$f''(q)=9,720q^{-3}$$
$$=\frac{9,720}{q^3}$$

At the critical value,

$$f''(18)=\frac{9,720}{(18)^3}$$
$$=1.667>0$$

Note that $f''(q)>0$ for q > 0. Therefore the graph of f is everywhere concave upward. Thus, the minimum value for f occurs when q = 18. Annual inventory costs will be minimized when 18 bicycles are ordered each time the retailer replenishes the supply.

(b) Minimum annual inventory costs are determined by calculating f(18), or



Inventory cost function

$$f(18)=\frac{4,860}{18}+15(18)+750,00=270+270+750,000=\$750,540$$

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